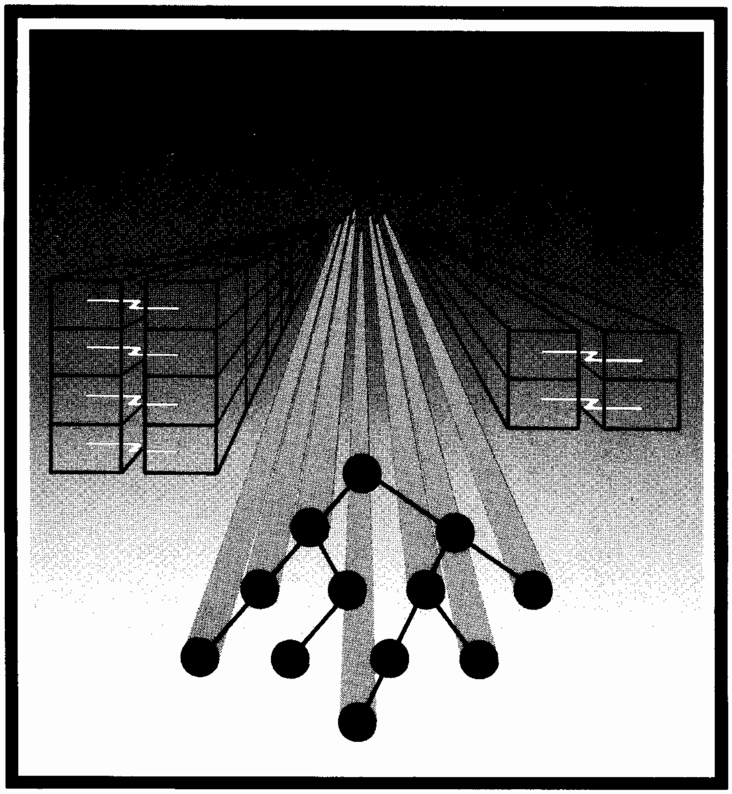
General Statistics

For the HP 9845



HEWLETT PACKARD

General Statistics

Part No. 09845-15131

Hewlett-Packard Desktop Computer Division
3404 East Harmony Road, Fort Collins, Colorado 80525

Printing History

New editions of this manual will incorporate all material updated since the previous edition. Update packages may be issued between editions and contain replacement and additional pages to be merged into the manual by the user. Each updated page will be indicated by a revision date at the bottom of the page. A vertical bar in the margin indicates the changes on each page. Note that pages which are rearranged due to changes on a previous page are not considered revised.

The manual printing date and part number indicate its current edition. The printing date changes when a new edition is printed. (Minor corrections and updates which are incorporated at reprint do not cause the date to change.) The manual part number changes when extensive technical changes are incorporated.

April 1979...Revision A.
May 1980...Revision B. Updated pages: 1
May 1981...Revision C. Updated pages: 292, 293, 294
August 1982...Updated pages: vii, 2 and 291. Deleted pages 292 thru 294.

Important

The tape cartridge or disc containing the programs is very reliable, but being a mechanical device, is subject to wear over a period of time. To avoid having to purchase a replacement medium, we recommend that you immediately duplicate the contents of the tape onto a permanent backup tape or disc. You should also keep backup copies of your important programs and data on a separate medium to minimize the risk of permanent loss.

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Commentary

The collection of statistical inferential procedures represents the latest addition to comprehensive data analysis programs we are developing for the Hewlett-Packard desktop computer systems. Included in these packages of programs are parametric and nonparametric procedures for analyzing one sample, two sample (both paired and independent), and multi-sample data sets. Almost all of these routines use the Data Manipulation routine as a "front end" for entering, structuring, and editing the data array.

Also included in this program package is a comprehensive collection of routines for generating probability or tabled values for a large number of statistical probability distributions. Many of the common continuous and discrete distributions are included in these routines. We have found that these routines are useful as a replacement for the usual tables located at the end of many standard statistical textbooks.

We are confident that you will find these routines useful in your data analysis. Please watch for further program packages to be introduced in the near future.

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Basic Statistics and Data Manipulation

Introduction

Description

This set of programs allows you to enter a data matrix into memory and to then perform various operations on the data. The data entry may be made via keyboard, mass storage, card reader, or other devices. The operations on the data set include editing, transforming, recoding, sorting, creating subfiles, naming, storing and listing.

Features include a provision for missing data values, a provision for incorporating a subfile structure, the ability to store the data matrix and related information, the ability to join two data sets, error detection, the ability to correct many possible errors, and YES / NO keys to speed program use.

More specific objectives and features are listed with the instructions for the individual programs.

System Configuration

Necessary:

9845B or 9845C opt. 100 Desktop Computer

Option 560	– Thermal line printer (or an external printer)

Optional:

Option 312	– I / O ROM for data entry from external devices
Option 311	– Graphics ROM (9845B only)
Option 600	– Secondary tape drive
Option 700	– Graphics Display Subsystem (9845B only)
9872A	– External plotter
9885M	– Flexible Disk Drive (requires Option 313 – Mass Storage ROM)
7900 Series	– Hard Disc (requires Option 313 – Mass Storage ROM and 98941A Disc Interface)

A sample data set is included on the tape in the scratch file "DATA". It is suggested that you first run through the various stat programs using this data set to familiarize yourself with the features available in this package. The simplest way to do this is to follow the user instructions contained in section "START".

Special Considerations

Data Matrix Configuration

The data matrix incorporated in this program should be thought of as a p by n array whose columns correspond to observations and whose rows correspond to variables as shown below.

OBSERVATIONS

		O_1	O_2	O_3	...	O_n
	V_1				...	
	V_2				...	
VARIABLES	V_3				...	
	.	.	.	.		.
	.	.	.	.		.
	.	.	.	.		.
	V_p				...	

Subfiles may be created, in which case the structure becomes only slightly more complex as shown below.

		SUBFILE 1	SUBFILE 2		SUBFILE S
		$O_1 O_2 \dots O_{n_1}$	$O_{n_1+1} O_{n_1+2} \dots O_{n_1+n_2}$	...	$O_{n_1+\dots+n_{s-1}+1} \dots O_{n_1+\dots+n_s}$
	V_1				
	V_2				
	.				
VARIABLES	.				
	.				
	V_p				

Limitations

The programs in this package have been designed to run on any 9845B regardless of memory size. They will operate on a maximum of 50 variables and a maximum of 20 subfiles. A total of 1500 data values may be input.

Missing Values

If a data value is missing, it can be so designated by pressing the special function key labeled M.V., when entering the data from the keyboard. The number used to designate a missing value is −9999999.99999. The justification for this number is that (besides seeming unlikely to occur as a legitimate data point) it is easily picked out in a listing of the data. If the input is to be via cards, it may be more desirable to designate a missing value by a more easily typed number, for example, by 0 if zero is not a legitimate data point. The zeros could then be converted to the missing value recognized by the programs via a user-defined transformation.

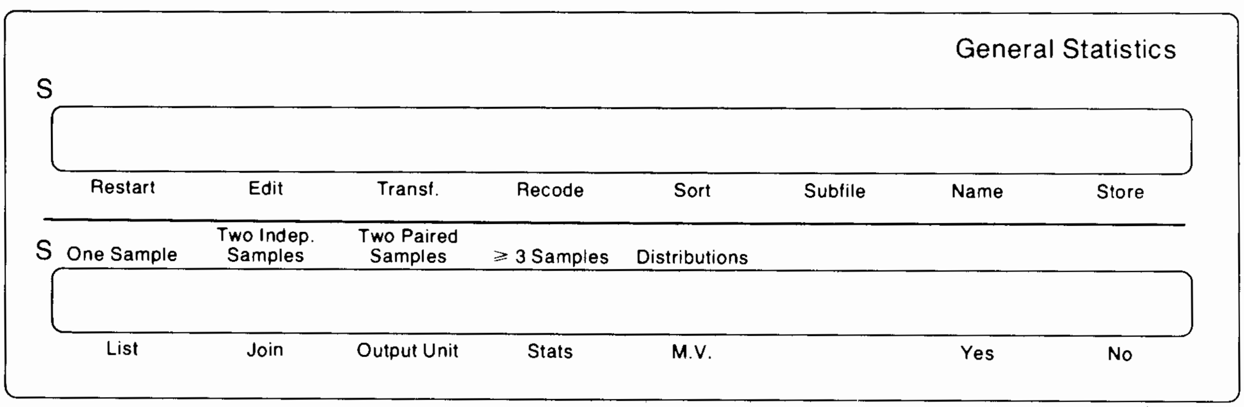
Incorrect Responses

If a response outside the range of plausible responses is input from the keyboard, a message so stating is displayed for about three seconds. Program execution is resumed by asking the question or a previous question again.

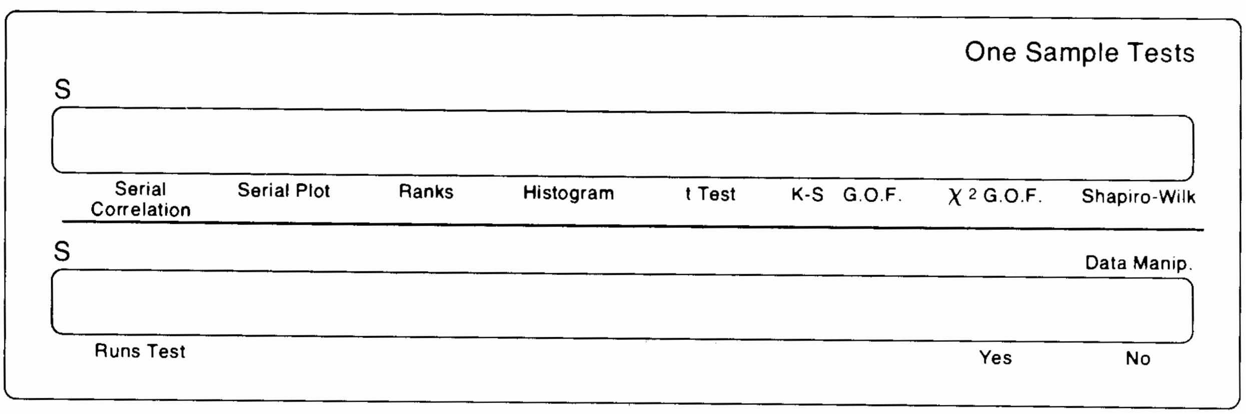
If a plausible response is given, but it is not correct, one of three possibilities exists. First, if an incorrect value has been entered for a data point, it may be corrected using the EDIT program. Second, in many cases, responses to several questions are printed on the CRT and then a question such as "Is the above information correct?" is asked. This allows any of the printed information to be changed. Third, if a YES / NO question is answered incorrectly or if the above options are not offered, the program can be re-started by pressing STOP, then RUN.

