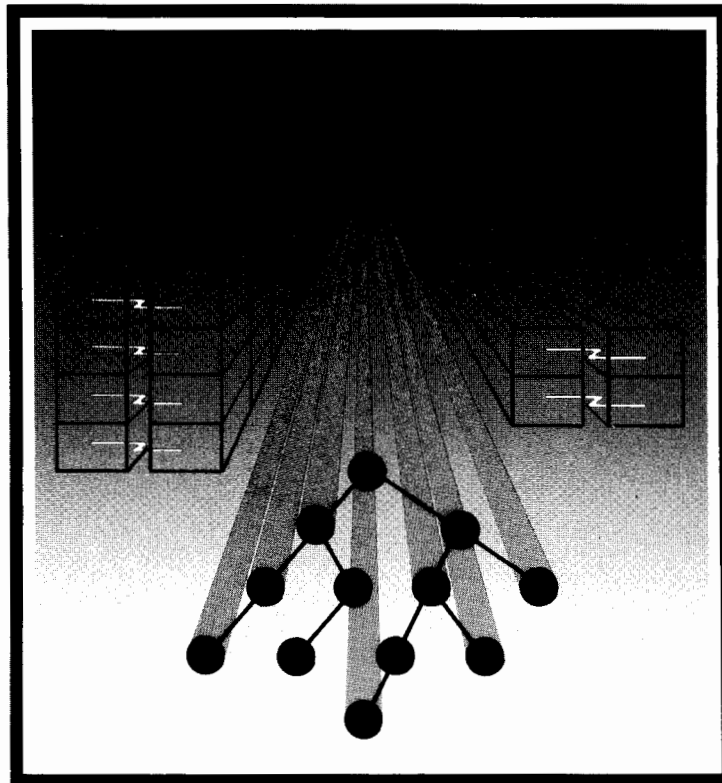


# Monte Carlo Simulation Utilities

*For the HP 9845*



# Monte Carlo Simulation Utilities



HP System 45 Desktop Computer

Hewlett-Packard Desktop Computer Division  
3404 East Harmony Road, Fort Collins, Colorado 80525

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# Forward

“Almost all good computer programs contain at least one random-number generator.”

Knuth, *The Art of Computer Programming*, Vol.2, p.157

“Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin.”

John Von Neumann (1951)



# Introduction



# Operating Specifications

## Introduction

### Description

The programs in this software package are meant primarily as a library of utility routines to be combined with the user's own programs. Hence, each routine is set up as an independent, modular unit with a standard set of input and output parameters. These subprograms contain no actual inputs or outputs, with the exception of error messages.

With each routine, the package provides a general-purpose front-end driver. In some cases, such as the Spectral and Run tests, the driver plus the routine make sense as a stand-alone unit. In other cases, such as the various random number deviates, the drivers are simply meant to introduce the user to the subprogram itself.

The software package **does not** establish the printers or the mass storage devices. It is the user's responsibility to select the printer and mass storage device **before** using any of these routines.

## SYSTEM CONFIGURATION:

### Necessary:

9845B Desktop Computer

Option 560 - Thermal line printer  
(or an external printer)

09845-15160 Monte Carlo Simulation Utilities Software Pack

09845-15161 Monte Carlo Simulation Utilities Manual

09845-15164 Goodness-of-Fit Cartridge

09845-15165 Random Number Cartridge

### Optional:

Option 204 - 187K bytes of Read/Write memory

Option 205 - 318K bytes of Read/Write memory

Option 206 - 448K bytes of Read/Write memory

Option 312 - I/O ROM for data entry from external devices

Option 311 - Graphics ROM

Option 600 - Secondary tape transport

Option 700 - Graphics Display Subsystem

9872A - External plotter

9885M - Flexible Disk Drive

(requires Option 313 - Mass Storage ROM)

7900 Series - Hard Disc

(requires Option 313 - Mass Storage ROM and 98041A  
Disc Interface)



# Operating Instructions



# Operating Instructions

## General Instructions



How Do I Load A Stand Alone Program?

1. Insert the Random Number cartridge into the tape transport.
2. None of the drivers ask for the desired printer or mass storage device. This must be set by the user from the keyboard.
3. Type: LOAD "File name",10  
Press: EXECUTE.
4. At this point, appropriate inputs are requested, computations are performed, and the results are printed or saved on a mass storage device.

How Do I Link One Of The Utility Subprograms Onto My Program?

Each program file has a driver and then one or more subprograms. If you want to incorporate just one of these subprograms into your routine, how do you do it?

The entire file needs to be loaded into memory first, and then the particular subprogram needs to be saved in a temporary file. Finally, after you have written your own code, you can link the temporary file containing the desired subprogram on after your code.

1. Insert the Random Number cartridge into the tape transport.
2. Type: LOAD "File name"  
Press: EXECUTE
3. After the program has been loaded,  
Type: EDITLINE  
Press: EXECUTE.

4. At this point, the screen looks as follows:

```
10 Beginning of driver program.  
20  
   .  
   .  
   .           Driver program  
   .  
END  
   .  
   .  
100 SUB Sub_to_be_linked  
   .  
   .  
   .  
SUBEND
```

5. If subprogram Sub\_to\_be\_linked is the one desired and it goes from line 100 to line 500, then  
Type: SAVE "TEMP",100,500  
PRESS: EXECUTE.
6. Type: SCRATCH (Don't type SCRATCH A. This will erase the binary.)  
Press: EXECUTE.
7. After you enter your program into memory, for this example assume that the last line of your code is line 2500. Then  
Type: LINK "TEMP",2510  
Press: EXECUTE.
8. The desired subprogram is then linked on behind your routine.

#### HOW DO I INCORPORATE THE BINARY ROUTINES INTO MY OWN PROGRAM?

Suppose you write your own routine and want to use the binary random number generator IRND. How is this done?

A binary program must first be loaded into memory before these statements can be used.

1. Type: LOAD BIN "RANDOM"  
Press: EXECUTE.
2. At this point, you may enter your program.

After the program has been entered into memory, how do you save it? There are two possibilities.

1. Type: STORE "File name"  
Press: EXECUTE.

The binary program is stored away with your program. Each time it is brought back into memory, the binary automatically comes with it.

or

2. Type: SAVE "File name"  
Press: EXECUTE.

The binary program will NOT be stored away. Each time the program is to be used, the binary program MUST be loaded first.

## Special Considerations

1. All the programs in this package have been set up using the random number generator IRND. This may be replaced by either RND or the super random generator contained in RSUPER.
2. You now have three different random number generators at your disposal.
  - IRND: a linear congruential generator. (See Appendix II for further details.)
  - RND: a randomly generated generator. (See Appendix I for further details.)
  - SUPER: a combination generator. (See "RSUPER" for further details.)

It is strongly suggested that any serious Monte Carlo simulation should be run with at least two of these different generators.

3. This package is meant to provide a set of subprogram utilities which you can combine to meet your particular needs. Each utility may be viewed as an independent modular unit. This allows you to combine these building blocks into your own program.
4. In order to get a feel for how each utility works and, in the case of the various generators, how much confidence you can place in them, driver routines have been provided. So, it is suggested that you first use these driver programs as is, and then later adapt them to your particular need.

5. In order to allow you the most flexibility, no references are made to printers or mass storage devices. Hence, to have a particular program run from a floppy disk having select code 8 and have all information printed on the CRT, you would type in the following before running your program:
  1. a. Type: MASS STORAGE IS ":F8"  
b. Press: EXECUTE
  2. a. Type: PRINTER IS 16  
b. Press: EXECUTE
6. Each of the driver programs for the random deviates allows you to:
  1. generate a set of random numbers to be printed or saved on a mass storage device.  
  
or
  2. get a feeling for the quality of the generator by running through some randomly generated tests.
7. There may be occasions where you will not have enough memory to store all the random numbers you would like to have. A number of possible tricks are available to you:
  - a. Presently all deviates are set up in full precision arrays. Can you store the deviates in an integer or short precision array? Where a full precision array requires 8 bytes per number, a short precision array needs only 4, and an integer only 2. Care must be taken here to dimension your array using an INTEGER or SHORT statement rather than a DIM. Also, the parameters in the SUB statement must be changed to INTEGER or SHORT.
  - b. Can you generate and use the random numbers in a partitioned fashion? For example, generate 1000 deviates, use them; generate 1000 more, use them; etc.
  - c. If b is not possible, can you make use of your mass storage device to recall the deviates as you need them? For example:
    - i. generate 1000 deviates; store them; generate 1000 more; store them; etc.
    - ii. bring first 1000 deviates into memory; use them; bring next 1000 in; use them; etc.

8. Entering a value of 0, or 16 for the printer's select code automatically causes the program to skip over the question requesting the printer's bus address.

## General References

1. Camp, Warren V. and Lewis, T.G., "Implementing a Pseudo-Random Number Generator on a Minicomputer", IEEE Transactions on Software Engineering, May, 1977.
2. Knuth, Donald E., The Art of Computer Programming, Volume 2: Seminumerical Algorithms, Addison-Wesley, Reading, Mass., 1969.
3. Learmonth, J. and Lewis, P.A.W., "Naval Postgraduate School Random Number Generator Package LLRANDOM", Naval Postgraduate School, Monterey, Calif., 1973.
4. Learmonth, J. and Lewis, P.A.W., "Statistical Tests of Some Widely Used and Recently Proposed Uniform Random Number Generators", Naval Postgraduate School, Monterey, Calif., 1973.
5. MacLauren, M.D. and Marsaglia, G., "Uniform Random Number Generators", JACM 12, Jan. 1965, p. 83-89.
6. Marsaglia, G. and Bray, T.A., "One-line Random Number Generators and Their Use in Combinations", CACM, Vol. II, 1968, p. 757-759.
7. Musyck, E., "Search For a Perfect Generator of Random Numbers", Studiecen-  
trum Voor Kernenergie, E. Plaskyiaan 144, Brussels 4, Belgium, January, 1977.
8. Reddy, Y.V., "PL/I Process Generators", SIMULETTER, Vol. III, Oct. 1976, p. 25-29.
9. Wheeler, Robert E., "Random Variable Generators", SIMULETTER, Vol. III, Oct. 1976, p. 16-23.





# Interface Specifications for Binary Routines





# Interface Specifications for Binary Routines

All binary commands can be either implemented under program control or executed directly from the keyboard.

The first set of statements is meant for the general user:

## IRND X

- X a full precision number.
- Input is ignored.
- On output, X contains a uniformly generated random number between 0 and 1.
- See Appendix II for details on how IRND was developed.

## SEED Seed

- Seed a full precision number.
- Resets the initial seed for the generator to Seed. Seed is transformed into a number in the range between 0 and 1 before use.
- This allows you to duplicate your experiment by using the same initial seed repeatedly.

The second set of statements is meant for a more sophisticated user.

The uniform random number generator uses the following theoretical model:

Let  $S(I)$  be the  $I$ th randomly generated number and  
 $S(I+1)$  be the  $(I+1)$ th.

Then  $S(I+1) = (A * S(I) + C) \text{ MOD } M$

where  $A$  and  $C$  are appropriately chosen constants and  $M$  is the word size of the computer, in this case  $10^{12}$ .

In the binary, the values of  $A$  and  $C$  are based on some standard theoretical considerations. (See Appendix II.) The following two statements allow you to change these values. This means a new uniform random number generator is produced.

**ACOEFX**

- X a full precision number.
- Resets the value of coefficient A in the above formula to X.
- The value of X is left unchanged.
- X must be chosen with great care. It must be a 12 digit whole number based on some theoretical considerations (See Appendix II). Unpredictable results will occur if this caution is not followed.

**CCOEFX**

- X a full precision number.
- Resets the value of constant C in the above formula to X.
- The value of X is left unchanged.

# Random Number Generators in BASIC





## Random Number Generators in BASIC

Subprograms with optional drivers are provided to generate random deviates on some standard statistical distributions.

The subprograms have been set up as independent modules. Hence, it is quite simple to use these routines in your own programs. Choose values for the required input parameters, call the subprogram and the resulting outputs are returned to you.

Optional drivers have also been set up for your use. In general, the drivers: i) allow you to directly generate a set of deviates to be printed or saved on a mass storage device; and ii) provide the ability to check out the particular generator through the use of some standard tests in order to get a feel for the quality of the deviates produced.





**(RBETA)**  
**Random Numbers Generated**  
**From a Beta Distribution**



# (RBETA)

## Random Numbers Generated from a Beta Distribution

### Description

Given a Beta distribution with  $V1$  and  $V2$  degrees of freedom respectively, this subprogram generates a set of random deviates.

### File Name

“RBETA”

### Calling Syntax

CALL Random\_beta (N,V1,V2,X(\*) )

### Input Parameters

N                                    number of deviates desired.  
V1, V2                                degrees of freedom on the Beta distribution.

### Output Parameters

X(\*)                                    array of dimension (1:N) containing the N deviates.

### Algorithm

This routine generates deviates for the beta distribution with  $v1$ ,  $v2$  degrees of freedom. The method used is valid for both integer and non-integer  $v1$  and  $v2$ :

1. Generate uniform random deviates  $u1$  and  $u2$ .
2. Set  $y1 = u1^{2/v1}$ ;  $y2 = u2^{2/v2}$ , repeating this process until finding  $y1 + y2 \leq 1$ .
3. Then  $x = y1 / (y1 + y2)$ .

### Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 (Seminumerical Algorithms), Reading, Mass.: Addison-Wesley, 1969, p. 115.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.
  - a. Type: LOAD "RBETA",10
  - b. Press: EXECUTE
3. The title "RANDOM BETA DEVIATES" will be printed along with the possible ways of using the program.
4. When "ENTER YOUR CHOICE. [EITHER 1 or 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.  
or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and the Beta parameters.
  - a. If the default values are satisfactory, simply press: CONT  
or
  - a. Change the values as desired.
  - b. Press: CONT
6. A set of deviates is produced for each pair of parameters. The starting seed, mean, variance and standard deviation are printed as well as the expected mean and variance.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press CONT
  - c. Go to 8.  
or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.

8. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, V1, V2, E.G., 10,PI,3,5” is displayed:
  - a. Enter values for the size of the set, the starting seed, and the Beta parameters.
  - b. Press: CONT
9. When “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.  
or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When “ENTER THE SELECT CODE OF THE PRINTER” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is then printed out on the appropriate device.
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.  
or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. The program is terminated at this point.

13. When "ENTER FILE NAME. FOR EXAMPLE: File:T14 OR DATA:F8" is displayed:
- Enter the desired file name.
  - Press: CONT
  - The data set is saved on your mass storage device as a vector and the program is terminated.

#### RANDOM BETA DEVIATES

This program generates sets of Beta (V1,V2) random deviates. You may use the program in two ways:

- Directly generate a set of deviates to be printed or saved on a mass storage device.
- Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of Beta (V1,V2) random deviates. It also compares the resulting data with the theoretical expectations. In particular, given degrees of freedom V1 and V2, the mean and variance of each set of Beta random numbers is calculated.

The expected value of the mean is:  $V1/(V1+V2)$ .

The expected value of the variance is:  $V1*V2/[(V1+V2+1)*(V1+V2)^2]$ .

The program is set up using the following default values:

- 100 deviates will be generated for each set of Beta parameters.
- Values for V1 and V2, the Beta parameters, are set at .50, 2.00, and 5.00.
- Results printed are:
  - mean, expected mean, and standard deviation for each set.
  - variance and expected variance for each set.

Default values may be changed from the keyboard as they come up.

```
# OF RANDOM DEVIATES: 1000
DEGREES OF FREEDOM (V1): 0.50, 2.00, 5.00
DEGREES OF FREEDOM (V2): 0.50, 2.00, 5.00
```

Number: 1000

|           |               |                    |      |                |      |
|-----------|---------------|--------------------|------|----------------|------|
| Seed:     | .292190093254 | V1:                | .50  | V2:            | .50  |
| Mean:     | .523          | Expected Mean:     | .500 | Standard Dev.: | .406 |
| Variance: | .165          | Expected Variance: | .125 |                |      |
| Seed:     | .731868426712 | V1:                | .50  | V2:            | 2.00 |
| Mean:     | .210          | Expected Mean:     | .200 | Standard Dev.: | .270 |
| Variance: | .073          | Expected Variance: | .046 |                |      |

|           |               |                    |      |                |      |
|-----------|---------------|--------------------|------|----------------|------|
| Seed:     | .501346863484 | V1:                | .50  | V2:            | 5.00 |
| Mean:     | .096          | Expected Mean:     | .091 | Standard Dev.: | .151 |
| Variance: | .023          | Expected Variance: | .013 |                |      |
| Seed:     | .632712189859 | V1:                | 2.00 | V2:            | .50  |
| Mean:     | .797          | Expected Mean:     | .800 | Standard Dev.: | .255 |
| Variance: | .071          | Expected Variance: | .046 |                |      |
| Seed:     | .110950998353 | V1:                | 2.00 | V2:            | 2.00 |
| Mean:     | .502          | Expected Mean:     | .500 | Standard Dev.: | .290 |
| Variance: | .084          | Expected Variance: | .050 |                |      |
| Seed:     | .309789614978 | V1:                | 2.00 | V2:            | 5.00 |
| Mean:     | .293          | Expected Mean:     | .286 | Standard Dev.: | .214 |
| Variance: | .046          | Expected Variance: | .026 |                |      |
| Seed:     | .533066196498 | V1:                | 5.00 | V2:            | .50  |
| Mean:     | .908          | Expected Mean:     | .909 | Standard Dev.: | .155 |
| Variance: | .024          | Expected Variance: | .013 |                |      |
| Seed:     | .308671253619 | V1:                | 5.00 | V2:            | 2.00 |
| Mean:     | .715          | Expected Mean:     | .714 | Standard Dev.: | .211 |
| Variance: | .044          | Expected Variance: | .026 |                |      |
| Seed:     | .592524293566 | V1:                | 5.00 | V2:            | 5.00 |
| Mean:     | .493          | Expected Mean:     | .500 | Standard Dev.: | .206 |
| Variance: | .043          | Expected Variance: | .023 |                |      |

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the Beta parameters, V1 and V2. For want of a better seed, PI is an excellent choice!

#### BETA DEVIATES:

|       |               |         |         |         |         |
|-------|---------------|---------|---------|---------|---------|
| Seed: | .017453292520 | V1:     | 3.0000  | V2:     | 5.0000  |
| I     | X(I)          | X(I+1)  | X(I+2)  | X(I+3)  | X(I+4)  |
| 1     | .542282       | .138571 | .441458 | .540076 | .367385 |
| 6     | .547167       | .314301 | .318138 | .238096 | .634416 |
| 11    | .143855       | .295088 | .286629 | .043278 | .282911 |
| 16    | .145880       | .879115 | .461297 | .201152 | .444524 |
| 21    | .203289       | .265061 | .420664 | .072694 | .239809 |





**(RBINOM)**  
**Random Integers Generated**  
**From a Binomial Distribution (T,P)**



# (RBINOM) Random Integers Generated From a Binomial Distribution (T,P)

## Description

Given that some event occurs with probability  $P$  and that we carry out  $T$  independent trials, this subprogram generates a set of integers with the binomial distribution  $(T,P)$ .

## File Name

“RBINOM”

## Calling Syntax

CALL Random\_binomial (N,P,T,X(\*) )

## Input Parameters

|   |                                     |
|---|-------------------------------------|
| N | number of deviates desired.         |
| P | probability of the event occurring. |
| T | number of independent trials.       |

## Output Parameters

|      |  |
|------|--|
| X(*) | array of dimension (1:N) containing integers randomly generated for the number of occurrences. |
|------|--|

## Algorithm

Given T and P:

1. Set Sum=0.
2. For I=1 to T.
3. Generate a uniform random deviate U.
4. If  $U \leq P$  then Sum=Sum+1.
5. Next I.
6. The binomial deviate is equal to Sum.

## Reference

1. Reddy, Y.V., “PL/I Process Generators”, SIMULETTER, Vol. III, Oct. 1976, p. 25-26.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.
  - a. Type: LOAD "RBINOM",10
  - b. Press: EXECUTE
3. The title "RANDOM BINOMIAL DEVIATES" is printed along with the options available to you in the program.
4. When "ENTER YOUR CHOICE. [EITHER 1 or 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and the probabilities.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. A set of deviates is produced for each combination of given probability  $P$  and number of trials  $T$ . After each data set is generated, the starting seed, the mean, variance and the standard deviation are printed as well as the theoretical expected mean and variance.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONTor
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.

8. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, TRIALS, PROB. E.G.100, PI,10,.8” is displayed:
  - a. Enter values for the size of the set, the starting seed, the number of trials and the probability.
  - b. Press: CONT
9. After the data set has been created, the prompt: “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.  
or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When “ENTER THE SELECT CODE OF THE PRINTER” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER” is displayed
  - a. Enter the bus address of the printer.
  - b. Press: CONT
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.  
or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
13. When “ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8” is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM BINOMIAL DEVIATES

This program generates sets of Binomial (T,P) random deviates. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of Binomial (T,P) random deviates. It also compares the resulting data with the theoretical expectations. In particular, given the number of independent trials T, and the probability P, the mean and variance of each set of Binomial random numbers is calculated.

The expected value of the mean is:  $T*P$ .

The expected value of the variance is:  $T*P*(1-P)$ .

The program is set up using the following default values:

1. 100 deviates will be generated for each set of Binomial parameters.
2. Values for T, the number of trials, are set at 5, 16 and 30.
3. Values for P, the probability, are set at 0.1, 0.5, and 0.9.
4. Results printed are:
  - mean, expected mean, and standard deviation for each set.
  - variance and expected variance for each set.

Default values may be changed from the keyboard as they come up.

```
# OF RANDOM DEVIATES: 1000
# OF INDEPENDENT TRIALS: 5, 16 ,30
PROBABILITIES: 0.1, 0.5, 0.9
```

Number: 1000

|           |               |                    |       |                |       |
|-----------|---------------|--------------------|-------|----------------|-------|
| Seed:     | .672190093205 | Trials:            | 5.00  | Prob:          | .10   |
| Mean:     | .511          | Expected Mean:     | .500  | Standard Dev.: | .677  |
| Variance: | .458          | Expected Variance: | .458  |                |       |
| Seed:     | .711868385227 | Trials:            | 5.00  | Prob:          | .50   |
| Mean:     | 2.477         | Expected Mean:     | 2.500 | Standard Dev.: | 1.101 |
| Variance: | 1.213         | Expected Variance: | 1.250 |                |       |
| Seed:     | .681311976483 | Trials:            | 5.00  | Prob:          | .90   |
| Mean:     | 4.510         | Expected Mean:     | 4.500 | Standard Dev.: | .667  |
| Variance: | .444          | Expected Variance: | .450  |                |       |
| Seed:     | .383372222047 | Trials:            | 16.00 | Prob:          | .10   |
| Mean:     | 1.576         | Expected Mean:     | 1.600 | Standard Dev.: | 1.225 |
| Variance: | 1.582         | Expected Variance: | 1.440 |                |       |

```

Seed:      .016038742609      Trails: 16.00      Prob:      .50
Mean:      7.994      Expected Mean: 8.000      Standard Dev.: 2.013
Variance:  4.052      Expected Variance: 4.000

```

```

Seed:      .468582534523      Trails: 16.00      Prob:      .90
Mean:      14.390      Expected Mean: 14.400      Standard Dev.: 1.226
Variance:  1.593      Expected Variance: 1.440

```

```

Seed:      .897911534893      Trails: 30.00      Prob:      .10
Mean:      2.949      Expected Mean: 3.000      Standard Dev.: 1.597
Variance:  2.549      Expected Variance: 2.700

```

```

Seed:      .143600845145      Trails: 30.00      Prob:      .50
Mean:      15.087      Expected Mean: 15.000      Standard Dev.: 2.792
Variance:  7.797      Expected Variance: 7.500

```

```

Seed:      .768310767024      Trails: 30.00      Prob:      .90
Mean:      26.943      Expected Mean: 27.000      Standard Dev.: 1.641
Variance:  2.694      Expected Variance: 2.700

```

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, the number of independent trials and the probability. For want of a better seed, PI is an excellent choice!

#### BINOMIAL DEVIATES:

```
Seed: .017453292520      # OF Trials: 10      Prob: .9800
```

| I  | X(I)     | X(I+1)   | X(I+2)    | X(I+3)    | X(I+4)    |
|----|----------|----------|-----------|-----------|-----------|
| 1  | 9.000000 | 8.000000 | 9.000000  | 8.000000  | 6.000000  |
| 6  | 8.000000 | 8.000000 | 8.000000  | 7.000000  | 6.000000  |
| 11 | 7.000000 | 7.000000 | 10.000000 | 10.000000 | 8.000000  |
| 16 | 8.000000 | 8.000000 | 10.000000 | 7.000000  | 9.000000  |
| 21 | 9.000000 | 7.000000 | 8.000000  | 9.000000  | 10.000000 |





**(RCHISQ)  
Random Numbers From a  
Chi-square Distribution**



# (RCHISQ) Random Numbers From a Chi-square Distribution

## Description

Given the number of degrees of freedom and the number of deviates desired, this subprogram generates a set of random numbers with the Chi-square distribution.

## File Name

“RCHISQ”

## Calling Syntax

CALL Random\_chi\_sq(N,V,X(\*) )

## Input Parameters

N                                      number of deviates desired.  
V                                      degrees of freedom.



## Output Parameters

X(\*)                                      array of dimension (1:N) containing the N deviates.

## Algorithm

This utility generates random deviates for the Chi-square distribution with  $v$  degrees of freedom.

For each deviate, if  $v=2*k$ , where  $k$  is an integer

set  $x=2*(y_1+y_2+\dots+y_k)$  where the  $y$ 's are independent random variables with the exponential distribution, each with mean=1.

If  $v=2*k+1$ ,

set  $x=2*(y_1+y_2+\dots+y_k)+z^2$  where the  $y$ 's are as before, and  $z$  is a random variable independent of the  $y$ 's, with the normal distribution (mean=0, standard deviation=1).

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass: Addison-Wesley, 1969, p. 115.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.
  - a. Type: LOAD "RCHISQ",10
  - b. Press: EXECUTE
3. The title "RANDOM CHI-SQUARE DEVIATES" is printed along with the options available to you in the program.
4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.

or

  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and the Chi-square parameter.
  - a. If the default values are satisfactory, simply press: CONT

or

  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates are produced for each sized set. The starting seed as well as the mean, variance and standard deviation of each data set are printed.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - c. Go to 8.

or

  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.

8. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, CHI-SQUARE PARAMETER. E.G., 10,PI,3” is displayed:
  - a. Enter values for the size of the set, the starting seed, and the Chi-square parameter.
  - b. Press: CONT
  
9. When “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.  
or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
  
10. When “ENTER THE SELECT CODE OF THE PRINTER” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
  
11. When “ENTER THE BUS ADDRESS OF THE PRINTER” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is then printed out on the appropriate device.
  
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.  
or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. The program is terminated at this point.

13. When "ENTER FILE NAME. FOR EXAMPLE: File:T14 OR DATA:F8" is displayed:
- Enter the desired file name.
  - Press: CONT
  - The data set is saved on your mass storage device as a vector and the program ends.

RANDOM CHI-SQUARE DEVIATES

This program generates sets of Chi-square (V) random deviates. You may use the program in two ways:

- Directly generate a set of deviates to be printed or saved on a mass storage device.
- Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

The program is set up using the following default values:

- Three sets of size 50 are produced in turn for each Chi-square parameter.
- Values for V, the Chi-square parameter, are set in turn at 1, 2, 4
- The mean, variance and standard deviation are then printed for each set.

Default values may be changed from the keyboard as they come up.

# OF RANDOM DEVIATES: 1000  
DEGREES OF FREEDOM: 3, 5, 8

CHI-SQUARE PARAMETER: 3

|       |               |           |       |                      |
|-------|---------------|-----------|-------|----------------------|
| Seed: | .246190093248 |           |       |                      |
| Mean: | 2.998         | Variance: | 5.467 | Standard Dev.: 2.338 |
| Seed: | .045868421511 |           |       |                      |
| Mean: | 2.898         | Variance: | 6.585 | Standard Dev.: 2.566 |
| Seed: | .575342490784 |           |       |                      |
| Mean: | 2.952         | Variance: | 5.779 | Standard Dev.: 2.404 |

CHI-SQUARE PARAMETER: 5

|       |               |           |        |                      |
|-------|---------------|-----------|--------|----------------------|
| Seed: | .063004740332 |           |        |                      |
| Mean: | 5.134         | Variance: | 10.858 | Standard Dev.: 3.295 |
| Seed: | .812223346477 |           |        |                      |
| Mean: | 5.051         | Variance: | 9.682  | Standard Dev.: 3.112 |

Seed: .079934386208  
 Mean: 4.892 Variance: 9.276 Standard Dev.: 3.046

CHI-SQUARE PARAMETER: 8

Seed: .140718800597  
 Mean: 8.189 Variance: 16.700 Standard Dev.: 4.087

Seed: .344511301952  
 Mean: 7.873 Variance: 15.289 Standard Dev.: 3.910

Seed: .734004941727  
 Mean: 8.062 Variance: 16.832 Standard Dev.: 4.103

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the chi-square parameter. For want of a better seed, PI is an excellent choice!

CHI-SQUARE DEVIATES:

Seed: .017453292528 V: 3.0000

| I  | X(I)     | X(I+1)   | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|----------|----------|----------|----------|
| 1  | 6.937335 | 1.327848 | 3.118890 | 5.009005 | 2.398437 |
| 6  | 2.617949 | 1.911048 | 1.933804 | 1.571041 | 8.919987 |
| 11 | .483658  | 1.500412 | .783231  | .238996  | 4.006474 |
| 16 | .804631  | 7.652400 | 4.543294 | 1.966788 | 2.652313 |
| 21 | 2.020308 | .794752  | 2.996792 | .468676  | 2.835952 |





**(REXPON)**  
**Random Numbers From an**  
**Exponential Distribution**



# (REXPON) Random Numbers From an Exponential Distribution

## Description

Given a mean, which you supply, this subprogram generates a set of exponential deviates.

## File Name

“REXPON”

## Calling Syntax

CALL Random\_expon (N,Mu,X(\*) )

## Input Parameters

|    |                             |
|----|-----------------------------|
| N  | number of deviates desired. |
| Mu | mean of the distribution.   |

## Output Parameters

|      |   |
|------|---|
| X(*) | array of dimension (1:N) containing the N deviates. |
|------|---|

## Algorithm

This routine uses the random minimization method (due to George Marsaglia) to compute an exponentially distributed variable without using the logarithm subroutine. Although this routine takes slightly more space, it is much faster than the traditional algorithm.

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 114.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2. a. Type: LOAD “REXPON”,10  
b. Press: EXECUTE
3. The title “RANDOM EXPONENTIAL DEVIATES” is printed along with the options available to you in the program.

4. When “ENTER YOUR CHOICE. [EITHER 1 OR 2]” is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and the means.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates are produced for each sized set. The starting seed as well as the mean will be printed. The corresponding Kolmogorov-Smirnov statistics are calculated for each set.
7. When “WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, MEAN. E.G.,100,PI,0.5” is displayed:
  - a. Enter values for the size of the set, the starting seed, and the mean.
  - b. Press: CONT

9. When “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When “ENTER THE SELECT CODE OF THE PRINTER” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is then printed out on the appropriate printer.
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. The program is terminated at this point.
13. When “ENTER FILE NAME. FOR EXAMPLE: File:T14 OR DATA:F8” is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program is terminated.

RANDOM EXPONENTIAL DEVIATES

This program generates sets of exponential random deviates. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of exponential random deviates. It also compares the resulting data with the theoretical expectations using the Kolmogorov-Smirnov test.

The program is set up using the following default values:

1. 0.5, 1.0, 2.5, and 5.0 are selected in turn as the mean.
2. For each parameter, 3 sets of 100 deviates are calculated.
3. The resulting positive and negative K-S statistics are then printed.

Default values may be changed from the keyboard as they come up.

# OF DEVIATES IN EACH SET: 1000  
MEANS: 0.5, 1.0, 2.5, 5.0

|                     |            |           |  |
|---------------------|------------|-----------|--|
| Mean: .5            |            |           |  |
| Seed: .723198093254 | Knp: .840  | Knn: .343 |  |
| Seed: .202868427612 | Knp: .602  | Knn: .602 |  |
| Seed: .612347621784 | Knp: .973  | Knn: .349 |  |
| Mean: 1.0           |            |           |  |
| Seed: .984349919381 | Knp: .917  | Knn: .470 |  |
| Seed: .838282198047 | Knp: .625  | Knn: .872 |  |
| Seed: .995328558683 | Knp: .342  | Knn: .882 |  |
| Mean: 2.5           |            |           |  |
| Seed: .071317852203 | Knp: .589  | Knn: .089 |  |
| Seed: .978313702577 | Knp: .580  | Knn: .501 |  |
| Seed: .761823866397 | Knp: .799  | Knn: .568 |  |
| Mean: 5.0           |            |           |  |
| Seed: .693871639521 | Knp: .407  | Knn: .739 |  |
| Seed: .546048836977 | Knp: 1.075 | Knn: .252 |  |
| Seed: .227071896716 | Knp: .628  | Knn: .500 |  |

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the mean. For want of a better seed, PI is an excellent choice.

EXPONENTIAL DEVIATES:

Seed: .017453292520      Mean: .5000

| I  | X(I)     | X(I+1)  | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|---------|----------|----------|----------|
| 1  | 1.046715 | .898935 | .014525  | .240158  | 1.377689 |
| 6  | .074588  | .371152 | .096440  | 1.170635 | .715012  |
| 11 | .237314  | .640684 | 1.233808 | .474077  | .554652  |
| 16 | .477486  | .426282 | .173493  | 2.229481 | .120429  |
| 21 | .168504  | .104212 | .010864  | .990982  | .036880  |





**(RF)**  
**Random Numbers Generated From an  
F-Distribution**



# (RF) Random Numbers Generated From an F-Distribution

## Description

Given an F-distribution (variance-ratio distribution) with  $V1$  and  $V2$  degrees of freedom, respectively, this subprogram generates a set of corresponding random deviates.

## File Name

“RF”

## Calling Syntax

CALL Random\_f (N,V1,V2,X(\*) )



## Input Parameters

$N$                                       number of deviates desired.  
 $V1, V2$                                     degrees of freedom on the F-distribution.

## Output Parameters

$X(*)$                                       array of dimension (1:N) containing the  $N$  random numbers.

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 116.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.    a. Type: LOAD “RF”,10  
       b. Press: EXECUTE
3. The title “RANDOM F DEVIATES” is printed along with the options available to you in the program.

4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and the F parameters.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. A set of deviates is produced for each pair of parameters. The starting seed, mean, and variance are printed as well as the expected mean and variance.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED, V1, V2.E.G., 10, P1,5,7" is displayed:
  - a. Enter values for the size of the set, the starting seed, and the F parameters.
  - b. Press: CONT

9. When “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When “ENTER THE SELECT CODE OF THE PRINTER” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is then printed out on the appropriate device.
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. The program is terminated at this point.
13. When “ENTER FILE NAME. FOR EXAMPLE: File:T14 OR DATA:F8” is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM F DEVIATES

This program generates sets of  $F(V_1, V_2)$  random deviates. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of  $F(V_1, V_2)$  random deviates. It also compares the resulting data with the theoretical expectations. In particular, given degrees of freedom  $V_1$  and  $V_2$ , the mean and variance of each set of  $F$  random numbers is calculated.

The expected value of the mean is:  
 $V_2 / (V_2 - 2)$  for  $V_2 > 2$ .

The expected value of the variance is:  
 $2 * V_2^2 * (V_1 + V_2 - 2) / (V_1 * (V_2 - 4) * (V_2 - 2)^2)$  for  $V_2 > 4$ .

The program is set up using the following default values:

1. 100 deviates will be generated for each set of  $F$  parameters.
2. Values for  $V_1$  and  $V_2$ , the  $F$  parameters, are set at 5, 10, 50.
3. Results printed are:
  - mean, expected mean, and standard deviation for each set.
  - variance and expected variance for each set.

Default values may be changed from the keyboard as they come up.

# OF RANDOM DEVIATES: 1000  
 DEGREES OF FREEDOM (V1): 5, 10, 50  
 DEGREES OF FREEDOM (V2): 5, 10, 50

Number: 1000

|           |               |                    |       |                |       |
|-----------|---------------|--------------------|-------|----------------|-------|
| Seed:     | .621900932048 | V1:                | 5.00  | V2:            | 5.00  |
| Mean:     | 1.762         | Expected Mean:     | 1.667 | Standard Dev.: | 2.463 |
| Variance: | 6.067         | Expected Variance: | 9.989 |                |       |
| Seed:     | .018683852607 | V1:                | 5.00  | V2:            | 10.00 |
| Mean:     | 1.337         | Expected Mean:     | 1.250 | Standard Dev.: | 1.253 |
| Variance: | 1.570         | Expected Variance: | 1.354 |                |       |
| Seed:     | .713120042531 | V1:                | 5.00  | V2:            | 50.00 |
| Mean:     | 1.073         | Expected Mean:     | 1.042 | Standard Dev.: | .754  |
| Variance: | .568          | Expected Variance: | .500  |                |       |

```

Seed:      .733955768681          V1:  10.00          V2:  5.00
Mean:     1.565          Expected Mean:  1.667          Standard Dev.:  1.841
Variance: 3.399          Expected Variance:  7.222

Seed:     .256801459317          V1:  10.00          V2: 10.00
Mean:     1.269          Expected Mean:  1.250          Standard Dev.:   .955
Variance: .914          Expected Variance:   .938

Seed:     .970027285542          V1:  10.00          V2: 50.00

Mean:     1.037          Expected Mean:  1.042          Standard Dev.:   .547
Variance: .299          Expected Variance:   .274

Seed:     .792947140391          V1:  50.00          V2:  5.00
Mean:     1.755          Expected Mean:  1.667          Standard Dev.:  2.788
Variance: 7.775          Expected Variance:  5.889

Seed:     .868545067767          V1:  50.00          V2: 10.00
Mean:     1.258          Expected Mean:  1.250          Standard Dev.:   .762
Variance: .581          Expected Variance:   .604

Seed:     .446401990886          V1:  50.00          V2: 50.00
Mean:     1.028          Expected Mean:  1.042          Standard Dev.:   .302
Variance: .091          Expected Variance:   .092

```

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the two F parameters. For want of a better seed, PI is an excellent choice!

#### F DEVIATES:

```
Seed:  .017453292520          V1:  5.0000          V2:  7.0000
```

| I  | X(I)     | X(I+1)   | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|----------|----------|----------|----------|
| 1  | 1.926214 | 1.334536 | 1.448056 | .976033  | 1.028440 |
| 6  | .414127  | .206951  | .976291  | 2.265742 | 1.718479 |
| 11 | .094033  | .562395  | 1.693379 | .500013  | 1.372930 |
| 16 | 1.372621 | 7.519611 | .966455  | .352944  | 1.147062 |
| 21 | .798963  | 1.640397 | .619726  | .496223  | .839288  |





**(RGAMM1)**  
**Random Integers Generated**  
**From a Gamma (Alpha) Distribution**



# (RGAMM1) Random Integers Generated From a Gamma (Alpha) Distribution

## Description

This subprogram generates a set of Gamma (Alpha) deviates.

## File Name

“RGAMM1”

## Calling Syntax

CALL Random\_gamma1 (N,Alpha,X(\*) )

## Input Parameters

|       |                                   |
|-------|-----------------------------------|
| N     | number of random numbers desired. |
| Alpha | Gamma parameter.                  |

## Output Parameters

|      |   |
|------|---|
| X(*) | array of dimension (1:N) containing numbers randomly generated with the given Gamma distribution. |
|------|---|

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.
  - a. Type: LOAD “RGAMM1”,10
  - b. Press: EXECUTE
3. The title “RANDOM GAMMA (Alpha) DEVIATES” is printed along with options available to you in the program.
4. When “ENTER YOUR CHOICE. [EITHER 1 OR 2]” is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.

or

  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.

5. Default values are supplied for the size of the set of deviates, and the probabilities.
  - a. If the default values are satisfactory, simply press: CONT
  - or
  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates with different starting seeds are produced for each parameter. After each data set is generated, the starting seed, the parameter, and the mean and variance of each set is printed.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED, GAMMA PARAMETER. E.G., 100,PI,3.5" is displayed:
  - a. Enter values for the size of the set, the starting seed, and the Gamma parameter.
  - b. Press: CONT
9. After the data set has been created, the prompt: "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.
  - or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT

Alpha: 10.25

|       |               |           |        |                      |
|-------|---------------|-----------|--------|----------------------|
| Seed: | .697946620926 |           |        |                      |
| Mean: | 10.224        | Variance: | 10.071 | Standard Dev.: 3.173 |
| Seed: | .973108200152 |           |        |                      |
| Mean: | 9.934         | Variance: | 12.735 | Standard Dev.: 3.569 |
| Seed: | .383996327637 |           |        |                      |
| Mean: | 10.559        | Variance: | 11.641 | Standard Dev.: 3.412 |

At this point, you may select a particular data set which you can have printed out or stored on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the Gamma parameter. For want of a better seed, PI is an excellent choice!