Special Function Keys

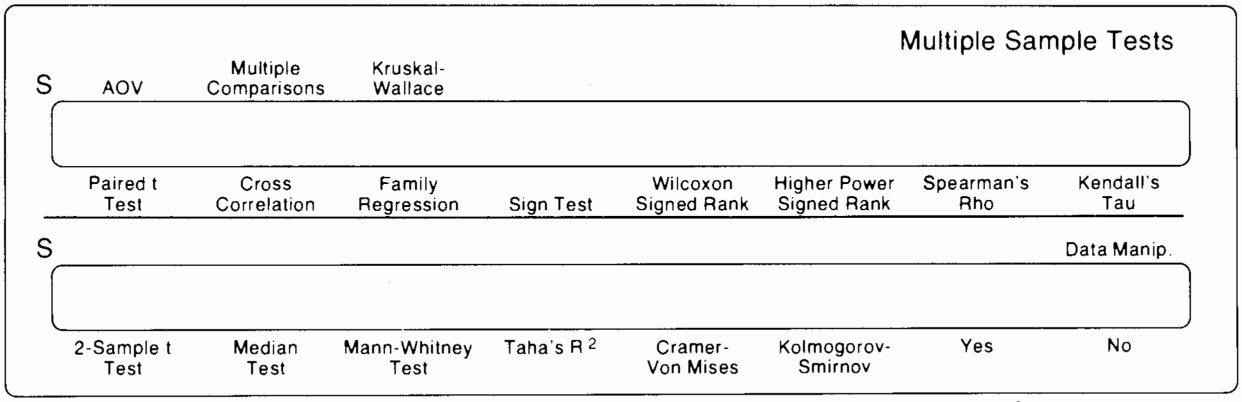
Most of the operations are selected by pressing the appropriate special function key, shown below. Two of the keys those labeled YES and NO, may be used as responses to any question requiring a yes or no answer.



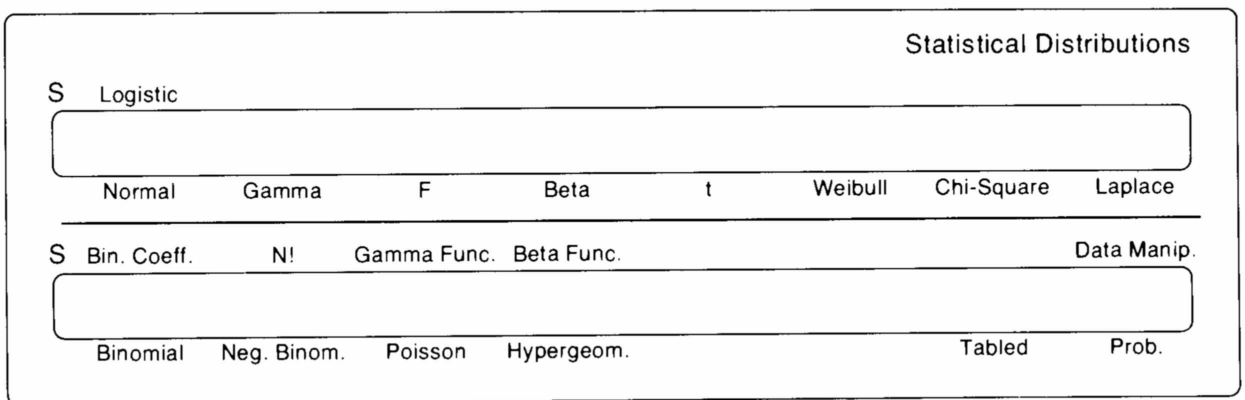
Part# 7120-6407



Part# 7120-6933



Part# 7120-6406



Part# 7120-6932

Part Numbers

The component parts of the GENERAL STATISTICS package may be ordered separately:

User Instructions		09845-15131
Cartridges		09845-15134
		09845-15135
Key Overlays:	"General Statistics"	7210-~~7219~~ 6407
	"One Sample Tests"	7120-~~7220~~ 6933
	"Multiple Sample Tests"	7120-~~7221~~ 6406
	"Statistical Distributions"	7120-~~7222~~ 6932

Start

Object of Program

This program allows you to place a data matrix into memory. The data may be entered from the keyboard, from cards, or from some other input device. Conversely, the data may have been entered previously and stored in the program medium's scratch file ("DATA") or in a user created file on a tape cartridge, flexible disk or hard disk. In this case, the function of this program is to retrieve the previously stored data and place it into memory so that further operations can be performed. After the data is in memory, a listing option is provided.

Special Considerations

- The prompts concerning the data medium and program medium may cause confusion. The word "medium" is used since the set of programs making up the "BASIC STATISTICS AND DATA MANIPULATION" package may be tape, floppy disk or hard disk. Thus, the "program medium" refers to either the cartridge or the disk on which the programs making up this package are stored. Conversely, the "data medium" refers either to the cartridge or to the disk on which the file containing the data matrix resides. In some cases, the program medium and the data medium may be the same. However, this cannot be determined by the program and hence, the prompts are still displayed to make sure the correct medium is in the correct device.
- When entering data from the keyboard, an option to enter data one case at a time is offered. The following example will serve to explain this feature. Suppose an investigator has collected four observations on each of three variables. He has the following data matrix:

		Variable 1	Variable 2	Variable 3
Observation	1	10	2	5
	2	11	2	6
	3	9	3	7
	4	9	2	6

He elects to enter the data one case at a time. Then, when the prompt "Observation #, all variables (separated by commas)=?" is displayed, he enters 10, 2, 5 and presses CONT, etc. This allows for quick entry of the data.

- Because of the wide variety of formats that could be used when entering data via the card reader or "other" device, no attempt was made to program in the necessary statements. It will be necessary for you to provide the statements before using the program. The following procedure should be followed:

  For card reader input:

  1. Type: LOAD "START"
  2. Press: EXECUTE
  3. Type: EDITLINE Card
  4. Press: EXECUTE
  5. Type in and store the appropriate statements for card reader input.

  For "other" input:

  1. Type: LOAD "START"
  2. Press: EXECUTE
  3. Type: EDITLINE Other
  4. Press: EXECUTE
  5. Type in and store the appropriate statements for "other" input.

User Instructions

Getting Started

1. With the machine turned on, insert the "GENERAL STATISTICS" cartridge into the tape drive.

2. Load the program into memory:

   a. Type: LOAD "AUTOST"
   b. Press: EXECUTE

3. Press: RUN
4. When "Is 'GENERAL STATISTICS' overlay placed on keys?" is displayed:

   a. Press: YES when the overlay is in place.

Hard-Copy Options

5. When "HARD-COPY OPTIONS" is printed and "Option number=?" is displayed:

   a. Enter: 1, if no hard copy is desired.

   b. Press: CONT

   c. Go to step 9.

   or

   a. Enter: 2, if hard copy is desired from the machines's internal printer.

   b. Press: CONT

   c. Go to step 9.

   or

   a. Enter: 3, if hard copy is desired from an external printer.

   b. Press: CONT

6. When "Are you using an HP-IB Printer?" is displayed:

   a. Press: YES if an HP-IB Printer is being used.

   b. Go to step 7.

   or

   a. Press: NO if the printer is not HP-IB.

   b. Go to step 8.

7. When "Printer select code, bus address=?" [for example, 7,1] is displayed:

   a. Enter the select code, bus address for the HP-IB printer.

   b. Press: CONT

   c. Go to step 9.

8. When "Printer select code=?" is displayed:

   a. Enter the select code of the printer desired for hard copy.

   b. Press: CONT

Data Entry Modes

9. When "DATA ENTRY MODES" is printed and "Mode number=?" is displayed:

   a. Enter: 1, if data is to be entered from the keyboard.

   b. Press: CONT

   c. Go to step 10.

   or

   a. Enter: 2,if data is to be entered from a cartridge or a disk.

   b. Press: CONT

   c. Go to step 15.

   or

   a. Enter: 3, if data is to be entered via cards.

   b. Press: CONT

   c. Go to step 10.

   or

   a. Enter: 4, if data is to be entered from some other device.

   b. Press: CONT

Creating the Data Set

10. When "Project title for this data set (<= 80 characters)=?" is displayed:

    a. Enter up to 80 characters to be used as the name of the data set.

    b. Press: CONT

11. When "Number of variables=?" is displayed:

    a. Enter the number of variables you want in the data set.

    b. Press: CONT

12. When "Number of observations/variable=?" is displayed:

    a. Enter the number of observations for each variable in the data set.

    b. Press: CONT

13. When "VARIABLE NAMES" is printed and "Variable #I name (<= 10 characters)=?" is displayed:

    a. Enter the name of the Ith variable - it may be up to 10 characters long.

    b. Press: CONT

    c. Repeat steps a and b until all variables have been named, then go to step 14.

14. When "Is above information correct" is displayed:

    a. Press: YES if the variable names are as desired.
    b. Go to step 21.

    or

    a. Press: NO if changes to the variable names are desired.
    b. Go to step 13.

Using an Existing Data Set

15. When "Is data stored on the program medium's scratch file (DATA)?" is displayed:

    a. Press: YES if the data is stored in "DATA", the scratch file on the program tape.
    b. The data and related information are loaded into memory at this point.
    c. Go to step 21.

    or

    a. Press: NO if the data is stored on a user-created file.