GAMMA DEVIATES:

Seed: .017453292520                      Alpha: 3.5000

| I  | X(I)     | X(I+1)   | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|----------|----------|----------|----------|
| 1  | 4.229690 | 1.309675 | 4.406097 | 3.134771 | 5.225892 |
| 6  | 4.719702 | .873601  | 3.696969 | 2.242268 | 4.142567 |
| 11 | 4.628855 | 4.743018 | 5.467311 | 3.777862 | 3.119722 |
| 16 | 2.274009 | 2.568017 | 6.147779 | .325210  | 4.059675 |
| 21 | 2.393629 | 2.748339 | 1.689142 | 6.686583 | 4.246621 |



**(RGAMM2)**  
**Random Numbers Generated From**  
**a Gamma (A,B) Distribution**





## (RGAMM2)

# Random Numbers Generated From a Gamma (A,B) Distribution

### Description

This subprogram generates a set of Gamma (A,B) random deviates.

### File Name

“RGAMM2”

### Calling Syntax

CALL Random\_gamma2 (N,A,B,X(\*) )

### Input Parameters

N                                      number of random deviates desired.  
A,B                                      Gamma parameters.

### Output Parameters

X(\*)                                      array of dimension (1:N) containing deviates randomly generated with the Gamma distribution.

### Algorithm

1. Given Gamma parameters A and B, generate B independent exponential deviates with mean=A.
2. The corresponding Gamma deviate is equal to the sum of the B exponential deviates.

### Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.   a. Type: LOAD “RGAMM2”,10  
   b. Press: EXECUTE
3. The title “RANDOM GAMMA DEVIATES” is printed along with options available to you in the program.

4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and the parameters.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. One set of deviates is produced with a different starting seed for each set of parameters. After each data set is generated, the starting seed, the parameters, and the mean and variance of the set are printed as well as the expected mean and variance.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONTor
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED, A,B. E.G., 100,PI,3.5" is displayed:
  - a. Enter values for the size of the set, the starting seed, and the appropriate parameters.
  - b. Press: CONT
9. After the data set has been created, the prompt: "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" is displayed:

- a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.
- or
- a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When “ENTER THE SELECT CODE OF THE PRINTER.” is displayed:
- a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER.” is displayed:
- a. Enter the bus address of the printer.
  - b. Press: CONT
12. When “WOULD YOU LIKE TO SAVE YOUR DATA SET? (Y/N)” is displayed:
- a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.
- or
- a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
13. When “ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8” is displayed.
- a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM GAMMA DEVIATES

This program generates sets of Gamma (A,B) random deviates. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of Gamma (A,B) random deviates. It also compares the resulting data with the theoretical expectations. In particular, given the parameters A and B, the mean and variance of each set of Gamma deviates is calculated.

The expected value of the mean is:  $A*B$ .

The expected value of the variance is:  $A*A*B$ .

The program is set up using the following default values:

1. 50 deviates will be generated for each set of Gamma parameters.
  2. Values for A are set at .2, and 1.
  3. Values for B are set at 2, and 5.
  4. Results printed are:
    - mean, expected mean, and standard deviation for each set.
    - variance and expected variance for each set.
- Default values may be changed from the keyboard as they come up.

```
# OF RANDOM DEVIATES: 1000
A PARAMETERS: .2, 1
B PARAMETERS: 2, 5
```

```
Number: 1000
```

|           |               |                    |       |                |       |
|-----------|---------------|--------------------|-------|----------------|-------|
| Seed:     | .882198893254 | A:                 | .20   | B:             | 2.00  |
| Mean:     | .401          | Expected Mean:     | .400  | Standard Dev.: | .263  |
| Variance: | .088          | Expected Variance: | .080  |                |       |
| Seed:     | .921868427612 | A:                 | .20   | B:             | 5.00  |
| Mean:     | 1.012         | Expected Mean:     | 1.000 | Standard Dev.: | .457  |
| Variance: | .209          | Expected Variance: | .200  |                |       |
| Seed:     | .291347620384 | A:                 | 1.00  | B:             | 2.00  |
| Mean:     | 2.000         | Expected Mean:     | 2.000 | Standard Dev.: | 1.404 |
| Variance: | 1.970         | Expected Variance: | 2.000 |                |       |

```

Seed:      .023348743108          A:      1.00          B:      5.00
Mean:      5.100          Expected Mean:  5.000          Standard Dev.:  2.361
Variance:  5.572          Expected Variance: 5.000

```

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter

the following values: the size of the data set, the starting seed, and the values of the parameters A and B. For want of a better seed,  $\pi/180$  is an excellent choice!

GAMMA DEVIATES:

```

Seed:      .017453292520          A:      .2000          B:      8.0000

```

| I  | X(I)     | X(I+1)   | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|----------|----------|----------|----------|
| 1  | 1.648088 | 2.201227 | 1.689699 | 1.694490 | 1.143761 |
| 6  | 1.996747 | 1.423744 | 1.531012 | 1.056391 | 1.397026 |
| 11 | 1.799022 | 1.344000 | 1.601870 | 1.315438 | 1.134806 |
| 16 | 1.962800 | 1.509895 | 1.679329 | 2.984781 | 1.419627 |
| 21 | 2.039519 | 1.797392 | 1.764438 | 2.161114 | 1.718534 |



**(RGEOM)**  
**Random Integers Generated**  
**From a Geometric Distribution**





# (RGEOM) Random Integers Generated From a Geometric Distribution

## Description

Given that a certain event occurs with probability  $P$ , this subprogram generates  $N$  random integers with the appropriate Geometric distribution; that is, each random integer represents the number of individual trials needed until the given event first occurs (or between occurrences of the event).

## File Name

“RGEOM”

## Calling Syntax

Call Random\_geom (N,P,Integer(\*) )



## Input Parameters

$N$                                       number of random integers desired.  
 $P$     probability of a given event occurring.

## Output Parameters

Integer(\*)                                      array of dimension (1:N) containing integers randomly generated for the number of independent trials needed until the given event occurs.

## Algorithm

The probability of the event first occurring on the  $R$ th trial is  $P \cdot (1-P)^{(R-1)}$ .

A convenient way to generate a variable with this distribution when  $P$  is small is to set  $R =$  the least integer function of  $[\ln(U)/\ln(1-P)]$  where  $U$  is a uniformly generated random number.

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms, Reading, Mass.: Addison-Wesley, p. 116.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.   a. Type: LOAD “RGEOM”,10  
      b. Press: EXECUTE
3. The title “RANDOM GEOMETRIC DEVIATES” is printed along with the options available to you in the program.

4. When “ENTER YOUR CHOICE. [EITHER 1 or 2]” is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the set of deviates, and the probabilities.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates are produced for each sized set. The starting seed, as well as the probability, is printed. The mean and variance of each data set are also printed as well as the expected mean and variance.
7. When “WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONTor
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, PROBABILITY. E.G., 100, PI,.8” is displayed:
  - a. Enter values for the size of the set, the starting seed, and the probability.
  - b. Press: CONT

9. After the data set has been created, the prompt: “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When “ENTER THE SELECT CODE OF THE PRINTER.” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER.” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
13. When “ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8” is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM GEOMETRIC DEVIATES

This program generates sets of Geometric (P) random deviates. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of Geometric (P) random deviates. It also compares the resulting data with the theoretical expectations. In particular, given a probability P, the mean and variance of each set of Geometric random numbers is calculated.

The expected value of the mean is:  $1/P$ .

The expected value of the variance is:  $(1-P)/(P*P)$ .

The program is set up using the following default values:

1. 100 deviates will be generated for each Geometric probability.
2. Values for the probability are set at 0.10, 0.50, and 0.75.
3. Results printed are:
  - mean, expected mean, and standard deviation for each set.
  - variance and expected variance for each set.

Default values may be changed from the keyboard as they come up.

# OF RANDOM DEVIATES: 1000  
PROBABILITIES: 0.10, 0.50, 0.75

Prob.: .100

|           |               |                    |        |                      |
|-----------|---------------|--------------------|--------|----------------------|
| Seed:     | .982190093254 |                    |        |                      |
| Mean:     | 9.574         | Expected Mean:     | 10.000 | Standard Dev.: 9.172 |
| Variance: | 84.119        | Expected Variance: | 90.000 |                      |

|           |               |                    |        |                      |
|-----------|---------------|--------------------|--------|----------------------|
| Seed:     | .021868427601 |                    |        |                      |
| Mean:     | 10.288        | Expected Mean:     | 10.000 | Standard Dev.: 9.885 |
| Variance: | 97.717        | Expected Variance: | 90.000 |                      |

|           |               |                    |        |                       |
|-----------|---------------|--------------------|--------|-----------------------|
| Seed:     | .391347612684 |                    |        |                       |
| Mean:     | 10.421        | Expected Mean:     | 10.000 | Standard Dev.: 10.303 |
| Variance: | 106.148       | Expected Variance: | 90.000 |                       |

Prob.: .500

|           |               |                    |       |                |       |
|-----------|---------------|--------------------|-------|----------------|-------|
| Seed:     | .123342266280 |                    |       |                |       |
| Mean:     | 1.967         | Expected Mean:     | 2.000 | Standard Dev.: | 1.437 |
| Variance: | 2.056         | Expected Variance: | 2.000 |                |       |
| Seed:     | .730945941525 |                    |       |                |       |
| Mean:     | 2.011         | Expected Mean:     | 2.000 | Standard Dev.: | 1.374 |
| Variance: | 1.887         | Expected Variance: | 2.000 |                |       |
| Seed:     | .641436821832 |                    |       |                |       |
| Mean:     | 1.980         | Expected Mean:     | 2.000 | Standard Dev.: | 1.380 |
| Variance: | 1.904         | Expected Variance: | 2.000 |                |       |

Prob.: .750

|           |               |                    |       |                |      |
|-----------|---------------|--------------------|-------|----------------|------|
| Seed:     | .448367159967 |                    |       |                |      |
| Mean:     | 1.325         | Expected Mean:     | 1.333 | Standard Dev.: | .632 |
| Variance: | .400          | Expected Variance: | .444  |                |      |
| Seed:     | .076781531006 |                    |       |                |      |
| Mean:     | 1.326         | Expected Mean:     | 1.333 | Standard Dev.: | .650 |
| Variance: | .422          | Expected Variance: | .444  |                |      |
| Seed:     | .573267576264 |                    |       |                |      |
| Mean:     | 1.329         | Expected Mean:     | 1.333 | Standard Dev.: | .644 |
| Variance: | .415          | Expected Variance: | .444  |                |      |

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the probability. For want of a better seed, PI is an excellent choice!

#### GEOMETRIC DEVIATES:

Seed: .017453292520      Prob: .8000

| I  | X(I)     | X(I+1)   | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|----------|----------|----------|----------|
| 1  | 1.000000 | 2.000000 | 1.000000 | 1.000000 | 1.000000 |
| 6  | 1.000000 | 1.000000 | 3.000000 | 2.000000 | 1.000000 |
| 11 | 1.000000 | 2.000000 | 5.000000 | 1.000000 | 1.000000 |
| 16 | 1.000000 | 2.000000 | 1.000000 | 1.000000 | 2.000000 |
| 21 | 1.000000 | 1.000000 | 3.000000 | 1.000000 | 2.000000 |



**(RLNORM)**  
**Random Lognormal Deviates**





# (RLNORM) Random Lognormal Deviates

## Description

This subprogram generates a set of random lognormal deviates with mean= $\mu$  and standard deviation= $\sigma$ .

## File Name

“RLNORM”

## Calling Syntax

CALL Random\_lognorm (N,Mu,Sigma,X(\*) )

## Input Parameters

|       |   |
|-------|---|
| N     | number of deviates desired.             |
| Mu    | mean of the distribution.               |
| Sigma | standard deviation of the distribution. |

## Output Parameters

|      |  |
|------|--|
| X(*) | array of dimension(1:N) containing the N lognormal deviates. |
|------|--|

## Algorithm

1. Let  $S = \log [(\sigma^2)/(\mu^2) + 1]$ .
2. Let  $U = \log (\mu) - 0.5*S$ .
3. Generate a normal deviate A, with mean = U and standard deviation = Square Root of(S).
4. Then the lognormal deviate is equal to  $\exp (A)$ .

## Reference

1. Reddy, Y.V., “PL/I Process Generators”, SIMULETTER, Vol. III, Oct. 1976, p. 27.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2. a. Type: LOAD “RLNORM”,10  
b. Press: EXECUTE
3. The title “RANDOM LOGNORMAL DEVIATES” is printed along with the options available to you in the program.

4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, the means and standard deviations.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates are produced for each sized set. The starting seed as well as the mean and standard deviation are printed and can be compared with the user specified values.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED, MEAN, SD. E.G., 100,PI,5,2.5" is displayed:
  - a. Enter values for the size of the set, the starting seed, the mean and standard deviation.
  - b. Press: CONT

9. After the data set has been created, the prompt: "WOULD YOU LIKE THE DATA SET STORED ON A MASS STORAGE DEVICE? (Y/N)" will be displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. Go to 11.
10. When "ENTER THE FILE NAME. FOR EXAMPLE: File:T14 OR DATA:F8" is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector.
11. When "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 12.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. The program is terminated at this point.
12. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
13. When "ENTER THE BUS ADDRESS OF THE PRINTER." is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is then printed out on the appropriate printer.
  - d. The program is terminated at this point.

RANDOM LOGNORMAL DEVIATES

This program generates sets of Log Normal random deviates with user-defined mean and standard deviation. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

The program is set up using the following default values:

1. Five different sets of means and standard deviations are used: (.5,.5), (2,.5), (.5,2), (10,1), and (1,10).
2. Three tests are performed for each mean and standard deviation using 100 log normal random numbers for each set.
3. For each test, the mean and standard deviation are calculated. These values can then be compared with the expected mean and standard deviation.

Default values may be changed from the keyboard as they come up.

# OF DIFFERENT MEANS AND SDs: 5

|                     |                           |                      |  |
|---------------------|---------------------------|----------------------|--|
| Mean: .50           | Standard Deviation: .50   |                      |  |
| Seed: .822190093254 | Mean: .428                | Standard Dev.: .382  |  |
| Seed: .461858427612 | Mean: .568                | Standard Dev.: .524  |  |
| Seed: .431347620384 | Mean: .544                | Standard Dev.: .444  |  |
| Mean: 2.00          | Standard Deviation: .50   |                      |  |
| Seed: .763348741980 | Mean: 1.998               | Standard Dev.: .519  |  |
| Seed: .976292004613 | Mean: 1.987               | Standard Dev.: .488  |  |
| Seed: .061575880207 | Mean: 2.028               | Standard Dev.: .473  |  |
| Mean: .50           | Standard Deviation: 2.00  |                      |  |
| Seed: .785315254000 | Mean: .492                | Standard Dev.: .629  |  |
| Seed: .450128614042 | Mean: .498                | Standard Dev.: 1.102 |  |
| Seed: .558164408898 | Mean: .349                | Standard Dev.: .684  |  |
| Mean: 10.00         | Standard Deviation: 1.00  |                      |  |
| Seed: .416267382067 | Mean: 10.334              | Standard Dev.: 1.153 |  |
| Seed: .061288817109 | Mean: 9.954               | Standard Dev.: .927  |  |
| Seed: .363895188375 | Mean: 9.949               | Standard Dev.: .985  |  |
| Mean: 1.00          | Standard Deviation: 10.00 |                      |  |
| Seed: .035853424106 | Mean: .784                | Standard Dev.: 2.425 |  |
| Seed: .152729673092 | Mean: .511                | Standard Dev.: .875  |  |
| Seed: .445655070460 | Mean: 1.185               | Standard Dev.: 4.566 |  |

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, the mean and the standard deviation. For want of a better seed, PI is an excellent choice!

LOGNORMAL DEVIATES:

Seed: .017453292520      Mean: 5.0000      Sd: 3.0000

| I  | X(I)     | X(I+1)   | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|----------|----------|----------|----------|
| 1  | 3.674948 | 3.048059 | 6.483291 | 3.687992 | 6.351107 |
| 6  | 6.086736 | 4.367232 | 3.288814 | 2.550134 | 4.180853 |
| 11 | 4.183955 | 2.589870 | 3.065009 | 5.478875 | 4.887000 |
| 16 | 2.735173 | 3.100603 | 9.897996 | 3.224541 | 4.471662 |
| 21 | 1.986578 | 4.558286 | 2.445821 | 3.258126 | 5.840190 |



**(RNEGBI)  
Random Numbers Generated  
From a Negative  
Binomial Distribution**





# (RNEGBI) Random Numbers Generated From a Negative Binomial Distribution

## Description

This subprogram generates a set of Negative Binomial random deviates.

## File Name

“RNEGBI”

## Calling Syntax

CALL Random\_neg\_bin (N,R,P,X(\*) )



## Input Parameters

|   |                                    |
|---|------------------------------------|
| N | number of random integers desired. |
| R | failure value.                     |
| P | probability.                       |

## Output Parameters

|      |  |
|------|--|
| X(*) | array of dimension (1:N) containing integers randomly generated with the Negative Binomial distribution. |
|------|--|

## Algorithm

1. Given parameters R and P, generate R random geometric deviates with parameter P.
  - a. Change the values as desired.
  - b. Press: CONT
2. The corresponding Negative Binomial Deviate is equal to the sum of the R geometric deviates.

## Reference

1. Wheeler, R. E., “Random Variable Generators”, SIMULETTER, Vol. IV, April, 1973, p. 22.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.
  - a. Type: LOAD “RNEGBI”,10
  - b. Press: EXECUTE
3. The title “RANDOM NEGATIVE BINOMIAL DEVIATES” is printed along with the options available to you in the program.

4. When “ENTER YOUR CHOICE. [EITHER 1 OR 2]” is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.  
or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates and the parameters.
  - a. If the default values are satisfactory, simply press: CONT  
or
  - a. Change the values as desired.
  - b. Press: CONT
6. One set of deviates is produced with a different starting seed for each set of parameters. After each data set is generated, the starting seed, the parameters, and the mean and variance of the set are printed as well as the expected mean and variance.
7. When “WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT  
or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, R, PROBABILITY. E.G., 100,PI,4,.8” is displayed:
  - a. Enter values for the size of the set, the starting seed, and the appropriate parameters.
  - b. Press: CONT

9. After the data set has been created, the prompt: "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When "ENTER THE BUS ADDRESS OF THE PRINTER." is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
12. When "WOULD YOU LIKE TO SAVE YOUR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
13. When "ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8" is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM NEGATIVE BINOMIAL DEVIATES

This program generates sets of Negative Binomial (R,P) random deviates. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of Negative Binomial (R,P) random deviates. It also compares the resulting data with the theoretical expectations. In particular, given a probability P and a failure value R, the mean and variance of each set of deviates, as well as the expected mean and variance is calculated.

The expected value of the mean is:  $R/P$ .

The expected value of the variance is:  $R*(1-P)/(P*P)$ .

The program is set up using the following default values:

1. 100 deviates will be generated for each Negative binomial probability and failure value R.
2. Values for the probability are set at 0.10, 0.50, and 0.75. Values for R are set at 2, 5, and 10.
3. Results printed are:
  - mean, expected mean, and standard deviation for each set.
  - variance and expected variance for each set.
 Default values may be changed from the keyboard as they come up.

# OF RANDOM DEVIATES: 1000  
 FAILURE VALUES: 2, 5, 10  
 PROBABILITIES: 0.10, 0.50, 0.75

Failure: 2 Prob.: .100

|           |               |                    |         |
|-----------|---------------|--------------------|---------|
| Seed:     | .692190893254 |                    |         |
| Mean:     | 20.971        | Expected Mean:     | 20.000  |
| Variance: | 196.284       | Expected Variance: | 180.000 |
|           |               | Standard Dev.:     | 14.010  |

Failure: 2 Prob.: .500

|           |               |                    |       |
|-----------|---------------|--------------------|-------|
| Seed:     | .331868427612 |                    |       |
| Mean:     | 4.022         | Expected Mean:     | 4.000 |
| Variance: | 4.080         | Expected Variance: | 4.000 |
|           |               | Standard Dev.:     | 2.000 |

|           |               |                    |         |                |        |
|-----------|---------------|--------------------|---------|----------------|--------|
| Failure:  | 2             | Prob.:             | .750    |                |        |
| Seed:     | .101347621784 |                    |         |                |        |
| Mean:     | 2.682         | Expected Mean:     | 2.667   | Standard Dev.: | .929   |
| Variance: | .864          | Expected Variance: | .889    |                |        |
| Failure:  | 5             | Prob.:             | .100    |                |        |
| Seed:     | .233349920581 |                    |         |                |        |
| Mean:     | 49.398        | Expected Mean:     | 50.000  | Standard Dev.: | 20.303 |
| Variance: | 412.218       | Expected Variance: | 450.000 |                |        |
| Failure:  | 5             | Prob.:             | .500    |                |        |
| Seed:     | .247283208747 |                    |         |                |        |
| Mean:     | 9.982         | Expected Mean:     | 10.000  | Standard Dev.: | 3.148  |
| Variance: | 9.912         | Expected Variance: | 10.000  |                |        |
| Failure:  | 5             | Prob.:             | .750    |                |        |
| Seed:     | .965178556113 |                    |         |                |        |
| Mean:     | 6.657         | Expected Mean:     | 6.667   | Standard Dev.: | 1.503  |
| Variance: | 2.258         | Expected Variance: | 2.222   |                |        |
| Failure:  | 10            | Prob.:             | .100    |                |        |
| Seed:     | .715165691778 |                    |         |                |        |
| Mean:     | 100.759       | Expected Mean:     | 100.000 | Standard Dev.: | 30.737 |
| Variance: | 944.754       | Expected Variance: | 900.000 |                |        |
| Failure:  | 10            | Prob.:             | .500    |                |        |
| Seed:     | .454346786405 |                    |         |                |        |
| Mean:     | 20.071        | Expected Mean:     | 20.000  | Standard Dev.: | 4.330  |
| Variance: | 18.745        | Expected Variance: | 20.000  |                |        |
| Failure:  | 10            | Prob.:             | .750    |                |        |
| Seed:     | .105647365356 |                    |         |                |        |
| Mean:     | 13.309        | Expected Mean:     | 13.333  | Standard Dev.: | 2.090  |
| Variance: | 4.368         | Expected Variance: | 4.444   |                |        |

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, the failure value and the probability. For want of a better seed, PI is an excellent choice!

NEGATIVE BINOMIAL DEVIATES:

Seed: .017453292520      Failure: 4      Prob: .9000

| I  | X(I)     | X(I+1)   | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|----------|----------|----------|----------|
| 1  | 5.000000 | 6.000000 | 6.000000 | 8.000000 | 6.000000 |
| 6  | 6.000000 | 6.000000 | 4.000000 | 4.000000 | 4.000000 |
| 11 | 4.000000 | 5.000000 | 5.000000 | 6.000000 | 5.000000 |
| 16 | 5.000000 | 4.000000 | 4.000000 | 5.000000 | 6.000000 |
| 21 | 4.000000 | 6.000000 | 4.000000 | 5.000000 | 4.000000 |

**(RNORM)**  
**Normal Random Deviates With**  
**Mean=0 and Standard Deviation=1**





# (RNORM)

## Normal Random Deviates With Mean=0 And Standard Deviation=1

### Description

This subprogram calculates an even number of normally distributed variables with mean=0 and standard deviation=1.

### File Name

“RNORM”

### Calling Syntax

CALL Random\_normal (N,X(\*) )

### Input Parameters

N                                    number of normal deviates desired. N must be even.

### Output Parameters

X(\*)                                 array of dimension (1:N) containing the N normal deviates.

### Algorithm

This utility generates random deviates for the normal distribution with mean=0 and standard deviation=1. An adapted form of the Polar Method is used. (See Reference 1.)

### Special Considerations

1. Due to the nature of the algorithm used, this routine generates an even number of normal deviates. If an odd number is requested, an error message is printed and the routine has to be re-entered again.
2. This method is rather slow, but it has essentially perfect accuracy and takes a minimum of storage space.

### Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 104.

### Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.     a. Type: LOAD “RNORM”,10  
       b. Press: EXECUTE
3. The title “RANDOM NORMAL DEVIATES” is printed along with the options available to you in the program.

4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates are produced for each sized set. The starting seed as well as the mean and standard deviation are printed and can be compared with the expected values of zero and one respectively.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED. E.G., 1000,PI" is displayed:
  - a. Enter values for the size of the set and the starting seed.
  - b. Press: CONT

NOTE: set size must be even.

9. When “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When “ENTER THE SELECT CODE OF THE PRINTER.” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER.” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is then printed on the appropriate device.
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like to save it.
  - b. Press: CONT
  - c. The program is terminated at this point.
13. When “ENTER FILE NAME. FOR EXAMPLE: File:T14 OR DATA:F8” is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM NORMAL DEVIATES

This program generates sets of Normal random deviates with mean=0 and standard deviation=1. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

The program is set up using the following default values:

1. Sets of size 100, 500, and 1000 are selected in turn.
2. Three tests are performed for each sized set.
3. For each set of data, the mean and standard deviation are calculated. These values can then be compared with the expected mean of 0 and the standard deviation of 1.

Default values may be changed from the keyboard as they come up.

SIZE OF RANDOM DEVIATE SETS: 100, 500, 1000

|              |               |       |       |                      |
|--------------|---------------|-------|-------|----------------------|
| Size of Set: | 100           |       |       |                      |
| Seed:        | .772190093254 | Mean: | -.018 | Standard Dev.: .920  |
| Seed:        | .411868427612 | Mean: | .166  | Standard Dev.: 1.048 |
| Seed:        | .381347620384 | Mean: | .211  | Standard Dev.: .979  |
| Size of Set: | 500           |       |       |                      |
| Seed:        | .713348741909 | Mean: | -.013 | Standard Dev.: 1.012 |
| Seed:        | .926291946613 | Mean: | -.078 | Standard Dev.: 1.015 |
| Seed:        | .011527102207 | Mean: | .049  | Standard Dev.: 1.026 |
| Size of Set: | 1000          |       |       |                      |
| Seed:        | .694292955785 | Mean: | .037  | Standard Dev.: 1.020 |
| Seed:        | .900375816295 | Mean: | -.017 | Standard Dev.: .984  |
| Seed:        | .216861505498 | Mean: | -.047 | Standard Dev.: .996  |

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, and the starting seed. For want of a better seed, PI is an excellent choice.

NORMAL DEVIATES:

Seed: .017458292520

| I  | X(I)      | X(I+1)   | X(I+2)    | X(I+3)   | X(I+4)   |
|----|-----------|----------|-----------|----------|----------|
| 1  | -.278004  | -.615298 | .745758   | -.271614 | .708610  |
| 6  | .631935   | .033243  | -.479201  | -.936948 | -.045410 |
| 11 | -.844072  | -.909064 | -.605297  | .442198  | .206267  |
| 16 | -.810623  | -.584475 | 1.508777  | -.513793 | .075859  |
| 21 | -1.387311 | .110459  | -1.012266 | -.495107 | .557368  |



**(RNORM1)**  
**Normal Random Deviates With**  
**Specified Mean And**  
**Standard Deviation**





## (RNORM1) Normal Random Deviates With Specified Mean And Standard Deviation

### Description

This subprogram generates a set of normal random deviates with mean=Mu and standard deviation=Sigma.

### File Name

“RNORM1”

### Calling Syntax

CALL Random\_normal1 (N,Mu,Sigma,X(\*) )

### Input Parameters

|       |   |
|-------|---|
| N     | number of deviates desired.                                 |
| Mu    | assume a normal distribution with mean=Mu.                  |
| Sigma | assume a normal distribution with Standard Deviation=Sigma. |

### Output Parameters

|      |  |
|------|--|
| X(*) | array of dimension (1:N) containing the N normal deviates. |
|------|--|

### Algorithm

Given a mean = u and standard deviation = s,

1. Generate a deviate x with a normal distribution with mean 0 and standard deviation =1.
2. Then  $y = u + s * x$ .

### Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 ,Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 113.

### Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.
  - a. Type: LOAD “RNORM1”,10
  - b. Press: EXECUTE

3. The title "RANDOM NORMAL DEVIATES" is printed along with the options available to you in the program.
4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.

or

  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, the means and standard deviations.
  - a. If the default values are satisfactory, simply press: CONT

or

  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates are produced for each sized set. The starting seed as well as the mean and standard deviation are printed and can be compared with the user specified values.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - c. Go to 8.

or

  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED, MEAN, SD. E.G., 100,P1,5,2.5" is displayed:
  - a. Enter values for the size of the set, the starting seed, the mean and standard deviation.
  - b. Press: CONT

9. When “WOULD YOU LIKE A PRINTOUT OF THE DATA (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. The program is terminated at this point.
10. When “ENTER THE SELECT CODE OF THE PRINTER.” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER.” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is printed out on the appropriate printer.
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. The program terminates at this point.
13. When “ENTER FILE NAME. FOR EXAMPLE: File:T14 OR DATA:F8” is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM NORMAL DEVIATES

This program generates sets of Normal random deviates with user-defined mean and standard deviation. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

The program is set up using the following default values:

1. Five different sets of means and standard deviations are used: (.5,.5), (2,.5), (.5,2), (10,1), and (1,10).
2. Three tests are performed for each mean and standard deviation using 100 normal random numbers for each set.
3. For each test, the mean and standard deviation are calculated. These values can then be compared with the expected mean and standard deviation.

Default values may be changed from the keyboard as they come up.

# OF DIFFERENT MEANS AND SDs: 5

|       |                |                     |       |                |        |
|-------|----------------|---------------------|-------|----------------|--------|
| Mean: | .50            | Standard Deviation: | .50   |                |        |
| Seed: | .882196893254  | Mean:               | .530  | Standard Dev.: | .524   |
| Seed: | .921868427612  | Mean:               | .558  | Standard Dev.: | .481   |
| Seed: | .291347620384  | Mean:               | .570  | Standard Dev.: | .513   |
| Mean: | 2.00           | Standard Deviation: | .50   |                |        |
| Seed: | .0233346743100 | Mean:               | 2.062 | Standard Dev.: | .541   |
| Seed: | .636292953813  | Mean:               | 2.067 | Standard Dev.: | .526   |
| Seed: | .122374157495  | Mean:               | 1.969 | Standard Dev.: | .500   |
| Mean: | .50            | Standard Deviation: | 2.00  |                |        |
| Seed: | .916666453150  | Mean:               | .782  | Standard Dev.: | 2.249  |
| Seed: | .916487897727  | Mean:               | .683  | Standard Dev.: | 1.999  |
| Seed: | .765649188925  | Mean:               | .495  | Standard Dev.: | 2.394  |
| Mean: | 10.00          | Standard Deviation: | 1.00  |                |        |
| Seed: | .918967885226  | Mean:               | 9.976 | Standard Dev.: | 1.051  |
| Seed: | .123991476468  | Mean:               | 9.847 | Standard Dev.: | 1.001  |
| Seed: | .276831709552  | Mean:               | 9.929 | Standard Dev.: | .958   |
| Mean: | 1.00           | Standard Deviation: | 10.00 |                |        |
| Seed: | .815467733295  | Mean:               | -.773 | Standard Dev.: | 10.448 |
| Seed: | .809363702464  | Mean:               | 2.434 | Standard Dev.: | 8.962  |
| Seed: | .823873773547  | Mean:               | 2.005 | Standard Dev.: | 9.335  |

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the desired mean and standard deviation. For want of a better seed, PI is an excellent choice!

NORMAL DEVIATES:

Seed: .017453292529

Mean: 5.0000

Sd: 2.5000

| I  | X(I)     | X(I+1)   | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|----------|----------|----------|----------|
| 1  | 4.384991 | 3.461756 | 6.964396 | 4.328965 | 6.771525 |
| 6  | 6.579839 | 5.083188 | 3.884498 | 2.657631 | 4.886475 |
| 11 | 4.889828 | 2.727348 | 3.486757 | 6.165494 | 5.515667 |
| 16 | 2.973442 | 3.538813 | 8.771944 | 3.718517 | 5.189646 |
| 21 | 1.531723 | 5.276148 | 2.469335 | 3.762231 | 6.393428 |



**(RNORM2)**  
**Dependent Normally Distributed**  
**Random Variables**





## (RNORM2)

# Dependent Normally Distributed Random Variables (Bivariate Normal Deviates)

### Description

This subprogram generates two dependent random variables, normally distributed with mean= $\mu_1, \mu_2$ , standard deviation = $\sigma_1, \sigma_2$ , and Correlation Coefficient= $\rho$ .

### File Name

“RNORM2”

### Calling Syntax

CALL Random\_normal2 ( $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho, X1(*), X2(*)$ )

### Input Parameters

|                      |                                  |
|----------------------|----------------------------------|
| $\mu_1, \mu_2$       | initial means.                   |
| $\sigma_1, \sigma_2$ | initial standard deviations.     |
| $\rho$               | initial correlation coefficient. |

### Output Parameters

$X1(*), X2(*)$  two vectors of dependent normally distributed random variables.

### Algorithm

If  $x_1$  and  $x_2$  are independent normal deviates with mean=0 and standard deviation=1, and if  $y_1 = \mu_1 + \sigma_1 * x_1$  and  $y_2 = \mu_2 + \sigma_2 * (\rho * x_1 + \sqrt{1 - \rho^2} * x_2)$

then  $y_1$  and  $y_2$  are dependent random variables, normally distributed with means  $\mu_1, \mu_2$  and standard deviations  $\sigma_1$  and  $\sigma_2$ , and with correlation coefficient  $\rho$ .

### Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 113.

### Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.
  - a. Type: LOAD “RNORM2”,10
  - b. Press: EXECUTE

3. The title "DEPENDENT NORMAL RANDOM DEVIATES" is printed along with the options available to you in the program.
4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, the means, standard deviations and correlation coefficient.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates are produced for each sized set. The starting seed, means, standard deviations and correlation coefficient are printed as well as the calculated means, standard deviations and correlation coefficient between the two sets of deviates.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED, CORRELATION COEFFICIENT. E.G., 100,PI,.7" is displayed:
  - a. Enter values for the size of the set, the starting seed, and the correlation coefficient.
  - b. Press: CONT

9. When “ENTER VALUES FOR THE FIRST MEAN, STANDARD DEVIATION. E.G., 5,1” is displayed:
  - a. Enter values for the first mean and standard deviation.
  - b. Press: CONT
10. When “ENTER VALUES FOR THE SECOND MEAN, STANDARD DEVIATION. E.G., 12,3.5” is displayed:
  - a. Enter values for the second mean and standard deviation.
  - b. Press: CONT
11. After the data set has been created, the prompt: “WOULD YOU LIKE THE DATA SET STORED ON A MASS STORAGE DEVICE? (Y/N)” will be displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 12.  
or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 14.
12. When “ENTER THE SELECT CODE OF THE PRINTER,” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
13. When “ENTER THE BUS ADDRESS OF THE PRINTER.” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is then printed out on the appropriate device.

14. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
- a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 15.
- or
- a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. The program is terminated at this point.
15. When “ENTER FILE NAME. FOR EXAMPLE:T14 OR DATA:F8” is displayed:
- a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

#### DEPENDENT NORMAL RANDOM DEVIATES

This program generates a set of dependent Normal random deviates with user defined means and standard deviations. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

The program is set up using the following default values:

1. Mean1 = 0.0                      Standard Dev1 = 1.0  
    Mean2 = 2.0                      Standard Dev2 = 5.0  
    Correlation Coefficient = 0.7
2. Three tests, using 100 deviates from each set will be performed.
3. For each test, the means and standard deviations are calculated, as well as the coefficient of correlation.

Default values may be changed from the keyboard as they come up.

# OF DEVIATES FOR EACH SET: 1000  
 MEAN1: 0.000 SD1: 1.000  
 MEAN2: 5.000 SD2: 2.000  
 CORRELATION COEFFICIENT: .5

Seed: .712190093254 Mean1: .011 Standard Dev1: 1.023  
 Mean2: 4.915 Standard Dev2: 2.048  
 Correlation Coefficient: .498

Seed: .951868427612 Mean1: .077 Standard Dev1: .941  
 Mean2: 4.994 Standard Dev2: 1.948  
 Correlation Coefficient: .479

Seed: .521347628384 Mean1: .015 Standard Dev1: 1.035  
 Mean2: 5.100 Standard Dev2: 2.035  
 Correlation Coefficient: .530

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, the two means and standard deviations, and the correlation coefficient. For want of a better seed, PI is an excellent choice!

DEPENDENT NORMAL DEVIATES:

Seed: .017453292520  
 Mean1: 5.0000 Sd1: 1.0000  
 Mean2: 12.0000 Sd2: 3.5000 Rho: .7000

| I  | X(I)       | X(I+1)    | X(I+2)    | X(I+3)    | X(I+4)     |
|----|------------|-----------|-----------|-----------|------------|
| 1  | 1.663956   | -2.383572 | 13.949101 | 1.740634  | 13.503322  |
| 6  | 12.583226  | 5.398920  | -.738410  | -6.243373 | 4.455081   |
| 11 | 4.471136   | -5.908770 | -2.263564 | 10.306370 | 7.475200   |
| 16 | -4.727476  | -2.013696 | 23.105329 | -1.165518 | 5.910302   |
| 21 | -11.647728 | 6.325512  | -7.147192 | -.941289  | -12.209863 |

| I  | Y(I)       | Y(I+1)     | Y(I+2)     | Y(I+3)     | Y(I+4)     |
|----|------------|------------|------------|------------|------------|
| 1  | 6.469833   | -4.434210  | 33.568919  | 4.471587   | 34.794196  |
| 6  | 33.155234  | 11.758055  | -7.325937  | -16.525786 | 10.264808  |
| 11 | 14.649641  | -14.484984 | -4.698050  | 28.247726  | 18.309951  |
| 16 | -8.855412  | -4.711909  | 57.046057  | .454459    | 13.826361  |
| 21 | -26.804736 | 16.456228  | -18.109722 | -1.783696  | -32.829985 |



**(RPAR1)**  
**Random Pareto Generator**  
**Of the First Kind**







# (RPAR1) Random Pareto Generator Of The First Kind

## Description

This program generates sets of random Pareto deviates of the first kind. The density function is defined as follows:

$$f(X) = [N \cdot A^N] / X^{(N+1)} \quad \text{for } X > A$$

## File Name

“RPAR1”

## Calling Syntax

CALL Random\_pareto1 (Number,A,N,X(\*) )

## Input Parameters

|        |                                    |
|--------|------------------------------------|
| Number | number of random deviates desired. |
| A,N    | Pareto parameters.                 |

## Output Parameters

|      |  |
|------|--|
| X(*) | array of dimension (1:N) containing N Pareto deviates of the first kind. |
|------|--|

## Algorithm

1. Given parameters A and N, generate a uniform deviate U.
2. Then the Pareto deviate is equal to:  $A / (1-U)^{(1/N)}$ .

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.
  - a. Type: LOAD “RPAR1”,10
  - b. Press: EXECUTE
3. The title “RANDOM PARETO DEVIATES OF THE FIRST KIND” is printed along with some introductory remarks.
4. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, A, N. E.G. 100, PI,10,.8” is displayed:
  - a. Enter values for the size of the set, the starting seed, and the Pareto parameters A and N.
  - b. Press: CONT

5. After the data set has been created, the prompt: "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 6.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 8.
6. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
7. When "ENTER THE BUS ADDRESS OF THE PRINTER." is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
8. When "WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 9.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
9. When "ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8" is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM PARETO DEVIATES OF THE FIRST KIND

This program generates sets of Pareto deviates of the First kind. The density function is defined as follows:  $f(X)=[N \cdot A^N] / X^{N+1}$  for  $X > A$ . The corresponding distribution function is:  $F(X)=1-(A/X)^N$  for  $X > A$ .

In this program, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the parameters A and N. For want of a better seed, PI is an excellent choice!

PARETO I DEVIATES:

Seed: .017453292520

A: 10.000

N: .800

| I  | X(I)       | X(I+1)    | X(I+2)     | X(I+3)    | X(I+4)     |
|----|------------|-----------|------------|-----------|------------|
| 1  | 16.084997  | 11.304425 | 180.885864 | 17.260754 | 73.783520  |
| 5  | 61.325641  | 25.885165 | 10.375376  | 11.395074 | 22.677573  |
| 11 | 22.663491  | 11.313336 | 10.006642  | 58.127092 | 291.523952 |
| 16 | 344.635942 | 12.237704 | 58.400460  | 31.784333 | 11.300812  |
| 21 | 19.181864  | 54.464768 | 10.490970  | 28.574731 | 13.071547  |



**(RPAR2)**  
**Random Pareto Generator of  
the Second Kind**



## (RPAR2)

# Random Pareto Generator Of The Second Kind

### Description

This program generates sets of random Pareto deviates of the second kind. The density function is defined as follows:

$$f(X) = [N \cdot B^N] / [B + X]^{(N+1)} \quad \text{for } x > 0.$$

### File Name

“RPAR2”

### Calling Syntax

CALL Random\_pareto2 (Number,B,N,X(\*) )

### Input Parameters

Number                                      number of random deviates desired.  
 B,N    Pareto parameters.