16. When file name specifications are printed and "Data file name=?" is displayed:

    a. Enter the data file name (less than or equal to six characters) followed by a colon and the mass storage unit specifier.
    b. Press: CONT

17. When "Was data stored by this program?" is displayed:

    a. Press: YES if the data file was created by the "BASIC STATISTICS AND DATA MANIPULATION" routine or by any routine which stores a header file with the data (see Special Considerations).
    b. Go to step 18.

    or

    a. Press: NO if the file was created by a program which does not store header information along with the data.
    b. Go to step 10.

18. When "Is data medium placed in device?" is displayed:

    a. Press: YES when the data medium is in place.

19. The data and related information are loaded into memory at this point.
20. When "Is program medium replaced in device?" is displayed:

    a. Press: YES when the program medium is in place (or if it was never removed).

Summary of the Data Set

21. At this point a summary of the data set is printed.
22. If the input mode is via keyboard, card reader or "OTHER", go to step 25.
23. When "List data?" is displayed:

    a. Press: YES if a hard-copy listing of the data is desired.

    b. Go to step 1 of the user instructions for the LIST program.

    or

    a. Press: NO if a hard-copy listing is not desired.

Selecting a Program

24. When "SELECT ANY KEY" is displayed:

    a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.

    b. Go to the user instructions for the selected key.

Data Input

25. At this point the subprogram "Input" is called. A subprogram is used here for the sole purpose of variable dimensioning.
26. When "DATA INPUT" is printed:

    a. If the data is being entered from the keyboard, go to step 32.

    or

    a. If the data is being entered from cartridge or disk, go to step 32.

    or

    a. If the data is being entered via cards, go to step 40.

    or

    a. If the data is being entered from some other device, go to step 41.

27. When the program notes are printed:

    a. If there is only one variable in the data set, go to step 28.

    or

    a. If there are two or more variables in the data set, go to step 29.

28. When the heading is printed and "Observation #I" is displayed:

    a. Enter the value of the Ith observation.

b. Press: CONT

c. Repeat steps a and b until all observations have been entered.

d. Go to step 42.

29. When "Enter data one case at a time, that is, by observation?" is displayed:

a. Press: YES if you desire to key in the values of all variables for a given observation at once.

b. Go to step 31.

or

a. Press: NO if you desire to key in the value of one variable at a time for a given observation.

30. When "Observation #I, Variable #J" is displayed:

a. Enter the value of the Ith observation, Jth variable.

b. Press: CONT

c. Repeat steps a and b until all variables for all observations are entered.

d. Go to step 42.

31. When "Observation #I, all variables (separated by commas)=?" is displayed:

a. Enter variable 1, comma, variable 2, comma, . . ., variable P for Observation #I.

b. Press: CONT

c. Repeat steps a and b until all variables for all observations are entered.

d. Go to step 42.

32. When "Are MISSING VALUES denoted by −9999999.99999?" is displayed:

a. YES if missing values were specified by pressing MISSING VALUE (M.V.) when the data was originally entered, or if there are no missing values.

b. Go to step 34.

or

a. Press: NO if missing values are specified by some other value.

33. When "Missing value=?" is displayed:

a. Enter the value that has been used to specify a missing data point.

b. Press: CONT

34. When "Is data in proper configuration, that is, variables = rows, observations = columns?" is displayed:

    a. Press: YES if the data matrix contains a variable in each row and an observation in each column.

    b. Go to step 36.

    or

    a. Press: NO if the data matrix is in a different configuration.

35. When "Data stored as contiguous array with observation = rows, variables = columns?" is displayed:

    a. Press: YES if the data matrix contains an observation in each row and a variable in each column.

    b. Go to step 36.

    or

    a. Press: NO if the data is configured in a different manner.

    b. Execution stops - the data set is unacceptable for use by this program and therefore must be keyed in.

36. When "Is data medium placed in device?" is displayed:

    a. Press: YES when data medium is in place.

37. At this point the data is loaded into memory.

38. When "Is program medium replaced in device?" is displayed:

    a. Press: YES when program medium is in place (or if it was never removed).

39. At this point the data is stored on the scratch file of the program medium (DATA). Go to step 42.

40. See Special Considerations.

41. See Special Considerations.

42. When "List data?" is displayed:

    a. Press: YES if a hard-copy listing of the data is desired.

    b. Go to step 1 of the user instructions for LIST.

    or

    a. Press: NO if no listing is desired.

Selecting a Program

43. When "SELECT ANY KEY" is displayed:

    a. Press: Any of the keys defined on the "BASIC STATISTICS AND DATA MANIPULATION" overlay.

    b. Go to step 1 of the user instructions for the selected key.

Example

The data listed below provides an example of most of the operations that can be performed by this package. There are five variables:

1. average monthly temperature (°C)
2. monthly production (kg.)
3. number of working days
4. number of people on the payroll
5. monthly water use (litres)

There are 17 observations, one per month.

```
********************************************************************************
*                            DATA MANIPULATION                                 *
********************************************************************************
                                   DATA

Data file name:
Number of observations: 17
Number of variables: 4

Variables names:
   1. AGE
   2. HEIGHT
   3. WEIGHT
   4. PRESSURE
```

DATA

OBS#	Variable # 1	Variable # 2	Variable # 3	Variable # 4
1	24.00000	172.50000	66.10000	76.60000
2	41.00000	196.00000	55.60000	74.20000
3	28.00000	189.80000	69.80000	72.40000
4	46.00000	149.50000	71.70000	83.30000
5	46.00000	145.60000	66.50000	77.80000
6	52.00000	155.30000	56.80000	72.70000
7	45.00000	158.80000	67.60000	82.90000
8	43.00000	172.10000	68.70000	82.20000
9	40.00000	182.30000	77.20000	82.40000
10	34.00000	187.40000	70.90000	81.70000
11	27.00000	181.80000	74.60000	73.50000
12	28.00000	183.30000	74.00000	75.50000
13	30.00000	165.40000	62.30000	87.20000
14	40.00000	177.60000	59.80000	79.30000
15	36.00000	202.90000	78.90000	78.20000
16	44.00000	147.50000	64.50000	75.10000
17	37.00000	176.30000	80.50000	81.00000

Edit

Object of Program

This program is designed to allow you to perform a variety of editing procedures on the data matrix. The editing capabilities include: the changing of an incorrect data value, the deletion of a variable, the deletion of an observation, the addition of an observation, the insertion of an observation (if the data is ordered), and the addition of a variable. All of the above operations can be performed repeatedly; for example, three variables could be deleted in succession. After the data matrix has been edited, you are given the option of listing the edited data.

Special Considerations

Order of Corrections

As stated in the program note printed on the screen, the data is renumbered after deletions or insertions are performed. For this reason, if more than one deletion (insertion) is to be performed, it is recommended that the highest-numbered observation (or variable) be deleted, then the next highest-numbered, etc. For example, if observations three and eight are to be deleted, then it is recommended to delete observation eight first, then observation three. Notice that if observation three were deleted first, the subsequent renumbering would move observation eight to position seven. The recommendation is meant to alleviate confusion which may occur due to the renumbering.

Subfiles

Insertions or deletions of observations will affect the content of subfiles which are in existence at the time of editing; for example, if subfile one consists of the first 10 observations while subfile two consists of the last 20 and if observation five is deleted, then observation ten (formerly numbered 11) will have jumped from subfile two to subfile one. Thus, it may be necessary to change the subfile structure after editing. It is recommended that subfiles be created only after all editing has been performed.

Methods and Formulae

The data matrix is redimensioned into a row vector to facilitate the shuffling of elements necessitated by the editing operations. The vector contains all the observations of variable one, followed by the observations of variable two, etc. When an observation is inserted, for example, the elements of the data vector are shuffled one at a time to make room for the incoming observation. Similarly, when an observation is deleted, the remaining observations are "packed" together so that the resultant data vector has no "holes" between observations.

User Instructions

Press: The special function key labeled Edit.

Correcting a Data Value

1. When "Correct a data value?" is displayed:

   a. Press: NO if there are no data values present that need to be changed.

   b. Go to step 6.

   or

   a. Press: YES if there is a data point in memory that needs to be changed.

2. When "Observation number=?" is displayed:

   a. Enter the observation number of the data value that is to be changed.

   b. Press: CONT

   c. If there is only one variable in the data set, go to step 4.

3. When "Variable number=?" is displayed:

   a. Enter the variable number of the data value that is to be changed.

   b. Press: CONT

4. When "Old value='.' −correct value=?" is displayed:
   (The old value is displayed so one can be sure that the correction is being made as anticipated.)

   a. Enter the correct data value.

   b. Press: CONT

   c. A hard-copy note of the change is printed.

5. When "Correct another value?" is displayed:

   a. Press: YES if there is another data point in memory that needs to be changed.

   b. Go to step 2.

   or

   a. Press: NO if there are no more changes to be made to existing data points.

6. If there is only one variable in the data set, go to step 11.

Deleting a Variable

7. When "Delete a variable?" is displayed:

   a. Press: NO if no variables are to be removed from the data set.

   b. Go to step 11.

   or

   a. Press: YES if you desire to remove one or more variables from the data set.

8. When the program note is printed, and "Number of the variable to be deleted=?" is displayed:

   a. Enter the highest-numbered variable to be deleted. For example, if variables 2 and 7 are to deleted, enter 7 first (since the variables are renumbered after a deletion, and if 2 were deleted first, then variable 7 would be referred to as variable 6).

   b. Press: CONT

   c. A hard-copy note of the deletion is printed.

9. If there is only one variable remaining in the data set, go to step 11.

10. When "Delete another variable?" is displayed:

    a. Press: YES if you desire to remove another variable from the data set.

    b. Go to step 8.

    or

    a. Press: NO if no more variables are to be deleted from the data set.

Deleting an Observation

11. When "Delete an observation?" is displayed:

    a. Press: NO if no observations are to be removed from the data set.

    b. Go to step 14.

    or

    a. Press: YES if you desire to remove one or more observations from the data set.

12. When the program note is printed and "Number of the observation to be deleted=?" is displayed:

    a. Enter the highest-numbered observation to be deleted; for example, if observations 2 and 5 are to be deleted, enter 5 first (since the observations are renumbered after each deletion, if 2 were deleted first, then observation 5 would become observation 4).

    b. Press: CONT

    c. A hard-copy note of the deletion is printed.

13. When "Delete another observation?" is displayed:

    a. Press: YES if it is desired to remove another observation from the data set.

    b. Go to step 12.

    or

    a. Press: NO if no more observations are to be removed from the data set.

Adding an Observation

14. If the addition of one more observation will exceed available memory, go to step 23.

15. When "Add an observation?" is displayed:

    a. Press: NO if no observations are to be added to the end of the data set or inserted into the data set.

    b. Go to step 23.

    or

    a. Press: YES if you desire to add more observations to the end of the data set or to insert observations into the data set.

16. When "Are observations ordered, that is, should additions be inserted?" is displayed:

    a. Press: YES if the data follows some type of order and if you desire to insert an observation, for example, between observations 4 and 5.

    b. Go to step 19.

    or

    a. Press: NO if the observations are to be added to the end of the data set.

17. When "How many observations are to be added?" is displayed:

    a. Enter the number of observations to be added to the "end" of the data set.

    b. Press: CONT

18. When "Observation #'I', Variable #'J'=?" is displayed:

    a. Enter the data point corresponding to the Jth variable of the Ith observation.

    b. Press: CONT

    c. A hard-copy note of the addition is printed.

    d. Repeat steps a-c until data has been entered for all variables of each of the observations added.

    e. Go to step 23.

19. When the program note is printed and "Insertion to precede observation #?" is displayed:

    a. Enter the number of the observation which the insertion will precede. For example, if an observation is to be inserted between observations 8 and 9, enter 9. Since the observations are renumbered after each insertion, insertions should be made between the highest-numbered observations first. For example, if insertions are to be made between observations 4 and 5 as well as between observation 8 and 9, the latter should be performed first, that is, 9 should be entered first.

    b. Press: CONT

20. When "Observation #'I', Variable #'J'=?" is displayed:

    a. Enter the data point corresponding to variable J of the inserted observation.

    b. Press: CONT

    c. A hard-copy note of the insertion is printed.

    d. Repeat steps a-c until data has been entered for all variables.

21. If the addition of one more observation will exceed program limitations, go to step 23.
22. When "Insert another observation?" is displayed:

    a. Press: YES if you desire to insert another observation.

    b. Go to step 19.

    or

    a. Press: NO if no more observations are to be inserted.

Adding a Variable

23. If the addition of a variable will exceed program limitations, go to step 29.
24. When "Add a variable?" is displayed:

    a. Press: NO if no variables are to be added from the keyboard.

    b. Go to step 29.

    or

    a. Press: YES if you desire to add one or more variables from the keyboard.

25. When "Variable name (<= 10 characters)" is displayed:

    a. Enter the name of the variable to be added - it may be up to 10 characters long.

    b. Press: CONT

26. When "Variable #'I', Observation #'J'=?" is displayed:

    a. Enter the Jth observation of the variable being added.

    b. Press: CONT

    c. A hard-copy note of the addition is printed.

    d. Repeat steps a-c until all observations have been entered for the added variable.

27. If the addition of one more variable will exceed available memory, go to step 29.

28. When "Add another variable?" is displayed:

    a. Press: YES if you desire to add another variable from the keyboard.

    b. Go to step 25.

    or

    a. Press: NO if no more variables are to be added from the keyboard.

29. When "More corrections?" is displayed:

    a. Press: YES if more corrections to the data matrix are required.

    b. Go to step 1.

    or

    a. Press: NO if no more corrections to the data set are necessary.

30. At this point the corrected data matrix and related information are recorded in file "DATA" of the program medium.

31. "List data?" is displayed:

    a. Press: YES if a hard-copy listing of the corrected data set is desired.

    b. Go to step 1 of the user instructions for LIST.

    or

    a. Press: NO if a hard-copy listing of the data is not required.

32. When "SELECT ANY KEY" is displayed:

    a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.

    b. Go to step 1 of the user instructions for the selected key.

Example

The program EDIT was run in the print all mode. The printout shows a correction, deletion of an observation, and addition of an observation. The edited data was then listed.

```
Correct a data value?
"YES"
Observation number = ?
11
Variable number = ?
2
Old value =  15022 -- Correct value =
?
15024
Observation # 11 Variable # 2 -- correct value =  15024
Correct another value?
"NO"

Delete a variable?
"NO"
Delete an observation?
"YES"
Number of the observation to be deleted = ?
10
Observation # 10 has been deleted, 16 observations remain.
Delete another observation?
"NO"

Add an observation?
"YES"
Are observations ordered, i.e., should additions be inserted?
"NO"
How many observations are to be added?
1
Observation # 17 Variable # 1  =
?
4.2
Observation # 17 Variable # 1  =  4.2
Observation # 17 Variable # 2  =
?
12707
Observation # 17 Variable # 2  =  12707
Observation # 17 Variable # 3  =
?
20
Observation # 17 Variable # 3  =  20
Observation # 17 Variable # 4  =
?
192
Observation # 17 Variable # 4  =  192
Observation # 17 Variable # 5  =
?
3614
Observation # 17 Variable # 5  =  3614

  Total number of observations now =  17

Add a variable?
"NO"
More corrections?
"NO"
List data?
"YES"
```

SAMPLE

OBS#	Variable # 1	Variable # 2	Variable # 3	Variable # 4	Variable # 5
1	14.90000	6396.00000	21.00000	134.00000	3373.00000
2	18.40000	5736.00000	22.00000	145.00000	3110.00000
3	21.60000	6116.00000	22.00000	158.00000	3180.00000
4	25.20000	8287.00000	20.00000	171.00000	3293.00000
5	26.30000	13313.00000	25.00000	198.00000	3390.00000
6	27.20000	13108.00000	23.00000	194.00000	4287.00000
7	22.20000	10768.00000	20.00000	180.00000	3852.00000
8	17.70000	12173.00000	23.00000	191.00000	3366.00000
9	12.50000	11390.00000	20.00000	195.00000	3532.00000
10	6.40000	15024.00000	22.00000	200.00000	3896.00000
11	13.30000	13114.00000	19.00000	211.00000	3437.00000
12	18.20000	12257.00000	22.00000	203.00000	3324.00000
13	22.80000	13118.00000	22.00000	197.00000	3214.00000
14	26.10000	13100.00000	21.00000	196.00000	4345.00000
15	26.30000	16716.00000	21.00000	205.00000	4936.00000
16	4.20000	14056.00000	22.00000	205.00000	3624.00000
17	4.20000	12707.00000	20.00000	192.00000	3614.00000

Transform

Object of Program

This program allows you to transform one or two variables in the data matrix via 16 pre-specified functions or through a function which you specify. The transformed data may then be treated as a new variable, or it may replace the elements of an existing variable. Hence, transformations on more than two variables may be made iteratively or via a user-defined transformation.

The transformations available are:

1. $a X^{b}+c$
2. $a \log(bX)+c$
3. $a \ln(bX)+c$
4. $a \exp(bX)+c$
5. $a (b^{cX})$
6. $a \cos(bX)+c$
7. $a \sin(bX)+c$
8. $a+\sqrt{\arcsin(bX)+c}$
9. $aX+bY+c$
10. $aX^{b}Y^{c}$
11. $a \log(bX+cY)$
12. $a \ln(bX+cY)$
13. $a \cos(bX+cY)$
14. $a \sin(bX+cY)$
15. PROUND(X,a) [round to specified power of 10]
16. DROUND(X,a) [round to specified no. of digits]
17. User Defined

Special Considerations

Missing Values

None of the 16 pre-specified transformations are applied to missing values. Thus, missing values are unaffected by these transformations. However, this is not necessarily the case with the user-defined transformation. If you define a transformation and there are missing values, you must make provisions to ensure that the transformation is not applied to the missing values (unless, of course, this is desired). This may be accomplished as explained below.

User-Defined Transformation

Up to 10 lines of code may be used to define a transformation, namely lines 1330 through 1339. These lines may be typed in and stored successively prior to pressing 'CONT'. (See user instructions.) The following example shows the form of a typical user-defined transformation. Suppose the data set consists of four variables with 20 observations each. There are missing values, which are to be left unchanged. You desire to form variable five as the sum of the exponentials of variables one and three. The following sequence should be carried out at step three of the TRANSFORM User Instructions.

a. Type: 1330 D(Z,I)=EXP(D(1,I))+EXP(D(3,I))

b. Press: STORE

c. Type: 1331 IF D(1,I)=−9999999.99999 OR D(3,I)=−9999999.99999 THEN D(5,I)=1

d. Press: STORE

e. Press: CONT

The variable Z is used to identify variable in which the result of the transformation is to be stored. Notice that the elements of D(.,.) are the data - the first subscript refers to variable number while the second subscript refers to observation number (and should always be 'I' here). Line 1331 sets D(5,I)=1 if a missing value is present in either of variables one or three. Notice that D(Z,I) or D(5,I) may be used interchangeably since Z has been assigned the value of 5 previously in the program.

User Instructions

Press: The special function key labeled Transf

1. When the program notes and transformations are printed and "Transformation number=?" is displayed:

   a. Enter the number of the transformation which is desired (1-17, inclusive).

   b. Press: CONT

2. If the selected transformation was not 'User defined', that is, not transformation number 17, go to step 4.
3. When the program notes are printed and "Ready to continue?" is displayed:

   a. Type: "1330 D(Z,I)='defined transformation' "; the form of the defined transformation is discussed in Special Considerations.

   b. Press: STORE

   c. Press: YES when ready to continue.

   d. Go to step 11.