### Output Parameters

X(\*)    array of dimension(1:N) containing N Pareto deviates of the second kind.

### Algorithm

1. Given parameters B and N, generate a uniform deviate U.
2. Then the Pareto deviate is equal to:  $B / (1-U)^{(1/N)} - B$ .

### Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.    a. Type: LOAD “RPAR2”,10  
       b. Press: EXECUTE
3. The title “RANDOM PARETO DEVIATES OF THE SECOND KIND” is printed along with some introductory remarks.
4. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, B, N. E.G. 100, PI,10,.8” is displayed:
  - a. Enter values for the size of the set, the starting seed, and the Pareto parameters B and N.
  - b. Press: CONT

5. After the data set has been created, the prompt: "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 6.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 8.
6. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
7. When "ENTER THE BUS ADDRESS OF THE PRINTER." is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
8. When "WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 9.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
9. When "ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8" is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.



RANDOM PARETO DEVIATES OF THE SECOND KIND

This program generates sets of Pareto deviates of the Second kind. The density function is defined as follows:  $f(X)=[N*B^N]/[B+X]^{(N+1)}$  for  $X>0$ . The corresponding distribution function is:  $F(X)=1-[B/(B+X)]^N$ .

In this program, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the parameters B and N. For want of a better seed, PI is an excellent choice!

PARETO II DEVIATES:

Seed: .017453292520

B: 10.000

N: .000

| I  | X(I)       | X(I+1)    | X(I+2)     | X(I+3)    | X(I+4)     |
|----|------------|-----------|------------|-----------|------------|
| 1  | 6.004997   | 1.304425  | 170.885864 | 7.260754  | 63.703520  |
| 6  | 51.325641  | 15.885165 | .375376    | 1.395074  | 12.677573  |
| 11 | 12.663491  | 1.313336  | .006642    | 48.127082 | 271.523852 |
| 16 | 334.635942 | 2.237704  | 48.400400  | 21.784333 | 1.300812   |
| 21 | 9.181864   | 44.464768 | .490970    | 19.574731 | 3.071547   |



**(RPOISS)**  
**Random Integers Generated**  
**From a Poisson Distribution**



# (RPOISS)

## Random Integers Generated From A Poisson Distribution

### Description

This subprogram generates a set of Poisson deviates with a specified mean.

### File Name

“RPOISS”

### Calling Syntax

CALL Random\_poisson (N,Mu,X(\*) )



### Input Parameters

N                                      number of random integers desired.  
Mu                                        mean of the Poisson distribution.

### Output Parameters

X(\*)                                      array of dimension (1:N) containing integers randomly generated with the given Poisson distribution.

### Algorithm

Given a mean of the distribution  $m$ ,

1. Set:  $P = \exp(-M)$   
 $N = 0$   
 $Q = 1$
2. Generate a random variable  $U$ , uniformly distributed between 0 and 1.
3. Set:  $Q = Q * U$
4. If  $Q > P$ , then set  $N = N + 1$  and return to step 2.  
     Else, terminate the algorithm with output  $N$ .

### Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 117.

### Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.

2.
  - a. Type: LOAD 'RPOISS',10
  - b. Press: EXECUTE
3. The title "RANDOM POISSON DEVIATES" is printed along with the options available to you in the program.
4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.

or

  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and the probabilities.
  - a. If the default values are satisfactory, simply press: CONT

or

  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates with different starting seeds are produced for each mean. After each data set is generated, the starting seed, the mean and the variance of the set are printed.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT

or

  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED, MEAN. E.G. 100, PI,5" is displayed:
  - a. Enter values for the size of the set, the starting seed, the mean of the distribution.
  - b. Press: CONT

9. After the data set has been created, the prompt: "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When "ENTER THE BUS ADDRESS OF THE PRINTER." is displayed:
  - a. Enter the bus address of the printer
  - b. Press: CONT
12. When "WOULD YOU LIKE TO SAVE YOUR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
13. When "ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8" is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM POISSON DEVIATES

This program generates sets of Poisson random deviates. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of Poisson random deviates. It also compares the resulting data with the theoretical expectations. In particular, the mean and variance of each set of Poisson random numbers is calculated. The expected value of both is the Poisson parameter from which the data was generated.

The program is set up using the following default values:

1. A random seed is generated.
2. 0.5, 3.5, 6.5, and 9.5 are selected in turn as the Poisson parameters.
3. For each parameter, 3 sets of 100 Poisson deviates along with their corresponding mean and variance are calculated.
4. The resulting data is printed for each set.

Default values may be changed from the keyboard as they come up.

The primary purpose of this driver program is to allow you to check out our Poisson subprogram. This subprogram may then be used in your own context.

# OF POISSON DEVIATES: 1000

POISSON PARAMETERS: 0.5, 3.5, 6.5, 9.5

Poisson Parameter: .5

|                     |            |                |
|---------------------|------------|----------------|
| Seed: .582198093254 | Mean: .478 | Variance: .488 |
| Seed: .621868427612 | Mean: .507 | Variance: .508 |
| Seed: .991347628344 | Mean: .483 | Variance: .488 |
| Mean St. Dev.:      | .016       |                |

Poisson Parameter: 3.5

|                     |             |                 |
|---------------------|-------------|-----------------|
| Seed: .723348710088 | Mean: 3.527 | Variance: 3.837 |
| Seed: .036265176712 | Mean: 3.534 | Variance: 3.454 |
| Seed: .799813614990 | Mean: 3.507 | Variance: 3.195 |
| Mean St. Dev.:      | .014        |                 |

Poisson Parameter: 6.5

|                     |             |                 |
|---------------------|-------------|-----------------|
| Seed: .970458286394 | Mean: 6.475 | Variance: 6.378 |
| Seed: .148623576655 | Mean: 6.563 | Variance: 6.773 |
| Seed: .992427965733 | Mean: 6.454 | Variance: 5.952 |
| Mean St. Dev.:      | .058        |                 |



Poisson Parameter: 9.5

|                     |             |                 |
|---------------------|-------------|-----------------|
| Seed: .631920064311 | Mean: 9.502 | Variance: 9.913 |
| Seed: .444774065081 | Mean: 9.518 | Variance: 9.247 |
| Seed: .655005551705 | Mean: 9.234 | Variance: 9.705 |
| Mean St. Dev.:      | .160        |                 |

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter

the following values: the size of the data set, the starting seed, and the Mean. For want of a better seed, PI is an excellent choice.

POISSON DEVIATES:

Seed: .017453292520 Mean: 5.0000

| I  | X(I)     | X(I+1)    | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|-----------|----------|----------|----------|
| 1  | 5.000000 | 2.000000  | 3.000000 | 6.000000 | 3.000000 |
| 6  | 3.000000 | 10.000000 | 5.000000 | 4.000000 | 4.000000 |
| 11 | 4.000000 | 4.000000  | 6.000000 | 5.000000 | 7.000000 |
| 16 | 5.000000 | 5.000000  | 7.000000 | 3.000000 | 5.000000 |
| 21 | 4.000000 | 4.000000  | 5.000000 | 2.000000 | 4.000000 |



**(RSPHER)**  
**Random Points on an M-dimensional  
Sphere of Radius One**



# (RSPHER)

## Random Points on an M-dimensional Sphere of Radius One

### Description

This subprogram generates a set of random points on an M-dimensional sphere of radius one.

### File Name

“RSPHER”

### Calling Syntax

CALL Random\_sphere (N,M,X(\*) )

### Input Parameters

N                                      number of random points desired.  
M                                      number of dimensions of the sphere.

### Output Parameters

X(\*)                                      array of dimension (1:N) containing the N random points.

### Algorithm

1. Let  $X_1, X_2, \dots, X_m$  be independent normal deviates (mean=0, standard deviation=1).
2. Let  $R = \text{SQR}(X_1^2 + X_2^2 + \dots + X_m^2)$ .
3. Then the point  $(X_1/R, X_2/R, \dots, X_m/R)$  is a random point on the M dimensional sphere of radius one.

### Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 116.

### Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.    a. Type: LOAD “RSPHER”,10  
      b. Press: EXECUTE
3. The title “RANDOM PTS. ON AN M-DIMENSIONAL SPHERE” is printed along with the options available to you in the program.

4. When “# OF POINTS DESIRED?” is displayed:
  - a. Enter the number of points desired.
  - b. Press: CONT
5. When “DIMENSIONS OF THE SPHERE?” is displayed:
  - a. Enter the dimensions of the sphere.
  - b. Press: CONT
6. When “ENTER A STARTING SEED.” is displayed:
  - a. Enter a starting seed. For want of a better seed,  $\pi/180$  is an excellent choice.
  - b. Press: CONT
7. When “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 8.  
or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 10.
8. When “ENTER THE SELECT CODE OF THE PRINTER.” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
9. When “ENTER THE BUS ADDRESS OF THE PRINTER.” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is then printed out on the appropriate printer.

10. When "WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)" is displayed:
- Enter Y if you would like it saved.
  - Press: CONT
  - Go to 11.
- or
- Enter N if you would not like it saved.
  - Press: CONT
  - The program is terminated at this point.
11. When "ENTER FILE NAME. FOR EXAMPLE: File:T14 OR DATA:F8" is displayed:
- Enter the desired file name.
  - Press: CONT
  - The data set is saved on your mass storage device as a vector, and the program ends.

RANDOM PTS. ON AN M-DIMENSIONAL SPHERE

This program generates sets of uniformly distributed random points on an M-dimensional sphere.

You will be required to enter the number of points desired and the dimensions of the sphere. You will then be given the options of printing out the generated data and storing it on a mass storage device.

# OF POINTS DESIRED: 12  
 DIMENSIONS OF THE SPHERE: 3  
 SEED: .882190093254

UNIFORM DEVIATES ON AN M-DIMENSIONAL SPHERE:

Seed: .882190093254

|    |          |          |          |
|----|----------|----------|----------|
| 1  | .946119  | -.295299 | .132880  |
| 2  | -.900914 | .178396  | .395637  |
| 3  | -.262349 | -.283895 | .922267  |
| 4  | .118382  | -.418917 | .900274  |
| 5  | -.058066 | .734255  | .676386  |
| 6  | .020370  | -.998962 | -.040748 |
| 7  | -.991123 | .098869  | -.088884 |
| 8  | -.040587 | .034705  | -.998573 |
| 9  | -.943735 | .222579  | -.244585 |
| 10 | .124431  | .570977  | .811481  |
| 11 | -.199613 | .979732  | -.016728 |
| 12 | -.683251 | .380686  | .623094  |





**(RSUPER)  
Super Uniform Random  
Number Generator**



# (RSUPER) Super Uniform Random Number Generator

## Description

Given methods for generating two random sequences, this algorithm successfully outputs the terms of a 'considerably more random' sequence. This routine uses both IRND and RND, generates 'super' random numbers and, due to the slow execution speed, should be used only in cases where no regular random number generator will do.

## File Name

"RSUPER"

## Calling Syntax

CALL Random\_super (N,X(\*) )

## Input Parameters

N                                      number of random deviates desired.

## Output Parameters

X(\*)                                      array of dimension (1:N) containing N uniformly generated random numbers on the range (0,1).

## Algorithm

This method has been suggested by MacLauren and Marsaglia (Ref. 1). Given methods for generating two pseudo-random sequences  $x_n$  and  $y_n$ , this routine will output terms of a 'considerably more random' sequence.

A temporary table  $V(1:100)$  is used as well as the modulus  $M$  used in the generation of sequence  $y_n$ .

1. Fill table  $V$  with the first 100 elements of sequence  $X_n$ .
2. Set  $X, Y$  equal to the next numbers of the sequences  $X_n, Y_n$  respectively.
3. Set  $J = \text{INT}(101 * Y / M + 1)$
4. Output  $V(J)$  and set  $V(J) = X$ .  
Go to step 2.

In our routine, sequence  $X_n$  is generated using RND and sequence  $Y_n$  using IRND.

Knuth contends that the sequence obtained by applying this algorithm will satisfy virtually anyone's requirements for randomness in a computer generated sequence.

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Vol. II, Seminumerical Algorithms Reading, Mass.: Addison-Wesley, 1969, p. 30.

## Special Considerations

1. As a result of our own tests, this generator comes highly recommended. It performed extremely well on all of our tests of randomness. In terms of execution speed and storage space, it is approximately three times as slow as either RND or IRND alone, plus it requires an extra 800 or so bytes for storage of the temporary array.
2. In using this routine, it is suggested that as many random deviates be generated on one call as is possible. Each time the subprogram is entered, 101 new table values are created.
3. If you are interested in repeatability of an experiment, remember that initial seeds must be set for both IRND (using SEED) and RND (using RANDOMIZE).
4. If you plan on calling this routine a large number of times, a significant amount of time would be saved if the table V is set up once in your calling routine and then passed as an additional parameter to Random\_super. This will avoid the overhead of redoing this table each time you enter the routine.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.
  - a. Type: LOAD "RSUPER",10
  - b. Press: EXECUTE
3. The title "SUPER RANDOM NUMBER GENERATOR" is printed along with options available to you in the program.
4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.

or

  - a. Enter 2 if you would like to check through some randomly generated examples using a standard Chi-square test.
  - b. Press: CONT
  - c. Go to 5.

5. Default values are supplied for the size of the sets of deviates, and number of categories to be used in the Chi-square test.
  - a. If the default values are satisfactory, simply press: CONT
  - or
  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates with different starting seeds are produced and a Chi-square test is performed for each data set. The starting seed, the Chi-square statistic and the right-tailed probability are printed.
7. When “WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED. E.G. 100, PI” is displayed:
  - a. Enter values for the size of the set and the starting seed.
  - b. Press: CONT
9. After the data set has been created, the prompt: “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.
  - or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.

10. When “ENTER THE SELECT CODE OF THE PRINTER.” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER.” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.  
or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
13. When “ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8” is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

SUPER RANDOM NUMBER GENERATOR

This program tests the super random number generator for randomness. This generator combines the two generators RND and IRND to produce a significantly more uniform pseudo-random sequence. It takes approximately 800 extra bytes and is about three times as slow as IRND.

The program may be used in the following ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

A series of Chi-square tests will be run on the generator. You will be required to enter values for the number of observations and the number of categories to be used in the Chi-square tests. Default values are provided for each input.

NUMBERS OF OBSERVATIONS: 1000  
NUMBER OF CATEGORIES: 100

The resulting Chi-square statistic and the corresponding right-tailed probability will be printed for each test.

```
# OF OBSERVATIONS: 1000
Seed: .622190890248      V:   79.4      Prob (X>V): .926
Seed: .431461189657      V:  113.4      Prob (X>V): .153
Seed: .086158972901      V:  106.8      Prob (X>V): .278
```

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the total number of observations and the starting seed. For want of a better seed, PI/180 is an excellent choice!

SUPER UNIFORM RANDOM DEVIATES:

Seed: .017453292520

| I  | X(I)    | X(I+1)  | X(I+2)  | X(I+3)  | X(I+4)  |
|----|---------|---------|---------|---------|---------|
| 1  | .907840 | .724974 | .908970 | .832369 | .249594 |
| 6  | .661944 | .447893 | .358119 | .703196 | .562812 |
| 11 | .523675 | .874582 | .987723 | .790323 | .383725 |
| 16 | .712651 | .815226 | .444412 | .170364 | .245539 |
| 21 | .171834 | .281558 | .178072 | .603049 | .399430 |





**(RT)**  
**Random Numbers Generated**  
**From a T-Distribution**





4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and the T parameter.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates are produced for each T parameter. The starting seed, mean, and variance are printed as well as the expected mean and variance.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED, DEGREE OF FREEDOM. E.G., 10,PI,5" is displayed:
  - a. Enter values for the size of the set, the starting seed, and the T parameter.
  - b. Press: CONT

9. When “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When “ENTER THE SELECT CODE OF THE PRINTER.” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When “ENTER THE BUS ADDRESS OF THE PRINTER.” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is then printed out on the appropriate device.
12. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. The program is terminated at this point.
13. When “ENTER FILE NAME. FOR EXAMPLE: File:T14 OR DATA:F8” is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector, and the program ends.

RANDOM T DEVIATES

This program generates sets of T(V) random deviates. You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of T(V) random deviates. It also compares the resulting data with the theoretical expectations. In particular, given V degrees of freedom, the mean and variance of each set of T random numbers is calculated.

The expected value of the mean is: 0

The expected value of the variance is:  $V/(V-2)$  for  $V > 2$ .

The program is set up using the following default values:

1. 50 deviates will be generated for each T parameter.
2. Values for V, the degrees of freedom, are set at 5, 10, and 50.
3. Results printed are:
  - mean, expected mean, and standard deviation for each set.
  - variance and expected variance for each set.

Default values may be changed from the keyboard as they come up.

# OF RANDOM DEVIATES: 1000  
DEGREES OF FREEDOM: 5, 10, 50

V: 5

|           |               |                    |       |                      |
|-----------|---------------|--------------------|-------|----------------------|
| Seed:     | .225190093254 |                    |       |                      |
| Mean:     | .044          | Expected Mean:     | 0.000 | Standard Dev.: 1.359 |
| Variance: | 1.846         | Expected Variance: | 1.667 |                      |
| Seed:     | .384868426712 |                    |       |                      |
| Mean:     | .043          | Expected Mean:     | 0.000 | Standard Dev.: 1.245 |
| Variance: | 1.551         | Expected Variance: | 1.667 |                      |
| Seed:     | .674346863484 |                    |       |                      |
| Mean:     | .041          | Expected Mean:     | 0.000 | Standard Dev.: 1.333 |
| Variance: | 1.778         | Expected Variance: | 1.667 |                      |

V: 10

|           |               |                    |       |                |       |
|-----------|---------------|--------------------|-------|----------------|-------|
| Seed:     | .125712189059 |                    |       |                |       |
| Mean:     | -.033         | Expected Mean:     | 0.000 | Standard Dev.: | 1.111 |
| Variance: | 1.234         | Expected Variance: | 1.250 |                |       |
| Seed:     | .723950998653 |                    |       |                |       |
| Mean:     | -.032         | Expected Mean:     | 0.000 | Standard Dev.: | 1.149 |
| Variance: | 1.319         | Expected Variance: | 1.250 |                |       |
| Seed:     | .842789866178 |                    |       |                |       |
| Mean:     | -.043         | Expected Mean:     | 0.000 | Standard Dev.: | 1.079 |
| Variance: | 1.164         | Expected Variance: | 1.250 |                |       |

V: 50

|           |               |                    |       |                |       |
|-----------|---------------|--------------------|-------|----------------|-------|
| Seed:     | .786277456806 |                    |       |                |       |
| Mean:     | -.016         | Expected Mean:     | 0.000 | Standard Dev.: | .996  |
| Variance: | .993          | Expected Variance: | 1.042 |                |       |
| Seed:     | .259341174646 |                    |       |                |       |
| Mean:     | -.020         | Expected Mean:     | 0.000 | Standard Dev.: | 1.060 |
| Variance: | 1.123         | Expected Variance: | 1.042 |                |       |
| Seed:     | .105927877294 |                    |       |                |       |
| Mean:     | .023          | Expected Mean:     | 0.000 | Standard Dev.: | 1.051 |
| Variance: | 1.104         | Expected Variance: | 1.042 |                |       |

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the degrees of freedom. For want of a better seed, PI is an excellent choice.

T DEViates:

Seed: .017453292520      V: 5.0000

| I  | X(I)      | X(I+1)    | X(I+2)    | X(I+3)    | X(I+4)   |
|----|-----------|-----------|-----------|-----------|----------|
| 1  | -.227298  | -1.009491 | .814428   | -.193426  | .685338  |
| 6  | .510023   | .042979   | -.685420  | -1.073209 | -.045602 |
| 11 | -.047114  | -.636961  | -1.055477 | .615446   | .171559  |
| 16 | -.974860  | -.579446  | 2.315553  | -.742191  | .090852  |
| 21 | -1.471052 | .110776   | -.853836  | -.596895  | .673159  |





**(RT1EXT)**  
**Random Type I Extreme-**  
**Value Generator**



# (RT1EXT) Random Type I Extreme-Value Generator

## Description

This program generates sets of random Type I Extreme-Value deviates. The distribution function is defined as follows:

$$F(X) = \exp(-\exp[-\text{Alpha}*(X - \text{Mu})])$$

## File Name

“RT1EXT”

## Calling Syntax

CALL Random\_type1ext (Number,Alpha,Mu,X(\*) ) .