4. When the selected transformation is printed and "Number of the variable corresponding to X=?" is displayed:

a. Enter the variable number corresponding to X, where X refers to the variable in the selected transformation.

b. Press: CONT

5. If the selected transformation number is less than 9 or greater than 14, go to step 7.
6. When "Number of the variable corresponding to Y=?" is displayed:

a. Enter the variable number corresponding to Y, where Y refers to the variable in the selected transformation.

b. Press: CONT

7. When "Parameter a=?" is displayed:

a. Enter the value of parameter a, where a refers to the parameter in the selected transformation.

b. Press: CONT

8. If the selected transformation number is greater than 14, go to step 11.
9. When "Parameter b=?" is displayed:

a. Enter the value of parameter b, where b refers to the parameter in the selected transformation.

b. Press: CONT

10. When "Parameter c=?" is displayed:

a. Enter the value of parameter c, where c refers to the parameter in the selected transformation.

b. Press: CONT

11. When "Store transformed data in variable # (<='I')" is displayed:

a. Enter the number of the variable in which the transformed data is to be stored - it may be less than or equal to I.

b. Press: CONT

12. If the variable specified in step 11 existed previously, that is, if the transformed data is being stored in place of previously existing data, go to step 14.
13. When "Variable name (<= 10 characters)=?" is displayed:

a. Enter the name of the variable to contain the transformed data - it may be up to 10 characters long.

b. Press: CONT

14. If the transformation was 'User defined', go to step 16.

15. When "Is above information correct?" is displayed:

    a. Press: NO if a mistake has been made during entry of the information printed.

    b. Go to step 1.

    or

    a. Press: YES if no corrections are to be made.

16. At this point a hard-copy summary of the transformation is made (unless it was user defined), then the transformation is carried out.

17. When "More transformations?" is displayed:

    a. Press: YES if more transformations are desired.

    b. Go to step 1.

    or

    a. Press: NO if no more transformations are desired.

18. At this point the altered data matrix and related information are stored on file "DATA" of the program medium.

19. When "List data?" is displayed:

    a. Press: YES if a hard-copy listing of the data is desired.

    b. Go to step 1 of the user instructions for LIST.

    or

    a. Press: NO if a hard-copy listing of the data is not required.

20. When "SELECT ANY KEY" is displayed:

    a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.

    b. Go to step 1 of the user instructions for the selected key.

Example

Variable five, Water Use, was tranformed, converting litres to U.S. gallons. Transformation 1 was used, with a=0.2642, b=1,c=0.


```
Transformation number = ?
1
Number of the Variable number corresponding to X = ?
5
Parameter a = ?
.2642
Parameter b = ?
1
Parameter c = ?
0
Store transformed data in Variable # ( <= 6 )
?
5
Is above information correct?
"YES"
The following transformation was performed: a*(X^b)+c
   where a =  .2642
         b =  1
         c =  0
         X is Variable # 5
         Transformed data is stored in Variable # 5 (Water Use).


More transformations?
"NO"
List data?
"YES"
```

SAMPLE

OBS#	Variable # 1	Variable # 2	Variable # 3	Variable # 4	Variable # 5
1	14.90000	6396.00000	21.00000	134.00000	891.14660
2	18.40000	5736.00000	22.00000	146.00000	821.66200
3	21.60000	6116.00000	22.00000	158.00000	840.15600
4	25.20000	8287.00000	20.00000	171.00000	870.01060
5	26.30000	13313.00000	25.00000	198.00000	895.63800
6	27.20000	13108.00000	23.00000	194.00000	1132.62540
7	22.20000	10768.00000	20.00000	180.00000	1017.69840
8	17.70000	12173.00000	23.00000	191.00000	889.29720
9	12.50000	11390.00000	20.00000	195.00000	933.15440
10	6.40000	15024.00000	22.00000	200.00000	1029.32320
11	13.30000	13114.00000	19.00000	211.00000	908.05540
12	18.20000	12257.00000	22.00000	203.00000	878.20080
13	22.80000	13118.00000	22.00000	197.00000	849.13880
14	26.10000	13100.00000	21.00000	196.00000	1147.94900
15	26.30000	16716.00000	21.00000	205.00000	1304.09120
16	4.20000	14056.00000	22.00000	205.00000	957.46080
17	4.20000	12707.00000	20.00000	192.00000	954.81880

Recode

Object of Program

This program allows you to assign codes to various categories or classes of data. The categories are intervals along the real number line and 20 of these may be specified. The recoding is done on one variable at a time. The same coding scheme may be used iteratively on successive variables. A summary of the coding intervals, codes, and number of observations assigned to each code is printed as hard copy.

Special Considerations

Coding Schemes

Four coding schemes are available for the sole purpose of eliminating unnecessary entries from the keyboard. If the coding intervals are all of the same length and are contiguous, that is, together they form a connected interval, then the interval construction can be accomplished internally knowing only the interval length and lower limit for the first interval. Similarly, if the intervals are of equal length but noncontiguous, for example, [10,20), [25,35), [35,45), [50,60), then the lower limit of each interval needs to be specified but the upper limit may be computed internally. Hence, the coding schemes are meant only to minimize the amount of information which needs to be entered from the keyboard. Clearly, the coding intervals could all be constructed by requiring you to enter the lower and upper limits for each and every interval (which is necessary, and what is done if the intervals are unequal and non-contiguous).

Same Coding Scheme

The coding is carried out on one variable at a time. However, if you desire to code both variables one and two according to the same coding intervals, these intervals need to be contructed only once. A positive response to the option offered by "Use same coding scheme?" allows variable two to be coded according to the same scheme without constructing the intervals a second time. If, however, you desire to code variable two according to a different scheme, it is possible to construct a second set of coding intervals by giving a negative response to the above prompt.

Brackets

The brackets used to denote the coding intervals are meant to follow their usual mathematical interpretation, that is, the intervals are closed on the left and open on the right.

Observation not in an Interval

If an observation does not fall into any of the coding intervals, its value is not changed during the coding process.

User Instructions

Press: The special function key labeled Recode.

1. When the title is printed and "Store recoded data in variable # (<='I')?" is displayed:

   a. Enter the number of the variable in which the recoded data is to be stored - it may be less than or equal to I.

   b. Press: CONT

2. If the variable specified in step 1 existed previously, that is, if the recorded data is being stored in place of previously existing data, go to step 4.
3. When "Variable name (<= 10 characters)=?" is displayed:

   a. Enter the name of the variable which will contain the recoded data - it may be up to 10 characters long.

   b. Press: CONT

4. When "Number of the variable to be recoded=?" is displayed:

   a. Enter the number of the variable which is to be recoded.

   b. Press: CONT

5. When the data coding schemes are printed and "Option number=?" is displayed:

   a. Enter the option number associated with the desired recoding scheme.

   b. Press: CONT

6. When "Number of recoding intervals to be specified (<=20)=?" is displayed:

   a. Enter the number of categories into which the data will be divided, or equivalently, the number of codes which will be assigned.

   b. Press: CONT

7. If the specified coding scheme does not have equal intervals, go to step 9.
8. When "Length of each interval=?" is displayed:

   a. Enter the common increment to be used to generate the intervals.

   b. Press: CONT

9. If the data recoding scheme is non-contiguous, go to step 11.
10. When "Lower limit of first interval=?" is displayed:

    a. Enter the smallest number which will be assigned code #1.

    b. Press: CONT

11. Note: The following steps, 12 through 14, are repeated the number of times specified in step 6 until all information is input for each of the recoding intervals. Some of the steps are skipped for certain schemes, since the necessary information is computed internally. For scheme #1, only step 14 needs to be repeated. For scheme #2, only steps 13 and 14 need to be repeated. For scheme #3, only steps 12 and 14 need to be repeated. For scheme #4, steps 12-14 need to be repeated.

12. When "Lower limit of interval #'I'=?" is displayed:

    a. Enter the smallest number which will be assigned the code associated with interval #I.

    b. Press: CONT

13. When "Upper limit of interval #'I'=?" is displayed:

    a. Enter the upper bound of numbers which will be assigned the code associated with interval #I.

    b. Press: CONT

14. When "For data falling in interval #'I', code=?" is displayed:

    a. Enter the code which will be assigned to those data values falling in the Ith coding interval.

    b. Press: CONT

    c. Repeat as noted in instruction 11.

15. When "Is above information correct?" is displayed:

    a. Press: NO if a mistake has been made in entering the information.

    b. Go to step 5.

    or

    a. Press: YES if the recoding scheme is as desired.

16. At this point the recoding is carried out and a hard-copy note of the coding is printed.

17. When "Recode more data?" is displayed:

    a. Press: YES if you desire to recode more data.

    b. Go to step 18.

    or

    a. Press: NO if no more recoding is desired.

    b. Go to step 19.

18. When "Use same coding scheme?" is displayed:

    a. Press: YES if the same intervals and codes are desired, that is, if the identical recoding scheme is to be applied to another variable.

    b. Go to steps 1-4, then directly to 16.

    or

    a. Press: NO if a different recoding scheme is desired.

    b. Go to step 1.

19. At this point the altered data matrix and related information are stored on file "DATA" of the program medium.

20. When "List data?" is displayed:

    a. Press: YES if a hard-copy listing of the data is desired.

    b. Go to step 1 of the user instructions for LIST.

    or

    a. Press: NO if no listing is desired.

21. When "SELECT ANY KEY" is displayed:

    a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.

    b. Go to step 1 of the user instructions for the selected key.

Example

Using the original data, variable 5, Water Use, was recoded, and the resulting data was stored as a new variable, 6. The purpose of the recoding operation was to simplify the data, replacing entire intervals by single numbers.

```
Store recoded data in Variable # (<= 6 )
?
6
Variable name (<= 10 characters) = ?
Coded Use
Number of the variable to be recoded = ?
5
Option number = ?
1
Number of recoding intervals to be specified (<=20) = ?
5
Length of each interval = ?
400
Lower limit of first interval = ?
3000
For data falling in interval 1 , code =
?
30
For data falling in interval 2 , code =
?
34
For data falling in interval 3 , code =
?
38
For data falling in interval 4 , code =
?
42
For data falling in interval 5 , code =
?
46
Is above information correct?
"YES"
```

```
Variable # 5 is recoded into 5 categories, and the recoded
values are stored in Variable # 6 , where:


              CATEGORY BOUNDS              # OBS
         LOWER            UPPER            CODED         CODE
       3000.000         3400.000             8         30.000
       3400.000         3800.000             4         34.000
       3800.000         4200.000             2         38.000
       4200.000         4600.000             2         42.000
       4600.000         5000.000             1         46.000

Recode more data?
"YES"
Use same coding scheme?


Recode more data?
YES


Recode more data?
NO

                              SAMPLE

    Variable # 1   Variable # 2   Variable # 3   Variable # 4   Variable # 5
      Variable # 6
OBS#
   1
        14.90000     6396.00000       21.00000      134.00000     3373.00000
          30.00000
   2
        18.40000     5736.00000       22.00000      146.00000     3110.00000
          30.00000
   3
        21.60000     6116.00000       22.00000      158.00000     3180.00000
          30.00000
   4
        25.20000     8287.00000       20.00000      171.00000     3293.00000
          30.00000
   5
        26.30000    13313.00000       25.00000      198.00000     3390.00000
          30.00000
   6
        27.20000    13108.00000       23.00000      194.00000     4287.00000
          42.00000
   7
        22.20000    10768.00000       20.00000      180.00000     3852.00000
          38.00000
```

8	17.70000	12173.00000	23.00000	191.00000	3366.00000	30.00000
9	12.50000	11390.00000	20.00000	195.00000	3532.00000	34.00000
10	6.90000	12707.00000	20.00000	192.00000	3614.00000	34.00000
11	6.40000	15022.00000	22.00000	200.00000	3896.00000	38.00000
12	13.30000	13114.00000	19.00000	211.00000	3437.00000	34.00000
13	18.20000	12257.00000	22.00000	203.00000	3324.00000	30.00000
14	22.80000	13118.00000	22.00000	197.00000	3214.00000	30.00000
15	26.10000	13100.00000	21.00000	196.00000	4345.00000	42.00000
16	26.30000	16716.00000	21.00000	205.00000	4936.00000	46.00000
17	4.20000	14056.00000	22.00000	205.00000	3624.00000	34.00000

Sort

Object of Program

This program allows the data matrix, or subfiles thereof, to be sorted according to the values of one variable. For example, suppose an investigator has five observations of three variables, say height, weight and age and wanted to arrange the observations in ascending order according to age. This is accomplished by sorting the data matrix according to variable three.

Special Considerations

Subfile Structure Options

If subfiles are ignored, the entire data set will be sorted and, in the process, the composition of the subfiles is subject to change. The option of sorting certain subfiles may be used to sort a single subfile or a set of successive subfiles according to one variable. The option of sorting all subfiles may be used to sort every subfile. The options of sorting certain subfiles and sorting all subfiles treat each subfile as if it were a separate data set. Thus, the sort is done with respect to one subfile at a time.

What Happens

It is important to note that entire observations are moved when the sort is carried out. Thus, referring to the example given in Object of Program, a person's height and weight remain with the person's age as shown below.

Original Matrix

		Height	Weight	Age
	1	72	170	21
	2	70	165	25
OBSERVATION	3	69	150	20
	4	70	165	25
	5	73	160	19

Matrix Sorted By Age

VARIABLE

		Height	Weight	Age
	1	73	160	19
	2	69	150	20
OBSERVATION	3	72	170	21
	4	70	165	25
	5	70	165	25

User Instructions

Press: The special function key labeled Sort.

1. At this point the subprogram "Sort" is called. A subprogram is used here for the sole purpose of variable dimensioning.
2. When "Number of the Variable on which to sort=?" is displayed:

   a. Enter the number of the variable on which the matrix will be sorted; that is, the variable whose observations will be arranged in ascending order.

   b. Press: CONT

3. If the data set contains no subfiles, go to step 7.
4. When the subfile structure options are printed and "Option number=?" is displayed:

   a. Enter 1 if you desire to ignore the subfile structure, that is, to lump all the observations together for the sort. Note that this may shift observations from subfile to subfile.

   b. Press: CONT

   c. Go to step 7.

   or

   a. Enter 2 if you desire to sort a set of one or more successive subfiles, for example, to sort subfiles 3 through 5. The sort will be done by subfile: subfile 3 will be sorted, then 4, and finally 5.

   b. Press: CONT

   c. Go to step 5.

   or

   a. Enter 3 if you desire to sort all of the subfiles. Each subfile will be sorted as if it were a separate data set.

   b. Press: CONT

   c. Go to step 7.

5. When "Number of first subfile=?" is displayed:

   a. Enter the number of the first subfile to be sorted.

   b. Press: CONT

6. When "Number of last subfile=?" is displayed:

   a. Enter the number of the last subfile to be sorted. All subfiles between and including the first and last specified will be sorted as though they were separate data sets.

   b. Press: CONT

7. At this point the desired sorting is performed, a hard-copy note of the sort is printed, and the sorted data is stored on file "DATA" of the program medium.
8. When "List data?" is displayed:

   a. Press: YES if a hard-copy listing of the sorted data is desired.

   b. Go to step 1 of the user instructions for LIST.

   or

   a. Press: NO if no listing is desired.

9. When "SELECT ANY KEY" is displayed:

   a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.

   b. Go to step 1 of the user instructions for the selected key.

Example

The data was sorted by variable 2. That is, the observations were arranged so that monthly production was in ascending order. Notice that the other variables are carried by the sort operation, so that although the order of the observations has changed, each observation remains intact.

```
    Data set:
                                                SAMPLE
    has been arranged in ascending order according to Variable # 2
    (Subfiles ignored)

      Variable # 1    Variable # 2    Variable # 3    Variable # 4    Variable # 5
         Variable # 6
OBS#
   1
          18.40000     5736.00000       22.00000       146.00000      3110.00000
             30.00000
   2
          21.60000     6116.00000       22.00000       158.00000      3180.00000
             30.00000
   3
          14.90000     6396.00000       21.00000       134.00000      3373.00000
             30.00000
   4
          25.20000     8287.00000       20.00000       171.00000      3293.00000
             30.00000
   5
          22.20000    10768.00000       20.00000       180.00000      3852.00000
             38.00000
   6
          12.50000    11390.00000       20.00000       195.00000      3532.00000
             34.00000
```

7	17.70000	12173.00000	23.00000	191.00000	3366.00000	30.00000
8	18.20000	12257.00000	22.00000	203.00000	3324.00000	30.00000
9	6.90000	12707.00000	20.00000	192.00000	3614.00000	34.00000
10	26.10000	13100.00000	21.00000	196.00000	4345.00000	42.00000
11	27.20000	13108.00000	23.00000	194.00000	4287.00000	42.00000
12	13.30000	13114.00000	19.00000	211.00000	3437.00000	34.00000
13	22.80000	13116.00000	22.00000	197.00000	3214.00000	30.00000
14	26.30000	13313.00000	25.00000	198.00000	3390.00000	30.00000
15	4.20000	14056.00000	22.00000	205.00000	3624.00000	34.00000
16	6.40000	15022.00000	22.00000	200.00000	3896.00000	38.00000
17	26.30000	16716.00000	21.00000	205.00000	4936.00000	46.00000

Subfiles

Object of Program

This program allows you to specify subfiles or logical groupings of the observations. This may be accomplished by entering the number of observations in each subfile or by entering the observation number of the first observation in each subfile. Names for the subfiles are entered in both cases. A third option allows you to destroy the existing subfile structure.

Special Considerations

Use of Subfiles

Subfiles may be created in order to specify logical groupings of observations. A subfile structure allows you to consider each subfile as a separate data set or to lump all the subfiles together and analyze the overall data set. For example, suppose an investigator wished to measure several variables on 50 trout. He would like to analyze the data separately for each of the three varieties of the trout. He could form three separate data sets and do the individual analyses, then later join the three sets together for the overall analysis. However, since the same variables were measured on each variety of fish, this situation is well-handled by specifying a subfile for each variety. The subfile structure options make it possible to do the analysis by subfile as well as for the overall data set.

Editing and Sorting

Certain operations in the editing and sorting programs may cause observations to move from one subfile to another. To avoid undesired results such as this, it is recommended that subfiles be specified after any editing or sorting has been carried out.

User Instructions

Press: The special function key labeled Subfile.

1. When the subfile characterization options are printed and "Option number=?" is displayed:

   a. Enter 1 if you desire to specify subfiles by entering the number of observations in each subfile.

   b. Press: CONT

   c. Go to step 2.

   or

   a. Enter 2 if you desire to specify subfiles by entering the number of the first observation in each subfile.

   b. Press: CONT

   c. Go to step 2.

   or

a. Enter 3 if you desire to destroy the existing subfiles structure, that is, lump all the data together.

b. Press: CONT

c. Go to step 8.

2. When "Number of subfiles (<= 20)=?" is displayed:

   a. Enter the number of subfiles which will be specified.

   b. Press: CONT

3. When "Name of subfile #'I' (<= 10 characters)=?" is displayed:

   a. Enter the name of the Ith subfile - if may be up to 10 characters long.

   b. Press: CONT

4. If the option number you chose was 2, go to step 6.
5. When "Subfile #'I', number of observations=?" is displayed:

   a. Enter the number of observations which will be in the Ith subfile.

   b. Press: CONT

   c. Repeat steps 3 and 5 until all subfiles have been specified.

   d. Go to step 7.

6. When "Subfile 'I', number of first observation=?" is displayed:

   a. Enter the number of the first observation in the Ith subfile.

   b. Press: CONT

   c. Repeat steps 3 and 6 until all subfiles have been specified.

7. When "Is above information correct?" is displayed:

   a. Press: NO if a mistake has been made while entering the required information.

   b. Go to step 2.

   or

   a. Press: YES if the subfile characterizations are as desired.

8. At this point a hard-copy record of the subfiles will be printed, and the information will be printed on file "DATA" of the program medium.
9. When "SELECT ANY KEY" is displayed:

   a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.

   b. Go to step 1 of the user instructions for the selected key.