## Input Parameters

Number                                    number of random deviates desired.  
Alpha,Mu                                    Type I parameters.

## Output Parameters

X(\*)                                        array of dimension (1:N) containing N Type I deviates.

## Algorithm

1. Given parameters Alpha and Mu, generate a uniform deviate U.
2. Then the Type II deviate is equal to:  $-\log[-\log(U)]/\text{Alpha} + \text{Mu}$ .

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2.    a. Type: LOAD “RT1EXT”,10  
      b. Press: EXECUTE
3. The title “RANDOM TYPE I EXTREME-VALUE DEVIATES” is printed along with some introductory remarks.
4. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, Alpha, Mu. E.G. 100,PI,10,.8” is displayed:
  - a. Enter values for the size of the set, the starting seed, and the Type I parameters Alpha and Mu.
  - b. Press: CONT

5. After the data set has been created, the prompt: “WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)” is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 6.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 8.
6. When “ENTER THE SELECT CODE OF THE PRINTER.” is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
7. When “ENTER THE BUS ADDRESS OF THE PRINTER.” is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
8. When “WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 9.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
9. When “ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8” is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM TYPE I EXTREME-VALUE DEVIATES

This program generates sets of Type I extreme-value deviates. The distribution function is:  $F(X) = \text{EXP}(-\text{EXPI}-\text{Alpha}*(X-\text{Mu})^3)$ .

In this program, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the parameters Alpha and Mu. For want of a better seed, PI is an excellent choice!

TYPE I EXTREME-VALUE DEVIATES:

|       |               |         |          |         |          |
|-------|---------------|---------|----------|---------|----------|
| Seed: | .017453292520 | Alpha:  | 10.000   | Mu:     | .800     |
| I     | X(I)          | X(I+1)  | X(I+2)   | X(I+3)  | X(I+4)   |
| 1     | .785932       | .713688 | 1.026475 | .796178 | .948806  |
| 6     | .932034       | .846248 | .673623  | .716250 | .831892  |
| 11    | .831019       | .713946 | .597968  | .927105 | 1.063444 |
| 16    | 1.000172      | .735676 | .927538  | .868322 | .713583  |
| 21    | .810417       | .921067 | .681207  | .857064 | .750104  |



**(RT2EXT)  
Random Type II  
Extreme-Value Generator**





# (RT2EXT) Random Type II Extreme-Value Generator

## Description

This program generates sets of random Type II Extreme-Value deviates. The distribution function is defined as follows:

$$F(X) = \exp[-(V/X)^K]$$

## File Name

'RT2EXT'

## Calling Syntax

CALL Random\_type2ext (Number,V,K,X(\*) )

## Input Parameters

Number                                    number of random deviates desired.  
V,K                                        Type II parameters.

## Output Parameters

X(\*)                                        array of dimension (1:N) containing N Type II deviates.

## Algorithm

1. Given parameters V and K, generate a uniform deviate U.
2. Then the Type II deviate is equal to:  $V * [-\log(U)]^{(-1/K)}$ .

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2. a. Type: LOAD "RT2EXT",10  
b. Press: EXECUTE
3. The title "RANDOM TYPE II EXTREME-VALUE DEVIATES" is printed along with some introductory remarks.
4. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED, V, K. E.G. 100, PI,10,.8" is displayed:
  - a. Enter values for the size of the set, the starting seed, and the Type II parameters V and K.
  - b. Press: CONT

5. After the data set has been created, the prompt: "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 6.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 8.
6. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
7. When "ENTER THE BUS ADDRESS OF THE PRINTER." is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
8. When "WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 9.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
9. When "ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8" is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM TYPE II EXTREME-VALUE DEVIATES

This program generates sets of Type II extreme-value deviates. The distribution function is:  $F(X) = \exp[-(V/X)^K]$ .

In this program, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the parameters V AND K. For want of a better seed, PI is an excellent choice!

TYPE II EXTREME-VALUE DEVIATES:

Seed: .017453292520

V: 10.000

K: .900

| I  | X(I)       | X(I+1)    | X(I+2)     | X(I+3)    | X(I+4)     |
|----|------------|-----------|------------|-----------|------------|
| 1  | 8.387473   | 3.399702  | 169.612721 | 9.533464  | 64.242076  |
| 6  | 52.091762  | 17.826373 | 2.060339   | 3.510349  | 14.749902  |
| 11 | 14.736392  | 3.410663  | .800267    | 48.979000 | 269.248818 |
| 16 | 331.868801 | 4.475137  | 49.244942  | 23.490015 | 3.395254   |
| 21 | 11.390763  | 45.418888 | 2.265232   | 20.467208 | 5.364942   |



**(RUNIF)**  
**Uniform Random Number Generator**





4. When "ENTER YOUR CHOICE. [EITHER 1 OR 2]" is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples using a standard Chi-square test.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and number of categories to be used in the Chi-square test.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. Three sets of deviates with different starting seeds are produced and a Chi-square test is performed for each data set. The starting seed, the Chi-square statistic and the right-tailed probability will be printed.
7. When "WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONTor
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When "ENTER VALUES FOR THE SIZE OF THE SET, SEED. E.G. 100, PI" is displayed:
  - a. Enter values for the size of the set, the starting seed, the mean of the distribution.
  - b. Press: CONT



9. After the data set has been created, the prompt: "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" is displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When "ENTER THE BUS ADDRESS OF THE PRINTER." is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
12. When "WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
13. When "ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8" is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

UNIFORM RANDOM NUMBER GENERATOR

This program generates sets of uniformly distributed random numbers in the range (0,1) using the generator IRND. The program may be used in the following ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

A series of Chi-square tests will be run on the generator using different number of observations. You will be required to enter values for the number of observations and the number of categories to be used in the Chi-square tests. Default values are provided for each input.

NUMBERS OF OBSERVATIONS: 1000  
NUMBER OF CATEGORIES: 100

The resulting Chi-square statistic and the corresponding right-tailed probability will be printed for each test.

```
# OF OBSERVATIONS: 1000
Seed: .842198893254      V:      82.2      Prob (X>V): .889
Seed: .281868427612     V:     110.8     Prob (X>V): .196
Seed: .051347621758     V:      98.6     Prob (X>V): .764
```

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the total number of observations and the starting seed. For want of a better seed,  $\pi/199$  is an excellent choice!

UNIFORM RANDOM DEVIATES:

Seed: .017453292520

| I  | X(I)    | X(I+1)  | X(I+2)  | X(I+3)  | X(I+4)  |
|----|---------|---------|---------|---------|---------|
| 1  | .316384 | .093430 | .981355 | .353822 | .797869 |
| 5  | .765638 | .532739 | .029050 | .099204 | .480575 |
| 11 | .480317 | .094802 | .000531 | .755377 | .930755 |
| 16 | .941101 | .149175 | .756294 | .603512 | .093198 |
| 21 | .406136 | .742304 | .037618 | .568268 | .192879 |

**(RWEIBU)**  
**Random Integers Generated From**  
**a Weibull Distribution**



# (RWEIBU) Random Integers Generated From a Weibull Distribution

## Description

This subprogram generates a set of Weibull deviates.

## File Name

“RWEIBU”

## Calling Syntax

CALL Random\_weibull (N,Alpha,Beta,X(\*) )

## Input Parameters

|             |                                    |
|-------------|------------------------------------|
| N           | number of random deviates desired. |
| Alpha, Beta | Weibull parameters.                |

## Output Parameters

|      |  |
|------|--|
| X(*) | array of dimension (1:N) containing deviates randomly generated with the given Weibull distribution. |
|------|--|

## Reference

1. Wheeler, R.E., “Random Variable Generators”, SIMULETTER, Vol. IV, April 1973, p. 22.

## Instructions

1. Insert the RANDOM NUMBER cartridge with the machine turned on.
2. a. Type: LOAD “RWEIBU”,10  
b. Press: EXECUTE
3. The title “RANDOM WEIBULL DEVIATES” is printed along with options available to you in the program.

4. When “ENTER YOUR CHOICE. [EITHER 1 OR 2]” is displayed:
  - a. Enter 1 if you want to directly generate a set of deviates.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter 2 if you would like to check through some randomly generated examples.
  - b. Press: CONT
  - c. Go to 5.
5. Default values are supplied for the size of the sets of deviates, and for the parameters.
  - a. If the default values are satisfactory, simply press: CONTor
  - a. Change the values as desired.
  - b. Press: CONT
6. One set of deviates with a different starting seed is produced for each set of parameters. After each data set is generated, the starting seed, the parameters, and the mean and variance of the set are printed as well as the expected mean and variance.
7. When “WOULD YOU LIKE TO CONSIDER A PARTICULAR DATA SET? (Y/N)” is displayed:
  - a. Enter Y if you would like to generate a particular set to be printed out or saved on a mass storage device.
  - b. Press: CONT
  - c. Go to 8.or
  - a. Enter N if no particular set is desired.
  - b. Press: CONT
  - c. At this point, the program is terminated.
8. When “ENTER VALUES FOR THE SIZE OF THE SET, SEED, Alpha, Beta. E.G., 100,PI,3.5” is displayed:
  - a. Enter values for the size of the set, the starting seed, and the appropriate parameters.
  - b. Press: CONT

9. After the data set has been created, the prompt: "WOULD YOU LIKE A PRINTOUT OF THE DATA? (Y/N)" will be displayed:
  - a. Enter Y if a printout is desired.
  - b. Press: CONT
  - c. Go to 10.or
  - a. Enter N if a printout is not desired.
  - b. Press: CONT
  - c. Go to 12.
10. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
11. When "ENTER THE BUS ADDRESS OF THE PRINTER." is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is printed on the appropriate device.
12. When "WOULD YOU LIKE TO SAVE YOUR DATA SET? (Y/N)" is displayed:
  - a. Enter Y if you would like it saved.
  - b. Press: CONT
  - c. Go to 13.or
  - a. Enter N if you would not like it saved.
  - b. Press: CONT
  - c. At this point, the program terminates.
13. When "ENTER FILE NAME. FOR EXAMPLE, File:T14 OR DATA:F8" is displayed:
  - a. Enter the desired file name.
  - b. Press: CONT
  - c. The data set is saved on your mass storage device as a vector and the program ends.

RANDOM WEIBULL DEVIATES                  

This program generates sets of Weibull (Alpha,Beta) random deviates from the distribution:

$$F(X) = 1-EXP[-X^{(Beta)}/Alpha].$$

You may use the program in two ways:

1. Directly generate a set of deviates to be printed or saved on a mass storage device.
2. Check through some randomly generated tests in order to get a feel for the quality of the deviates produced. You would then have the option to create your own data set.

This program generates sets of Weibull(Alpha,Beta) random deviates. It also compares the resulting data with the theoretical expectations. In particular, given parameters Alpha and Beta, the mean and variance of each set of random numbers is calculated.

The program is set up using the following default values:

1. 100 deviates will be generated for each set of Weibull parameters.
2. Values for Alpha and Beta, the Weibull parameters, are set at 2,5,10 and .5,1.5,2 respectively.
3. Results printed are:
  - mean, expected mean, and standard deviation for each set.
  - variance and expected variance for each set.

Default values may be changed from the keyboard as they come up.

# OF RANDOM DEVIATES: 1000  
ALPHA PARAMETERS: 2, 5, 10  
BETA PARAMETERS: .5, 1.5, 2

Number: 1000

|           |               |                    |         |                |        |
|-----------|---------------|--------------------|---------|----------------|--------|
| Seed:     | .082198896205 | Alpha:             | 2.00    | Beta:          | .50    |
| Mean:     | 7.942         | Expected Mean:     | 8.000   | Standard Dev.: | 16.388 |
| Variance: | 268.567       | Expected Variance: | 320.000 |                |        |

|           |               |                    |       |                |       |
|-----------|---------------|--------------------|-------|----------------|-------|
| Seed:     | .121868385812 | Alpha:             | 2.00  | Beta:          | 1.50  |
| Mean:     | 1.464         | Expected Mean:     | 1.433 | Standard Dev.: | 1.819 |
| Variance: | 1.039         | Expected Variance: | .947  |                |       |

|           |               |                    |       |                |      |
|-----------|---------------|--------------------|-------|----------------|------|
| Seed:     | .491312467983 | Alpha:             | 2.00  | Beta:          | 2.00 |
| Mean:     | 1.222         | Expected Mean:     | 1.253 | Standard Dev.: | .654 |
| Variance: | .428          | Expected Variance: | .429  |                |      |



```

Seed:      .193785573561      Alpha:    5.00      Beta:      .50
Mean:      58.847      Expected Mean:    58.000      Standard Dev.:196.463
Variance: 11334.458      Expected Variance: 12500.000

```

```

Seed:      .973667364881      Alpha:    5.00      Beta:      1.50
Mean:      2.563      Expected Mean:    2.640      Standard Dev.: 1.755
Variance:  3.079      Expected Variance:  3.212

```

```

Seed:      .854253863553      Alpha:    5.00      Beta:      2.00
Mean:      2.004      Expected Mean:    1.982      Standard Dev.: 1.100
Variance:  1.218      Expected Variance:  1.073

```

```

Seed:      .427499247862      Alpha:   10.00      Beta:      .50
Mean:     198.173      Expected Mean:   200.000      Standard Dev.:427.305
Variance:182589.804      Expected Variance:200000.000

```

```

Seed:      .526866779278      Alpha:   10.00      Beta:      1.50
Mean:      4.241      Expected Mean:    4.193      Standard Dev.: 2.879
Variance:  8.298      Expected Variance:  8.094

```

```

Seed:      .094961373904      Alpha:   10.00      Beta:      2.00
Mean:      2.776      Expected Mean:    2.882      Standard Dev.: 1.533
Variance:  2.351      Expected Variance:  2.146

```

At this point, you may select a particular data set which you can have printed out or saved on a mass storage device. You will be required to enter the following values: the size of the data set, the starting seed, and the two Weibull parameters. For want of a better seed, PI is an excellent choice!

#### WEIBULL DEVIATES:

```
Seed: .017453292520      Alpha:  5.0000      Beta:  1.5000
```

| I  | X(I)     | X(I+1)   | X(I+2)   | X(I+3)   | X(I+4)   |
|----|----------|----------|----------|----------|----------|
| 1  | 1.534687 | .621902  | 5.118788 | 1.683819 | 3.998067 |
| 6  | 3.747455 | 2.436985 | .279040  | .648624  | 2.285407 |
| 11 | 2.204292 | .624564  | .019178  | 3.673297 | 5.627718 |
| 16 | 5.852853 | .867328  | 3.679823 | 2.776147 | .626821  |
| 21 | 1.893490 | 3.582183 | .332489  | 2.683849 | 1.047058 |



# Tests for Randomness in BASIC





## Tests for Randomness in BASIC

A standard set of Statistical tests for randomness is provided. These tests are designed as independent subprograms with optional drivers. These driver programs have been set up to test the binary random number generator IRND for randomness. The aim here is twofold: i) to actually allow you to check the randomness of IRND; and ii) to show you how a typical test might be set up.



**(TCHISQ)  
Chi-square Test**





## (TCHISQ) Chi-square Test

### Description

This subprogram performs a Chi-Square test on a set of observations placed in a set of categories with given probabilities.

### File Name

“TCHISQ”

### Calling Syntax

Call Chi\_sq\_test (N,Cats,Prob(\*),Obs(\*),V,P)

### Input Parameters

|         |  |
|---------|--|
| N       | number of observations. This should be at least 5 *Cats, but preferably much larger, for a valid test.   |
| Cats    | number of categories.  |
| Prob(*) | array of dimension (1:Cats) containing the probabilities of any event occurring in a particular category. Care must be taken to insure that no probability value is too small. |
| Obs(*)  | array of dimension (1:Cats) containing the number of observations occurring in each category.  |

### Output Parameters

|   |   |
|---|---|
| V | Chi-square statistic. V is expected to have the Chi-square distribution with (Cats-1) degrees of freedom. |
| P | right-tailed probability; Prob (X>V).   |

### Special Considerations

1. The Chi-square method can only be used with sets of independent observations.
2. The proper choice of N is somewhat obscure. Large values of N will tend to smooth out ‘locally’ nonrandom behavior, that is, blocks of numbers with a strong bias followed by blocks of numbers with the opposite bias. But, N should be large enough so that each of the expected values  $N \cdot \text{Prob} \geq 5$  for the probability associated with each category. Preferably, N should be taken much larger than this. So, the method should probably be used with a number of different values of N.
3. From the Chi-square formula, we can see that a very small probability value would severely influence the Chi-square statistic. Hence, it is suggested that categories with very small probabilities be grouped together into one larger category.

## Algorithm

A fairly large number,  $N$ , of independent observations is made. We count the number of observations falling into each of  $K$  categories, and compute the quantity

$$V = (1/N) * \sum_{i=1}^K ( (\text{observed}(I) \wedge 2) / \text{Prob}(I) ) - N$$

In the associated driver program, the right-tailed probability  $P(X > V)$  is then calculated using  $(K-1)$  as the number of degrees of freedom.

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 35-40.

## Instructions

1. Insert the GOODNESS-OF-FIT cartridge with the machine turned on.
2.
  - a. Type: LOAD "TCHISQ",10
  - b. Press: EXECUTE
3. The title "CHI-SQUARE TEST" is printed along with an explanation of the test. The driver program is set up to test the uniform random number generator: IRND. Default values are provided for each input.
4. When "NUMBERS OF OBSERVATIONS?" is displayed:
  - a. Enter the number of observations desired for each set size.  
Three independent tests are run on each sized set.
  - b. Press: CONT
5. When "NUMBER OF CATEGORIES?" is displayed:
  - a. Enter the number of categories into which the data set is to be partitioned. For example, to check the randomness of the first digit, 10 categories would be sufficient; to check the first two digits, 100 categories would suffice.
  - b. Press: CONT
6. The resulting Chi-square statistic and the corresponding right-tailed probability is printed for each set.

CHI-SQUARE TEST

This program performs a Chi-square goodness of fit test on a given set of data. The random number generator IRND will be tested using different numbers of observations.

You will be required to enter the values for the number of observations to be used and for the number of corresponding categories. Three tests will be run at each level. Default values are provided for each input.

First enter the number of observations for each set. This has been set up as: 1000, 5000, and 10000. You may edit these values from the keyboard. Then press 'CONT'.

NUMBERS OF OBSERVATIONS: 1000, 5000, 10000  
NUMBER OF CATEGORIES: 100

The resulting Chi-square statistic and the corresponding right-tailed probability will be printed for each test.

|                    |               |             |       |
|--------------------|---------------|-------------|-------|
| # OF OBSERVATIONS: | 1000          |             |       |
| Seed:              | .432190093254 | V:          | 95.4  |
|                    |               | Prob (X>V): | .584  |
| Seed:              | .471868427612 | V:          | 113.4 |
|                    |               | Prob (X>V): | .153  |
| Seed:              | .841347620384 | V:          | 89.6  |
|                    |               | Prob (X>V): | .740  |
| # OF OBSERVATIONS: | 5000          |             |       |
| Seed:              | .573348741980 | V:          | 108.2 |
|                    |               | Prob (X>V): | .248  |
| Seed:              | .136292004613 | V:          | 100.4 |
|                    |               | Prob (X>V): | .441  |
| Seed:              | .671575879668 | V:          | 113.7 |
|                    |               | Prob (X>V): | .149  |
| # OF OBSERVATIONS: | 10000         |             |       |
| Seed:              | .795314801600 | V:          | 103.6 |
|                    |               | Prob (X>V): | .357  |
| Seed:              | .859748145692 | V:          | 102.9 |
|                    |               | Prob (X>V): | .374  |
| Seed:              | .048190527910 | V:          | 82.0  |
|                    |               | Prob (X>V): | .891  |



(TKS)  
**Kolmogorov-Smirnov Test**





## Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 41-48.

## Instructions

1. Insert the GOODNESS-OF-FIT cartridge with the machine turned on.
2. a. Type: LOAD "TKS",10  
b. Press: EXECUTE
3. The title "KOLMOGOROV-SMIRNOV TEST" is printed along with an explanation of the test. The driver program is set up to test the uniform random number generator: IRND. Default values are provided for each input.
4. When "# OF SETS TO BE GENERATED?" is displayed:
  - a. Enter the number of sets desired.
  - b. Press: CONT
5. The resulting individual K-S statistics are printed after each set of data is produced. The mean and standard deviation of the set of K-S statistics is then calculated. Finally, a K-S test is then applied to the above individual statistics.



KOLMOGOROV-SMIRNOV TEST

This program uses the Kolmogorov-Smirnov Goodness-of-Fit test to evaluate the randomness of the uniform generator IRND. The test is set up as follows:

1. Sets of 1000 uniform deviates are generated.
2. For each set, the K-S statistics are calculated.
3. The means and standard deviations of the set of K-S statistics are also calculated.
4. The K-S test is then applied to the above K-S statistics. For large N, say N=1000, the distribution of the K-S statistics is closely approximated by:

$$F(X) = 1 - \text{EXP}(-2 * X * X) \quad \text{for } X \geq 0.$$

This method of using several tests for moderately sized N, then combining the observations later in another K-S test, will tend to detect both local and global nonrandom behavior.

# OF SETS TO BE GENERATED: 10

INDIVIDUAL K-S STATISTICS:

|     |                     |                  |                 |
|-----|---------------------|------------------|-----------------|
| # 1 | Seed: .872198093254 | Kp(1000)= .7894  | Kn(1000)= .5041 |
| # 2 | Seed: .511868427612 | Kp(1000)= .2272  | Kn(1000)= .9500 |
| # 3 | Seed: .481347620384 | Kp(1000)= .9574  | Kn(1000)= .2105 |
| # 4 | Seed: .813348741980 | Kp(1000)= .8219  | Kn(1000)= .5993 |
| # 5 | Seed: .026292004601 | Kp(1000)= .4776  | Kn(1000)= .5867 |
| # 6 | Seed: .111575869768 | Kp(1000)= .8574  | Kn(1000)= .2157 |
| # 7 | Seed: .835386474800 | Kp(1000)= .9830  | Kn(1000)= .1758 |
| # 8 | Seed: .492745386887 | Kp(1000)= .3526  | Kn(1000)= .7170 |
| # 9 | Seed: .398883091313 | Kp(1000)= 1.3983 | Kn(1000)= .2761 |
| #10 | Seed: .398399794989 | Kp(1000)= .4465  | Kn(1000)= .5556 |

MEAN & SD OF INDIVIDUAL TESTS:

|           |             |           |
|-----------|-------------|-----------|
| Kp(1000): | Mean= .7143 | Sd= .3463 |
| Kn(1000): | Mean= .4711 | Sd= .2526 |

COMBINED K-S STATISTICS:

|         |           |           |
|---------|-----------|-----------|
| Kp(10): | Kp= .1894 | Kn= .7621 |
| Kn(10): | Kp= .9892 | Kn= .1896 |



**(TMAXT)**  
**Maximum of T Test**



## (TMAXT) Maximum of T Test

### Description

This routine generates groups of uniform random numbers, finds the maximum of each group and then applies the Kolmogorov-Smirnov test to the resulting set of numbers.

### File Name

“TMAXT”

### Calling Syntax

CALL Max\_of\_t (N,T,Knp,Knn)

### Input Parameters

N                                    number of groups to be tested.  
T                                    size of each group.

### Output Parameters

Knp                                    positive Kolmogorov-Smirnov statistic.  
Knn                                    negative Kolmogorov-Smirnov statistic.

### Algorithm

For  $0 \leq j < n$ , let  $V_j = \max(U_{tj}, U_{tj+1}, \dots, U_{tj+t-1})$  where the  $U$ 's are uniformly distributed random numbers.

Now apply the Kolmogorov-Smirnov test to the sequence  $V_0, V_1, \dots, V_{n-1}$ , with the distribution function  $F(x) = X \wedge t$ , ( $0 \leq X \leq 1$ ).

### Reference

1. Knuth, Donald E., The Art of Computer Programming, Vol. II, Seminumerical Algorithms, Reading, Mass.: Addison-Wesley, 1969, p. 64.

### Instructions

1. Insert the GOODNESS-OF-FIT cartridge with the machine turned on.
2.    a. Type: LOAD “TMAXT”,10  
      b. Press: EXECUTE

3. The title "MAXIMUM OF T TEST" is printed along with an explanation of the test. The driver program is set up to test the uniform random number generator: IRND. Default values are provided for each input.
4. When "# OF GROUPS?" is displayed:
  - a. Enter the number of groups desired. This should be at least 200 for a reasonable test.
  - b. Press: CONT
5. When "SIZE OF EACH GROUP?" is displayed:
  - a. Enter the desired size of each group.
  - b. Press: CONT
6. When "INITIAL SEED?" is displayed:
  - a. Enter an initial seed. For want of a better value,  $\text{PI}/180$  is an excellent choice.
  - b. Press: CONT
7. The resulting K-S statistics are printed for each data set.

MAXIMUM-OF-T TEST

In the Maximum of T test, we take groups of random numbers from the sequence of numbers generated by IRND. We then look at the maximum value in each of these groups using the Kolmogorov-Smirnov test. The distribution function used is:  $F(X) = X^T$  where T is the number of elements in each group.

You will be required to enter the number of groups you would like to test as well as the size of each group. The program will supply a reasonable default value for each of these inputs. Three tests will then be performed.

```
# OF GROUPS: 200
SIZE OF EACH GROUP: 5
```

|                     |            |            |
|---------------------|------------|------------|
| Seed: .400190093254 | Knp: .4261 | Knn: .5414 |
| Seed: .241968427612 | Knp: .9818 | Knn: .1992 |
| Seed: .411347621744 | Knp: .3553 | Knn: .9431 |

**(TPOKER)  
Modified Poker Test**





## (TPOKER) Modified Poker Test

### Description

This subprogram calculates the number of distinct values in a given set of observations. A Chi-square test is then applied to the set of data.

### File Name

“TPOKER”

### Calling Syntax

CALL Poker\_test (K,N,Digits,V,P)

### Input Parameters

|        |  |
|--------|--|
| K      | number of possible different digits in a set. The degrees of freedom is then (K-1). A reasonable number here is 5.     |
| N      | number of test sets to be used. N should be at least 5*(K-1), but preferably much larger, for a valid Chi-square test. |
| Digits | range on the allowed digits, [0,Digits-1]. 13 or 10 would be reasonable values here.                                   |

### Output Parameters

|   |  |
|---|--|
| V | Chi-square statistic. V is expected to have the Chi-square distribution with (K-1) degrees of freedom. |
| P | right-tailed probability; Prob (X>V).  |

### Algorithm

In general, we look at n groups of k successive numbers. We count the number of k-tuples with r different values. For example, generate 1000 groups of 5 successive numbers, where the numbers range from 1 to 13. How many sets have all 5 numbers different? How many have 4 different? How many 3? 2? 1?

A Chi-square test is then made, using the probability

$$P(r) = d*(d-1)*...*(d-r+1)/(d^k)*S(k,r)$$

where d is the number of possible digits considered and S(k,r) is the standard Sterling number of k,r.

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 57-58.

## Instructions

1. Insert the GOODNESS-OF-FIT cartridge with the machine turned on.
2. a. Type: LOAD "TPOKER",10  
b. Press: EXECUTE
3. The title "MODIFIED POKER TEST" is printed along with an explanation of the test. The driver program is set up to test the uniform random number generator: IRND. Default values are provided for each input.
4. You will be required to enter a starting and ending value for the number of groups desired, as well as the increment between values. At each value, three independent tests are run.
5. When "STARTING VALUE: # OF GROUPS?" is displayed:
  - a. Enter the starting value for the number of groups desired.
  - b. Press: CONT
6. When "FINAL VALUE: # OF GROUPS?" is displayed:
  - a. Enter the final value for the number of groups desired.
  - b. Press: CONT
7. When "INCREMENT VALUE?" is displayed:
  - a. Enter the increment value.
  - b. Press: CONT
8. The number of possible digits has been set at 10 and the size of each hand has been set equal to 5.
9. The resulting Chi-square statistic and the corresponding right-tailed probability is printed for each set.

**(TRUNS)  
Runs Test**



# (TRUNS)

## Runs Test

### Description

This subprogram sets up  $N$  random numbers and calculates the number of ascending or descending runs in the sequence. A special Chi-square statistic is then produced.

### File Name

“TRUNS”

### Calling Syntax

CALL Runs-test (N,Direction,V,P)

### Input Parameters

$N$  number of random deviates used. The value of  $N$  should be 4000 or more.

Direction Direction = 1 => ascending run.  
Direction = -1 => descending run.

### Output Parameters

$V$  Chi-square statistic. Since adjacent runs are not independent, a standard Chi-square test cannot be used here. A special test, with 6 degrees of freedom is used instead.

$P$  Right-tailed probability; Prob ( $X > V$ ).

### Algorithm

In this algorithm, we examine the length of monotone subsequences of an original sequence of random numbers; that is, segments which are increasing or decreasing.

1. Calculate the increasing (or decreasing) run lengths and count how many runs have length 1, 2, ..., 6 or greater.
2. Since adjacent runs are not independent, we cannot apply a standard Chi-square test to the above data. Instead, we calculate a special statistic  $V$  (see Ref. 1, p. 61) which should have the Chi-square distribution with six degrees of freedom, when  $N$  is large. The value of  $N$  should be at least 4000 for a valid test. This test may also be used for decreasing runs.

### Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 60-61.

## Instructions

1. Insert the GOODNESS-OF-FIT cartridge with the machine turned on.
2.
  - a. Type: LOAD "TRUNS",10
  - b. Press: EXECUTE
3. The title "RUNS TEST" is printed along with an explanation of the test. The driver program is set up to test the uniform random number generator: IRND. Default values are provided for each input.
4. When "ENTER 1 FOR RUNS UP; -1 FOR RUNS DOWN." is displayed:
  - a. Enter 1 for a runs up test.
  - b. Press: CONTor
  - a. Enter -1 for a runs down test.
  - b. Press: CONT
5. When "STARTING VALUE: NUMBER OF OBSERVATIONS?" is displayed:
  - a. Enter the starting value for the number of observations.
  - b. Press: CONT
6. When "FINAL VALUE: NUMBER OF OBSERVATIONS?" is displayed:
  - a. Enter the final value for the number of observations.
  - b. Press: CONT
7. When "INCREMENT VALUE?" is displayed:
  - a. Enter the increment value.
  - b. Press: CONT
8. The resulting special Chi-square statistic and the corresponding right-tailed probability is printed for each set.

### RUNS TEST

This program tests the random number generator IRND for sequences of 'runs up' and 'runs down'. This means we examine the length of monotone subsequences of the original sequence, that is, segments which are increasing or decreasing.

Since a long run will tend to be followed by a short run, and vice versa, adjacent runs are not independent. Hence, we should not apply the standard Chi-Square test to the sampled data. Instead, a special statistic is generated which has the Chi-Square distribution with six degrees of freedom, when the number of samples is large, say, 4000, or more. (See the user's manual for further details.)

This program is set up to perform a series of runs tests. Simply enter the starting and ending values for the number of observations and the incremental value between tests. So, for example, a starting value of 4000, an ending value of 10000, and an increment of 1000, would generate tests on 4000 observations, 5000 observations, ..., and 10000 observations. At each level, 3 tests will be run. Default values are provided for each input.

RUNS UP

STARTING VALUE: # OF OBSERVATIONS = 4000

FINAL VALUE: # OF OBSERVATIONS = 10000

INCREMENT VALUE = 1000

# of Observations: 4000

|                     |         |                     |
|---------------------|---------|---------------------|
| Seed: .723190093254 | V: 2.9  | Prob (X>V): .823    |
| Seed: .202868427612 | V: 7.8  | Prob (X>V): .252    |
| Seed: .612347621784 | V: 14.7 | Prob (X>V): .023 ** |



# of Observations: 5000

|                     |        |                  |
|---------------------|--------|------------------|
| Seed: .984349919381 | V: 7.4 | Prob (X>V): .287 |
| Seed: .838282198047 | V: 4.8 | Prob (X>V): .565 |
| Seed: .995328558683 | V: 1.6 | Prob (X>V): .952 |

# of Observations: 6000

|                     |        |                  |
|---------------------|--------|------------------|
| Seed: .071317852203 | V: 5.4 | Prob (X>V): .500 |
| Seed: .978313702577 | V: 6.4 | Prob (X>V): .383 |
| Seed: .761823866397 | V: 5.6 | Prob (X>V): .473 |

# of Observations: 7000

|                     |        |                  |
|---------------------|--------|------------------|
| Seed: .693871639521 | V: 7.0 | Prob (X>V): .323 |
| Seed: .546048036977 | V: 3.2 | Prob (X>V): .778 |
| Seed: .227071896716 | V: 8.6 | Prob (X>V): .200 |

# of Observations: 8000

|                     |         |                    |
|---------------------|---------|--------------------|
| Seed: .967465138085 | V: 11.0 | Prob (X>V): .089 * |
| Seed: .638181130590 | V: 2.7  | Prob (X>V): .850   |
| Seed: .710330825952 | V: 6.1  | Prob (X>V): .411   |

# of Observations: 9000

|                     |        |                  |
|---------------------|--------|------------------|
| Seed: .389224625493 | V: 6.8 | Prob (X>V): .343 |
| Seed: .496910039714 | V: 7.4 | Prob (X>V): .283 |
| Seed: .901343399391 | V: 2.5 | Prob (X>V): .873 |

# of Observations: 10000

|                     |         |                     |
|---------------------|---------|---------------------|
| Seed: .029798886706 | V: 14.9 | Prob (X>V): .021 ** |
| Seed: .060863719607 | V: 2.9  | Prob (X>V): .823    |
| Seed: .106388189147 | V: 9.7  | Prob (X>V): .136    |





**(TSERAL)  
Serial Test**



## (TSERAL) Serial Test

### Description

This subprogram tests whether pairs of successive numbers are uniformly distributed in an independent manner.

### File Name

“TSERAL”

### Calling Syntax

CALL Serial\_test (N,D,D\_squared,V,P)

### Input Parameters

N                    number of uniform random numbers to be tested.  
 D                    number of digits permitted. 5 or 10 is a reasonable number here.  
 D\_squared             $D * D$ ; this must be passed as a parameter to allow for dynamic allocation of arrays.

### Output Parameters

V                    Chi-square statistic. V is expected to have the Chi-square distribution with  $(D * D - 1)$  degrees of freedom.  
 P                    right-tailed probability;  $\text{Prob}(X > V)$ .

### Algorithm

Given n =            total number of uniform random numbers.  
       d =            number of digits permitted; that is, the deviates created are used to create integers 1,2,..., d.  
       y<sub>j</sub> =            jth random integer.

Then for each pair of integers (q,r) with  $0 \leq q, r < d$  count the number of times the pair

$(y_{2j}, y_{2j+1}) = (q, r)$  occurs, for  $0 \leq j < n$ .

Finally, apply the Chi-square test to these  $k = d * d$  equi-probable categories with probability  $1 / (d * d)$  in each case.

## Special Considerations

1. The number of digits permitted may be chosen as any convenient number. But care must be taken since a valid Chi-square test should have  $n$  large compared to  $k$ ; that is,  $n > 5 * d * d$  at least.

So, if

$d = 10$     then     $n > 500$

$d = 20$     then     $n > 2000$

etc.

2. This test may easily be adapted to triples, quadruples, etc., instead of pairs. But the value of  $d$  must be severely limited in order to avoid having too many categories. Frequently, in this case, less exact tests, such as the poker test or the maximum of  $t$  test are used instead.

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Volume 2 Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 55-56.

## Instructions

1. Insert the GOODNESS-OF-FIT cartridge with the machine turned on.
2.
  - a. Type LOAD "TSERAL",10
  - b. Press: EXECUTE
3. The title "SERIAL TEST" is printed along with an explanation of the test. The driver program is set up to test the uniform random number generator: IRND. Default values are provided for each input.
4. You are required to enter a starting and ending value for the number of groups desired, as well as the increment between values. At each value, three independent tests are run.
5. When "STARTING VALUE: # OF GROUPS?" is displayed:
  - a. Enter the starting value for the number of groups desired.
  - b. Press: CONT
6. When "FINAL VALUE: # OF GROUPS?" is displayed:
  - a. Enter the final value for the number of groups desired.
  - b. Press: CONT
7. When "INCREMENT VALUE?" is displayed:
  - a. Enter the increment value.
  - b. Press: CONT

8. When “# OF DIGITS?” is displayed:
  - a. Enter the number of digits allowed. For example, if you enter 10, the digits 1,2,...,10 is used. This also sets the degrees of freedom in the Chi-square test to digit\*digit-1.
  - b. Press: CONT
9. The resulting Chi-square statistic and the corresponding right-tailed probability is printed for each set.

### SERIAL TEST

The Serial Test checks that pairs of successive numbers are uniformly distributed in an independent manner. The Chi-square test statistic, as well as the associated probability, are the resultant products.

This driver program is set up to perform a number of test runs on the uniform random number generator, IRND. For example, we can allow 5 possible digits, and perform the test with runs ranging in size from 200 to 1000, taking 3 cases at each size. To simplify the required data input, default values will be provided.

Tests of different sizes will be run. To automate the input, you will be asked the starting size value, the increment, and the ending size value. Again, these will default to appropriate values.

STARTING SIZE VALUE: 200  
 INCREMENTAL VALUE: 100  
 FINAL VALUE: 1000

In this test, we count the number of times a pair of random digits occurs. You will be required to enter the number of allowed digits. Care must be taken since a valid chi-square test should have a large number of runs compared to the number of permitted digits.

# OF DIGITS: 5                    DEGREES OF FREEDOM FOR CHI-SQUARE TEST: 24

SIZE OF TEST: 200

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .252190093254 | V: 28.8 | Prob (X>V): .230 |
| Seed: .091868426731 | V: 31.3 | Prob (X>V): .147 |
| Seed: .261346880884 | V: 31.0 | Prob (X>V): .154 |

SIZE OF TEST: 300

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .792726823659 | V: 22.0 | Prob (X>V): .579 |
| Seed: .683258696977 | V: 23.0 | Prob (X>V): .520 |
| Seed: .620564156702 | V: 19.8 | Prob (X>V): .706 |

## SIZE OF TEST: 400

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .894455787660 | V: 24.4 | Prob (X>V): .440 |
| Seed: .237317420917 | V: 24.3 | Prob (X>V): .447 |
| Seed: .583950991105 | V: 24.9 | Prob (X>V): .413 |

## SIZE OF TEST: 500

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .102783518078 | V: 29.8 | Prob (X>V): .191 |
| Seed: .440938703522 | V: 16.0 | Prob (X>V): .888 |
| Seed: .829449660922 | V: 21.1 | Prob (X>V): .633 |

## SIZE OF TEST: 600

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .567164834301 | V: 22.8 | Prob (X>V): .535 |
| Seed: .985625646380 | V: 25.9 | Prob (X>V): .357 |
| Seed: .911168605043 | V: 13.3 | Prob (X>V): .962 |

## SIZE OF TEST: 700

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .292796839889 | V: 28.3 | Prob (X>V): .248 |
| Seed: .242142346608 | V: 30.9 | Prob (X>V): .156 |

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .641713497280 | V: 21.7 | Prob (X>V): .596 |
|---------------------|---------|------------------|

## SIZE OF TEST: 800

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .681051211991 | V: 25.1 | Prob (X>V): .399 |
| Seed: .764059283385 | V: 32.0 | Prob (X>V): .127 |
| Seed: .582267327809 | V: 19.6 | Prob (X>V): .721 |

## SIZE OF TEST: 900

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .686822688552 | V: 21.5 | Prob (X>V): .609 |
| Seed: .617881072057 | V: 23.9 | Prob (X>V): .465 |
| Seed: .637981601351 | V: 26.0 | Prob (X>V): .353 |

## SIZE OF TEST: 1000

|                     |         |                  |
|---------------------|---------|------------------|
| Seed: .542526736866 | V: 22.2 | Prob (X>V): .570 |
| Seed: .264985703975 | V: 18.7 | Prob (X>V): .770 |
| Seed: .862977042885 | V: 27.8 | Prob (X>V): .271 |

**(SPCTRL)  
Spectral Test**





# (SPCTRL) Spectral Test

## Description

This test is used in theoretically determining the value of coefficient A, given the word size of the computer, M, in the linear congruential model described in Appendix II. The value of A is crucial in setting up a good uniform random number generator. This is by far the most powerful test currently available on any sized machine. It tends to measure the statistical independence of adjacent n-tuples of numbers and is generally applied for N=2,3,4 and perhaps a few higher values of N.