Example

The original data (before sorting and recoding) was grouped into two subfiles. The first 12 observations make up the first subfile (Fiscal year '76) and the remaining observations make up the second subfile (Fiscal year '77).

```
Option number = ?
2
Number of subfiles ( <=10 ) = ?
2
Name of Subfile # 1 ( <= 10 characters ) =
?
FY'76
Name of Subfile # 2 ( <= 10 characters ) =
?
FY'77
Subfile # 2 ; number of first observation =
?
13
Is above information correct?
"YES"
Subfile name:     beginning observation--number of observations
 1. FY'76                          1                        12
 2. FY'77                         13                         5

SELECT ANY KEY
```

Name

Object of Program

This program allows you to rename variables and/or to rename subfiles. These names are then stored, along with the data, on the program medium's scratch file ("DATA").

User Instructions

Press: The special function key labeled Name.

1. When "Rename data set?" is displayed:

   a. Press: NO to leave the data set name unchanged.

   b. Go to step 3.

   or

   a. Press: YES to change the data set name.

2. When "Name of data set (<= 80 characters)=?" is displayed:

   a. Enter the project title for the data set.

   b. Press: CONT

3. When the title is printed and "Name variables?" is displayed:

   a. Press: NO if no variables are to be renamed.

   b. Go to step 9.

   or

   a. Press: YES if you desire to rename some variables.

4. When "Name beginning with variable #?" is displayed:

   a. Enter the number of the first variable to be renamed.

   b. Press: CONT

5. When "Name ending with variable #?" is displayed:

   a. Enter the number of the last variable to be renamed.

   b. Press: CONT

6. When "Name of variable #'I' (<= 10 characters)=?" is displayed:

   a. Enter the name of the Ith variable - it may be up to 10 characters long.

   b. Press: CONT

   c. Repeat parts a and b until all specified variables have been renamed.

7. When "Is above information correct?" is displayed:

   a. Press: NO if a mistake has been made while entering variable names.

   b. Go to step 1.

   or

   a. Press: YES if the variable names are as desired.

8. At this point a hard-copy list of all the current variable names will be printed. If there are no subfiles in the data set, go to step 15.

9. When "Name subfiles?" is displayed:

   a. Press: NO if no subfiles are to be renamed.

   b. Go to step 15.

   or

   a. Press: YES if you desire to rename some subfiles.

10. When "Name beginning with subfile #?" is displayed:

    a. Enter the number of the first subfile to be renamed.

    b. Press: CONT

11. When "Name ending with subfile #?" is displayed:

    a. Enter the number of the last subfile to be renamed.

    b. Press: CONT

12. When "Name of subfile #'I' (<= 10 characters)=?" is displayed:

    a. Enter the name of the Ith subfile - it may be up to 10 characters long.

    b. Press: CONT

    c. Repeat parts a and b until all specified subfiles have been named.

13. When "Is above information correct?" is displayed:

    a. Press: NO if a mistake has been made while entering subfile names.

    b. Go to step 10.

Example

The name of variable four was changed from 'Payroll' to 'Payroll (#)'.

```
Name variables?
"YES"
Name beginning with Variable # ?
4
Name ending with Variable # ?
4
Name of Variable # 4 ( <= 10 characters ) =
?
Payroll(#)
Is above information correct?
"YES"

CURRENT VARIABLE NAMES
 1. Temp(C)
 2. Production
 3. Days
 4. Payroll(#)
 5. Water Use
 6. Coded Use

Name subfiles?
"NO"
SELECT ANY KEY
```

Store

Object of Program

This program allows you to store the data matrix and related information in a file so that it may be retrieved at a later date for further analysis. The program also allows you to specify the file name.

Special Considerations

Use of Program

The store feature will be useful in two different situations. First, if an investigator has a data set which he may want to analyze further at a later date, he may store it and retrieve it later via the BASIC STATISTICS AND DATA MANIPULATION routine. Secondly, if several people have access to the data input programs, it becomes mandatory that each be able to store his data set in a unique place. Note that if only one person uses the routine on one data set it is unnecessary to use the store feature since the data and related information are kept in "DATA" - the scratch file on the program medium.

Protecting Existing Data

The existence of a file is checked in the program in an attempt to avoid the accidental loss of existing data. Thus, when a file is specified to receive the data, an attempt is made to ensure that you are not accidentally storing the new data in a file which you did not know existed.

User Instructions

Press: The special function key labeled Store.

1. When the program notes are printed and "Name of data file=?" is displayed:

   a. Enter the name of the file in which the data matrix and related information are to be stored - refer to the program note for the proper form of the file name.

   b. Press: CONT

2. When "Is data medium placed in device 'I'?" is displayed:

   a. Press: YES when the medium on which the data and related information are to be stored is in place.

3. When "Does this file already exist?" is displayed:

   a. Press: YES if the file has been previously created (and is of the form specified in the Special Considerations) and you desire to "print over" the information that is currently in the file.

   b. Go to step 4.

   or

   a. Press: NO if the file has not been previously created.

NOTE

This step is an attempt to prevent you from accidentally writing over an existing file. If the response to the question in step 3 is negative and the program finds a file by this name, a note will be displayed and control will pass to step 1.

---

4. At this point, the file will be created if necessary and the data matrix along with the related information will be stored in the specified file. A hard-copy note of the storage will be printed.
5. When "Is program medium replaced in device?" is displayed:

   a. Press: YES when the program medium is in place (or if it was never removed).

6. When "SELECT ANY KEY" is displayed:

   a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.
   b. Go to step 1 of the user instructions for the selected key.

List

Object of Program

This program allows you to obtain a listing of the data matrix. The listing will appear on the device that has been specified for hard-copy in the START routine or in the output unit routine.

User Instructions

Press: The special function key labeled List.

1. When the data listing options are printed and "Option number=?" is displayed:

   a. Enter 1 if you desire to obtain a hard-copy listing of all data - the listing will be by observation.

   b. Press: CONT

   c. Go to step 6.

   or

   a. Enter 2 if you desire to obtain a hard-copy listing of data by variable - it may be all the data or just a set of successive variables.

   b. Press: CONT

   c. Go to step 2.

   or

   a. Enter 3 if you desire to obtain a hard-copy listing of data by observation - the observations are to be successive but need not be the entire data set.

   b. Press: CONT

   c. Go to step 4.

2. When "List beginning with variable #?" is displayed:

   a. Enter the number of the first variable to be included in the list.

   b. Press: CONT

3. When "Ending with Variable #?" is displayed:

   a. Enter the number of the last variable to be included in the list (all variables between the first and last variables specified will be included in the list).

   b. Press: CONT

   c. Go to step 6.

4. When "List beginning with Observation #?" is displayed:

   a. Enter the number of the first observation to be included in the list.

   b. Press: CONT

5. When "Ending with Observation #?" is displayed:

   a. Enter the number of the last observation to be included in the list.

6. At this point the hard-copy listing of the data will be printed.

7. When "SELECT ANY KEY" is displayed:

   a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.

   b. Go to step 1 of the user instructions for the selected key.

Join

Object of Program

This program allows you to join or combine two data sets into a single unit. One data set must be in memory and the other data set must have been previously stored by the BASIC STATISTICS AND DATA MANIPULATION program. Two options are available. First, observations may be added together (if both sets have the same number of variables). Second, variables may be added together (if both sets have the same number of observations).

Special Considerations

Adding Observations

Suppose data on six variables was gathered in each of the 52 weeks in 1975, analyzed, and stored on an auxiliary data tape. Suppose the same variables were measured in 1976, analyzed, and stored. If the investigator is interested in lumping the two sets of data together for an overall analysis, he may use the Add Observations option of the joining routine. One set of data must be retrieved via the START routine. Then, after pressing the JOIN key, the second set may be retrieved and the joining carried out. Notice that the variables must be in the same order in the two data sets.

Adding Variables

Suppose an investigator measured five variables on each of 50 mice in an experiment. These were analyzed and stored on disk. Later, he realized that three more variables were of interest. He measured these variables on the mice in the same order as before and analyzed them. All eight variables measured on each mouse could be combined into a single data set via the joining routine.

Subfiles

If variables are added, the subfile structure assigned to the resultant data set is the subfile structure of data set #1, that is, the data set that is in the machine prior to the joining operation. If observations are added, the following procedures are employed: 1) If no subfiles exist in either data set, the resultant set has no subfiles. 2) If data set #1 has no subfiles, but data set #2 does, then a subfile named "SET #1" is created which consists of data set #1 and the subfiles of data set #2 remain unchanged. 3) If data set #1 contains subfiles, but data set #2 does not, then a subfile named "SET #2" is created which consists of data set #2 and the subfiles of data set #1 remain unchanged. 4) If both data sets contain subfiles, all of the subfiles of data set #1 are retained and as many subfiles of data set #2 are retained as possible - the upper limit of total subfiles for the resultant set being 20.

User Instructions

Press: The special function key labeled Join.

1. At this point the subprogram "Join" is called. A subprogram is used here for the sole purpose of variable dimensioning.
2. When the joining options are printed and "Option number=?" is displayed:

   a. Enter 1 if you desire to add **variables** from an external data medium to those which are currently in memory.

   b. Press: CONT

   c. Go to step 3.

   or

   a. Enter 2 if you desire to add **observations** from an external data medium to those which are currently in memory.

   b. Press: CONT

3. When the requirements are printed and "OK to continue?" is displayed:

   a. Press: NO if the requirements have not been met.

   b. Go to step 16.

   or

   a. Press: YES if you feel that the requirements have been satisfied.

4. When "Project title for the combined data set (<= 80 characters)=?" is displayed:

   a. Enter up to 80 characters to be used as the name of the joined data set.

   b. Press: CONT

5. When the program note is printed and "File name of data set #2=?" is displayed:

   a. Enter the name of the data file in which the data to be joined to the set already in memory resides. Refer to the program note for the proper form of the file name.

   b. Press: CONT

6. When "Is data set #2 medium placed in device 'I'?" is displayed:

   a. Press: YES when the medium on which data set #2 resides in place.