## File Name

“SPCTRL”

## Calling Syntax

CALL Spectral (A,M,N,Info,Q,V,Cn)

## Input Parameters

|      |   |
|------|---|
| A    | the multiplier to be tested. It is essential that the linear congruential sequence be of maximal period.  |
| M    | modulus used in the model; in our case, $M=10^{12}$ , the word size of the computer. This cannot be changed if the binary random number generator IRND is being used.   |
| N    | size of n-tuple to be measured. This test is generally applied for N=2,3,4 and perhaps a few higher values of N.  |
| Info | intermediate information on program execution each time a particular section of code has been entered as well as total number of iterations required for convergence can be printed out at the user's option:<br>Info=1 => print out intermediate information.<br>Info=0 => do not print out the information. |

## Output Parameters

|    |   |
|----|---|
| Q  | $V \wedge 2$ .                                      |
| V  | smallest nonzero wave number in the spectrum.       |
| Cn | $= \frac{PI \wedge (N/2) * V \wedge N}{(N/2)! * M}$ |

## Special Considerations

1. Since BASIC string routines are used to perform the multi-precision arithmetic, this program is very slow.
2. The subprogram allows at most 12 digits for A and M. If larger numbers are desired, some parameters must be changed to strings before entering the routine.

```
Change:  SUB Spectral (A,M,N,Info,Q,V,Cn)
         DIM -----
         Coef$=VAL$(A)
         CALL Clean-up (Coef$)
         Base$=VAL$(M)
         CALL Clean-up (Base$)
         .
         .
         .
```

```
To:  SUB Spectral (Coef$,Base$,N,Info,Q,V,Cn)
```

The reason for this is that with the 9845 processor, a simple variable can hold at most 12 digits. All extra digits are lost.

3. As suggested in the literature, the driver has been set up for  $N=2,3,4,5,6$ .
4. The multi-precision arithmetic routines are set up as independent subprograms so that the user may apply them to other contexts as well. Presently, each of these routines allows for up to 90 digits of accuracy. This can be increased simply by changing the DIM statements at the beginning of each routine.

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Vol. II, Seminumerical Algorithms. Reading, Mass.: Addison-Wesley, 1969, p. 69-100.

## Instructions

1. Insert the GOODNESS-OF-FIT cartridge with the machine turned on.
2.
  - a. Type: LOAD "SPCTRL",10
  - b. Press: EXECUTE
3. When the title "SPECTRAL TEST" is printed and "HOW MANY PAIRS OF NUMBERS WOULD YOU LIKE TO CHECK?" is displayed:
  - a. Enter the desired number of pairs.
  - b. Press: CONT

---

### NOTE:

This test is quite slow. It is not unusual for it to run for a couple of hours with one pair.

---

4. When “A(I)?” and “M(I)?” are displayed:
  - a. Enter the desired A coefficient and the M word size.
  - b. Press: CONT
  - c. Repeat this step until all pairs have been entered.
5. When the pairs of numbers have been printed and “CHANGES?” (Y/N)” is displayed:
  - a. Enter Y if changes are desired.
  - b. Press: CONT
  - c. Go to 6.

or

  - a. Enter N if no changes are desired.
  - b. Press:CONT
  - c. Go to 8.
6. When “ENTER THE NUMBER OF THE PAIR TO BE CHANGED.” is displayed:
  - a. Enter the appropriate number.
  - b. Press: CONT
7. When “A(I)?” and “M(I)?” are displayed:
  - a. Enter the corrected values.
  - b. Press: CONT
  - c. Go to 5.
8. When “WOULD YOU LIKE THE INTERMEDIATE STEPS LISTED AS THEY ARE EXECUTED? (Y/N)” is displayed:
  - a. Enter Y if the intermediate steps are desired. This means that each time a particular section of code is entered, the label of that section is printed. Also, the total number of required iterations is output.
  - b. Press: CONT

or

  - a. Enter N if the listing is not desired.
  - b. Press: CONT
9. The program has been set up with n-tuples of size 2, 3, 4, 5 and 6. For each of these values, the quantity  $C_n$  is calculated. Large values of  $C_n$  correspond to randomness, small values correspond to nonrandomness. Knuth suggests that the multiplier A passes the spectral test if the  $C_n$  values are all greater than or equal to 0.1, and it passes the test with flying colors if all are greater than or equal to 1.

SPECTRAL TEST

A( 1): 314159262221      M( 1): 10000000000000  
 A( 2): 3954654621      M( 2): 10000000000000

|    |              |    |                |    |   |    |     |        |
|----|--------------|----|----------------|----|---|----|-----|--------|
| A: | 314159262221 | M: | 10000000000000 | N: | 2 | => | Cn: | 2.7079 |
| A: | 314159262221 | M: | 10000000000000 | N: | 3 | => | Cn: | 2.7397 |
| A: | 314159262221 | M: | 10000000000000 | N: | 4 | => | Cn: | 1.4861 |
| A: | 314159262221 | M: | 10000000000000 | N: | 5 | => | Cn: | 2.1204 |
| A: | 314159262221 | M: | 10000000000000 | N: | 6 | => | Cn: | 1.9864 |
| A: | 3954654621   | M: | 10000000000000 | N: | 2 | => | Cn: | .5075  |
| A: | 3954654621   | M: | 10000000000000 | N: | 3 | => | Cn: | .5234  |
| A: | 3954654621   | M: | 10000000000000 | N: | 4 | => | Cn: | .1800  |
| A: | 3954654621   | M: | 10000000000000 | N: | 5 | => | Cn: | .6891  |
| A: | 3954654621   | M: | 10000000000000 | N: | 6 | => | Cn: | .3263  |

# Elementary Sampling Techniques





## **Elementary Sampling Techniques in BASIC**

This section provides some elementary sampling and shuffling techniques. Independent subprograms with optional driver routines are provided.





**(SSEL)  
Selection Sampling**



## (SSEL) Selection Sampling

### Description

Given a set of  $N$  objects, this program will select  $n$  of them at random in an unbiased manner (a simple random style without replacement).

### File Name

“SSEL”

### Calling Syntax

CALL Sel\_sampling (T\_number,S\_number,X(\*) )

### Input Parameters

|          |                                     |
|----------|-------------------------------------|
| T_number | total number of records in the set. |
| S_number | number of records to be selected.   |

### Output Parameters

|      |  |
|------|--|
| X(*) | array of size (1:N) containing the index numbers of the records to be sampled. |
|------|--|

### Algorithm

To select  $n$  records at random from a set of  $N$ , where  $0 < n \leq N$ :

1. Set  $t=0, m=0$ .
2. Generate a random number  $U$ , uniformly distributed between zero and one.
3. If  $(N-t)*U \geq (n-m)$ , then go to step 5.  
Else go to step 4.
4. Select the next record index for the sample.  
 $m=m+1$ .  
 $t=t+1$ .  
 If  $m < n$  then go to step 2.  
 Else the sample is complete and the algorithm terminates.
5. Skip the next record index.  
 $t=t+1$ .  
 Go to step 2.

## Special Considerations

1. In order to avoid connections between samples obtained on different runs, care must be taken to use different starting seeds each time this program is run. Both IRND (using SEED) and RND (using RANDOMIZE) allow for this. The seed can either be initialized in the calling program or the subprogram itself.

A simple way of initializing different seeds for different runs is to do the following: use the digits from the month, day, and time that the program is run as the seed. For example, if you are running the program on June 19 at 9:47 am, then your seed would be .6190947.

## Reference

1. Knuth, Donald E., The Art of Computer Programming, Vol. II, Seminumerical Algorithms, Reading, Mass.: Addison-Wesley, 1969, p. 122.

## Instructions

1. Insert the GOODNESS-OF-FIT cartridge with the machine turned on.
2.
  - a. Type: LOAD "SSEL",10
  - b. Press: EXECUTE
3. The title "SELECTION SAMPLING TECHNIQUE" is printed along with an explanation of how the program works.
4. When "STARTING SEED?" is displayed:
  - a. Enter a starting seed for the random number calculations
  - b. Press: CONT
5. When "TOTAL # OF RECORDS IN SET?" is displayed:
  - a. Enter the total number of records in the set.
  - b. Press: CONT
6. When "# OF RECORDS TO BE SELECTED?" is displayed:
  - a. Enter the total number of records to be selected.
  - b. Press: CONT
7. When complete, the indices of the records to be chosen are printed. Since the actual records may be stored in a variety of formats, you are required to write your own routine to get the actual records themselves.

SELECTION SAMPLING TECHNIQUE

Given a set of N objects, this program will select n of them at random in an unbiased manner.

In order to avoid connections between samples obtained on different runs, care must be taken to use different starting seeds each time this program is used. A simple way of doing this is the following: use the month, day, hour, minute that you are starting the program as the initial seed. So, for example, if today were June 19 and it were 9:07am, then your starting seed would be 6190907.

STARTING SEED: 81478

TOTAL # OF RECORDS IN SET: 100

# OF RECORDS TO BE SELECTED: 13

RECORDS TO BE SELECTED:

15 16 20 21 22 49 57 67 73 78 88 89 90



**(SSHUFL)**  
**Shuffling**







3. The title "SHUFFLING" is printed along with an explanation of the options available to you in the program.
4. When "HOW MANY NUMBERS WOULD YOU LIKE TO SHUFFLE?" is displayed:
  - a. Enter the number of digits to be shuffled.
  - b. Press: CONT
5. When "HOW WOULD YOU LIKE TO ENTER YOUR DATA? [ENTER 1, 2 OR 3]" is displayed:
  - a. Enter 1 if each point is to be input individually.
  - b. Press: CONT
  - c. Go to step 6.  
or
    - a. Enter 2 if you wish to enter just the starting value, ending value, and the step size.
    - b. Press: CONT
    - c. Go to step 9.  
or
      - a. Enter 3 if the data is to be entered from a stored file. In this case, the data must be stored as a one-dimensional vector.
      - b. Press: CONT
      - c. Go to step 10.
6. When "X(I)?" is displayed (for I=1 to the number of points to be entered):
  - a. Enter X(I).
  - b. Press: CONT
  - c. Continue with this step until all your points have been entered.
  - d. Go to step 7.
7. When "WOULD YOU LIKE TO MAKE ANY CHANGES IN THE DATA: (Y/N)" is displayed:
  - a. Enter Y if you would like to make a change in the data.
  - b. Press: CONT
  - c. Go to step 8.  
or
    - a. Enter N if no changes are desired in the data.
    - b. Press: CONT
    - c. Go to step 11.

8. When “ENTER THE NUMBER OF THE ELEMENT TO BE CHANGED.” is displayed:
  - a. Enter the index number of the element to be changed.
  - b. Press: CONT
  - c. Enter the corrected value.
  - d. Press: CONT
  - e. Go to step 7.
  
9. When “ENTER STARTING VALUE, ENDING VALUE, STEP SIZE. [FOR EXAMPLE: 10,1000,10]” is displayed:
  - a. Enter a starting value, ending value and step size. In the example, these three values would replace entering: 10, 20, 30, ..., 1000.
  - b. Press: CONT
  - c. Go to step 11.
  
10. When “ENTER FILE NAME. [FOR EXAMPLE, File:T14 OR Data:F8]” is displayed:
  - a. Enter the name of the file from which you want to retrieve the data.
  - b. Press: CONT
  - c. The data is retrieved from your file.
  - d. Go to step 11.
  
11. After the data has been shuffled, the options available to you at this point in the program will be displayed. When “ENTER YOUR CHOICE. [EITHER 1, 2, or 3]” is displayed:
  - a. Enter 1 if you would like the shuffled data printed out.
  - b. Press: CONT
  - c. Go to step 12.  
or
  - a. Enter 2 if you would like the data saved on a mass storage device.
  - b. Press: CONT
  - c. Go to step 14.  
or
  - a. Enter 3 to end the program.
  - b. Press: CONT
  - c. The program is terminated.

12. When "ENTER THE SELECT CODE OF THE PRINTER." is displayed:
  - a. Enter the select code of the printer.
  - b. Press: CONT
  
13. When "ENTER THE BUS ADDRESS OF THE PRINTER." is displayed:
  - a. Enter the bus address of the printer.
  - b. Press: CONT
  - c. The data set is printed on the appropriate device.
  - d. Go to step 11.
  
14. When "ENTER THE FILE NAME. [FOR EXAMPLE: File:T14 OR DATA:F8]" is displayed:
  - a. Enter the file name on which you want the data stored.
  - b. Press: CONT
  - c. The data set is saved as a vector, and the program ends.

#### SHUFFLING

Given an array of numbers, this program will randomly shuffle the digits. You may enter your vector in any of the following manners:

1. Enter each point individually, for example: 5,2,6,4,2,1,5.
2. Enter the starting value, ending value, and the step size.  
For example: 1,10,1  
This would be equivalent to entering: 1,2,3,4,5,6,7,8,9,10
3. Enter the data from a stored file. In this case, the data must be stored as a one dimensional vector.

# OF ELEMENTS TO BE SHUFFLED: 52

STARTING VALUE: 1  
ENDING VALUE: 52  
STEP SIZE: 1

The data has been shuffled. The choices available now are:

1. Print the data out.
2. Store the data on a mass storage file.
3. End the program.

SHUFFLED DATA:

| I  | X(I)      | X(I+1)    | X(I+2)    | X(I+3)    | X(I+4)    |
|----|-----------|-----------|-----------|-----------|-----------|
| 1  | 34.000000 | 16.000000 | 24.000000 | 35.000000 | 41.000000 |
| 6  | 42.000000 | 50.000000 | 26.000000 | 9.000000  | 19.000000 |
| 11 | 2.000000  | 39.000000 | 48.000000 | 6.000000  | 36.000000 |
| 16 | 46.000000 | 51.000000 | 21.000000 | 11.000000 | 33.000000 |
| 21 | 14.000000 | 22.000000 | 17.000000 | 27.000000 | 44.000000 |
| 26 | 38.000000 | 52.000000 | 47.000000 | 7.000000  | 15.000000 |
| 31 | 45.000000 | 5.000000  | 3.000000  | 30.000000 | 8.000000  |
| 36 | 25.000000 | 29.000000 | 32.000000 | 49.000000 | 37.000000 |
| 41 | 1.000000  | 31.000000 | 43.000000 | 10.000000 | 23.000000 |
| 46 | 18.000000 | 13.000000 | 40.000000 | 4.000000  | 12.000000 |
| 51 | 20.000000 | 28.000000 |           |           |           |



# Appendix I

## 9845 Random Number Generator: RND

This generator uses a modification of the standard “multiplicative congruential generator”. In this generator, a starting value called the seed is multiplied by a positive integer constant, and the fractional part of the result becomes the new seed and the next random number in the generated sequence.

The algorithm used in the RND has a starting seed of  $\text{PI}/180=0.017453292520$ . This seed may be set by the program to any new value by using the RANDOMIZE statement.

In this routine, the value of 29 is used for the multiplier. To prevent undesired properties in the sequences of random numbers generated (i.e., pairwise correlation and non-random trailing digits), two modifications are made to the standard multiplicative congruential generator. First, the algorithm is performed twice before returning the next random number and setting the new seed. Second, two digits from the first application of the method are saved and appended to the end of the result of the second pass. The exact steps used in the algorithm, along with three examples, are presented below.

In general, the method used is not a very sophisticated one, and is intended only to provide a convenient source of random number sequences for such applications as game programs, random pattern generation, random testing sequences, and the like. Anyone doing sophisticated random statistical techniques or Monte Carlo evaluations would almost certainly want to use a random number generator that has been shown to have passed more severe tests for statistical randomness, such as IRND or RSUPER.

The algorithm below is the one used to generate the next random number in a sequence from the previous one (i.e., the seed) using RND:

1. Multiply the seed by 29.
2. Keep the fractional part of the result of step 1.
3. Counting from the first non-zero digit to the right of the decimal point, save digits 5 and 6 for use in step 6.
4. Multiply the result of step 2 by 29.
5. Keep the fractional part of the result of step 4.
6. Counting from the first non-zero digit to the right of the decimal point, replace digits 11 and 12 with the two digits saved in step 3.
7. Save the result of step 6 as the new seed, and return its value as the next random number in the sequence.

Below are three examples of the algorithm. In each case, the seeds chosen and the random numbers generated are from the actual sequence of numbers generated with a starting seed of  $\text{PI}/180$ .

| Seed:   | Example 1       | Example 2       | Example 3       |              |
|---------|-----------------|-----------------|-----------------|--------------|
| Step 1: | .017453292520   | .382186859051   | .315147299705   |              |
| Step 2: | .506145483080   | 11.083418912500 | 9.1392271691450 |              |
| Step 3: | .506145483080   | .083418912500   | .1392271681450  |              |
|         | 45 ←            | 89 ←            | 71 ←            | digits saved |
| Step 4: | 14.678219009300 | 2.4191484462500 | 4.038879052050  |              |
| Step 5: | .678219009300   | .419148462500   | .038879052050   |              |
| Step 6: | .678219009345   | .419148462589   | .0388790520571  |              |



# Appendix II



## Uniform Random Number Generator: IRND

By far the most successful random number generators known today are special cases of the Linear Congruential Generator Model. In this model, we choose

|       |                    |                           |
|-------|--------------------|---------------------------|
| $X_0$ | the starting value | $X_0 \geq 0$ .            |
| $A$   | the multiplier     | $A \geq 0$ .              |
| $C$   | the increment      | $C \geq 0$ .              |
| $M$   | the modulus        | $M > X_0, M > A, M > C$ . |

The desired sequence of random numbers  $X_n$  is then obtained by setting

$X_{n+1} = (A * X_n + C) \text{MOD } M, \quad n \geq 0$ . This is called a linear congruential sequence.

IRND is based on this linear congruential model. In designing IRND, the following choices were made:

1. The number  $X_0$ , the starting seed, has been set at  $\text{PI} / 180$ . This means that each time IRND is loaded into the machine,  $\text{PI} / 180$  is set as the seed. The statement SEED has been provided in the binary to allow you to change this starting value. Each time a new random number is generated, it becomes the new seed.
2. In order to make IRND as efficient as possible, the word size of the 9845 has been chosen as the modulus  $M$ . Hence,  $M = 10 \wedge 12$ . In the binary the computation of  $(A * X + C) \text{MOD } M$  is done exactly with no roundoff error. There is no way to change the value of  $M$ .
3. To ensure that the random number generator will produce all  $M$  different possible values of  $X$  before it starts to repeat and to ensure high potency,  $A$  has been chosen so that  $A \text{MOD } 200 = 21$ . In particular,  $A = 314159262221$ . If another value for  $A$  is desired, the ACOEF statement may be used. It is strongly suggested that  $A$  be tampered with only if you have an excellent understanding of the linear congruential model.
4. This choice of  $A$  along with  $M = 10 \wedge 12$  has passed the Spectral Test. A number of other values for the coefficient  $A$  have also done well on the test.

| <b>M = 10<sup>12</sup></b> |      |      | <b>Cn Values</b> |       |   |       |   |       |   |       |   |       |
|----------------------------|------|------|------------------|-------|---|-------|---|-------|---|-------|---|-------|
| <b>A-Coefficient</b>       |      |      | **               | N=2   | * | N=3   | * | N=4   | * | N=5   | * | N=6   |
| *****                      |      |      |                  |       |   |       |   |       |   |       |   |       |
| 3141                       | 5926 | 2221 | **               | 2.708 | * | 2.740 | * | 1.486 | * | 2.120 | * | 1.906 |
|                            |      |      | **               |       | * |       | * |       | * |       | * |       |
| 3954                       | 6546 | 5421 | **               | 2.778 | * | 1.407 | * | 4.505 | * | 1.250 | * | 3.735 |
|                            |      |      | **               |       | * |       | * |       | * |       | * |       |
| 3141                       | 3574 | 5221 | **               | 1.644 | * | 1.188 | * | 2.498 | * | 1.100 | * | 2.458 |
|                            |      |      | **               |       | * |       | * |       | * |       | * |       |
| 3147                       | 4376 | 9221 | **               | 2.006 | * | 1.663 | * | 1.270 | * | 1.095 | * | 1.568 |
|                            |      |      | **               |       | * |       | * |       | * |       | * |       |

5. The constant C has been chosen as 0.211324865407. This may be changed using the binary statement CCOEF.
6. The least significant (right-hand) digits of X are not very random, so decisions based on the number X should always be primarily influenced by the most significant digits.
7. For "high resolution" Monte Carlo applications, it is suggested that the super random number generator "RSUPER" which combines both IRND and RND be used.