7. At this point, the number of variables, number of observations, and subfile structures are printed for data sets 1 and 2 as well as for the resultant set. The requirements are checked internally and if they have not been met, control is passed to step 16.

8. When "OK to continue?" is displayed:

   a. Press: NO if, after reviewing the summary, you realize that the resultant data set will not turn out as expected (remember, the subfile structure may be changed at a later time).

   b. Go to step 12.

   or

   a. Press: YES if the resultant data set will be as expected.

9. At this point a summary of the variable names for data sets 1 and 2 as well as for the resulting joined data set are printed.

10. When "OK to continue?" is displayed:

    a. Press: NO if, after reviewing the summary, you desire to abort the joining operation (remember, the variable names may be changed at a later time).

    b. Go to step 12.

    or

    a. Press: YES if the joining operation is to be carried out.

11. At this point the joining operation is performed and a hard-copy summary of the resultant data set is printed.

12. When "Is program medium replaced in device?" is displayed:

    a. Press: YES when the program medium is in place (or if it was never removed).

13. If the joining operation was aborted, go to step 16.

14. At this point the resultant data set and related information are printed on file "DATA" of the program medium.

15. When "List data?" is displayed:

    a. Press: YES if a hard-copy listing of the resultant data set is desired.

    b. Go to step 1 of the user instructions for LIST.

    or

    a. Press: NO if no listing is desired.

16. When "SELECT ANY KEY" is displayed:

    a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.

    b. Go to step 1 of the user instructions for the selected key.

Example

Three observations were entered by keyboard and stored in file "TDATA5". Then the JOIN program added these observations to the end of the original data set to form a new set.

```
Option number = ?
2
OK to continue?
"YES"
Project title for combined data set (<= 80 characters) = ?
SAMPLE
File name of data set #2 = ?
TDATA5:F8
Is data set #2 medium placed in device F8
?
"YES"
OK to continue?
"YES"
OK to continue?
"YES"
Is program medium replaced in device?
"YES"
List data?
"YES"
```

SAMPLE

OBS#	Variable # 1	Variable # 2	Variable # 3	Variable # 4	Variable # 5
1	14.90000	6396.00000	21.00000	134.00000	3373.00000
2	18.40000	5736.00000	22.00000	146.00000	3110.00000
3	21.60000	6116.00000	22.00000	158.00000	3180.00000
4	25.20000	8287.00000	20.00000	171.00000	3293.00000
5	26.30000	13313.00000	25.00000	198.00000	3390.00000
6	27.20000	13108.00000	23.00000	194.00000	4287.00000
7	22.20000	10768.00000	20.00000	180.00000	3852.00000
8	17.70000	12173.00000	23.00000	191.00000	3366.00000
9	12.50000	11390.00000	20.00000	195.00000	3532.00000
10	6.90000	12707.00000	20.00000	192.00000	3614.00000
11	6.40000	15022.00000	22.00000	200.00000	3896.00000
12	13.30000	13114.00000	19.00000	211.00000	3437.00000
13	18.20000	12257.00000	22.00000	203.00000	3324.00000
14	22.80000	13118.00000	22.00000	197.00000	3214.00000
15	26.10000	13100.00000	21.00000	196.00000	4345.00000
16	26.30000	16716.00000	21.00000	205.00000	4936.00000
17	4.20000	14056.00000	22.00000	205.00000	3624.00000
18	25.30000	9315.00000	20.00000	183.00000	3356.00000
19	12.40000	11298.00000	19.00000	203.00000	4205.00000
20	18.60000	14653.00000	21.00000	189.00000	4256.00000

Output Unit

Object of Program

This program allows you to change the device on which the hard-copy output will be printed, or conversely, to specify that no hard-copy is desired.
In lieu of this routine, the hard-copy device may be changed at any time by the following procedure:

Non HP-IB Printer

1. Type: Hc=
2. Key in the select code of the printer desired.
3. Press: EXECUTE
4. Type: Hcbus=999
5. Press: EXECUTE

HP-IB Printer

1. Type: Hc=
2. Key in select code of the printer desired.
3. Press: EXECUTE
4. Type: Hcbus=
5. Key in the bus address of the HP-IB device.
6. Press: EXECUTE

User Instructions

Press: The special function key labeled Output Unit.

1. When the hard-copy options are printed and "Option number=?" is displayed:

    a. Enter 1 if no hard-copy is desired.
    b. Press: CONT
    c. Go to step 5.

    or

    a. Enter 2 if a hard-copy is desired from the computer's internal printer.
    b. Press: CONT
    c. Go to step 5.

    or

    a. Enter 3 if a hard-copy is desired from an external printer.
    b. Press: CONT

2. When "Are you using an HP-IB Printer?" is displayed:

   (See Appendix for explanation of HP-IB.)

   a. Press: YES if an HP-IB printer is being used.
   b. Go to step 3.

   or

   a. Press: NO if the printer is not HP-IB.
   b. Go to step 4.

3. When "Printer select code, bus address=? [for example, 7,1]" is displayed:

   a. Enter the select code, bus address for the HP-IB printer.
   b. Press: CONT
   c. Go to step 5.

4. When "Printer select code=?" is displayed:

   a. Enter the select c le of the printer desired for hard-copy.
   b. Press: CONT

5. When "SELECT ANY KEY" is displayed:

   a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.
   b. Go to step 1 of the user instructions for the selected key.

Basic Statistics

Object of Program

This program computes a variety of summary statistics for data which was entered via the BASIC STATISTICS AND DATA MANIPULATION program set. The statistics may be computed by subfile or for the entire data set (ignoring subfiles). Basic statistics which are computed include: number of observations, number of missing values, sum, mean, variance, standard deviation, coefficient of skewness, coefficient of kurtosis, coefficient of variation, standard error of the mean, and confidence interval of the mean. An option is available to obtain the correlation matrix. Order statistics computed include: the maximum, the minimum, range, and midrange. Additional order statistics which may be obtained include: the median, 25th percentile, 75th percentile, Tukey's middlemeans, and user-specified percentiles.

Special Considerations

Hard-copy Output

If a hard-copy of the statistics is not being made, the program halts occasionally so that you may study the results. In this case, it is necessary only to press 'CONT' to continue program execution.

Additional Order Statistics

If the option to obtain additional order statistics (Tukey's middlemeans and percentiles) is exercised, the data matrix is sorted and the observations of each variable are arranged in ascending order. At the end of the program the original data matrix is re-loaded. Thus, if the program is aborted, that is, if another key is pressed before the re-loading can occur, the data matrix will be in the sorted state. Hence, if the portion of the program used to calculate additional order statistics is accessed, abortion of the program should be discouraged.

Methods and Formulae

Ith Variable

Let N(J) be the number of observations of the Jth variable in the data set or subfile, whichever is applicable. Let D(I,J) be the Jth observations of the Ith variable. The following formulas are computed for the Ith variable.

- Sum: $S(I) = \sum_{J=1}^{N(I)} D(I,J)$

- Mean: $M(I) = \frac{S(I)}{N(I)}$

- Variance: $V(I) = \frac{\sum_{J=1}^{N(I)} (D(I,J))^2 - N(I)(M(I))^2}{N(I)-1}$

- Standard deviation: $Sd(I) = (V(I))^{1/2}$

- Second moment: $M_0(I) = \frac{(N(I)-1)V(I)}{N(I)}$

- Skewness: $Sk(I) = \frac{\sum_{J=1}^{N(I)} (D(I,J))^3 - 3M(I) \sum_{J=1}^{N(I)} (D(I,J))^2 + (2M(I))^3}{(M_0(I))^{3/2} N(I)}$

- Kurtosis:

$$K(I) = \frac{\sum_{J=1}^{N(I)} (D(I,J))^4 - 4M(I) \sum_{J=1}^{N(I)} (D(I,J))^3 + 6(M(I))^2 \sum_{J=1}^{N(I)} (D(I,J))^2 - 3(M(I))^4 N(I)}{(M_0(I))^2 N(I)} - 3$$

t-value

Let C be the confidence coefficient for a confidence interval on the mean. The following operations are used to obtain the desired t-value.

$$P = \frac{1-\frac{c}{100}}{2}$$

$$V = \left(\ln\left(\frac{1}{P^2}\right)\right)^{1/2}$$

$$X = 2.5155174+.802853V+.010328V^2$$

$$Y = 1 + 1.432788V + .189269V^2 + .001308V^3$$

$$Z = V-\frac{X}{Y}$$

$$M = N(J)-1$$

- Then the desired t-value is:

$$T = Z + \frac{Z^3+Z}{4M} + \frac{5Z^5+16Z^3+3Z}{96M^2} + \frac{3Z^7+19Z^5+17Z^3-15Z}{384M^3}$$

$$+ \frac{79Z^9+776Z^7+1482Z^5-1920Z^3-945Z}{92160M^4}$$

- Standard error: $Se(I) = \frac{(V(I))^{1/2}}{(N(I))^{1/2}}$

- Confidence interval on mean: $M(I) \pm T(Se(I))$

- Coefficient of variation: $Cv(I) = \left| \frac{(V(I))^{1/2}}{M(I)} \right| (100)$

- Correlations: Suppose we have the following data matrix:

OBSERVATION

		1	2	3	4	5
VARIABLE	1	5	M	3	4	5
	2	6	7	M	6	4
	3	1	3	2	1	1

An M denotes a missing value. When computing the correlation between variables 1 and 2, we discard observations 2 and 3 since variable 1 is missing a data value for observation 2 and variable 2 is missing the data value for observation 3. However, when computing the correlation between variables 1 and 3, we need only discard observation 2. Similarly, the correlation between 2 and 3 is computed by discarding only observation 3. Hence, the correlations may be based on different numbers of observations. An observation is thrown out if and only if a data value from that observation is missing from one of the two variables for which the correlation is being computed. With this in mind, let N(I,J) be the number of observations used to compute the correlation between variables I and J. Then, the correlation is:

$$C(I,J) = \frac{\sum_{K=1}^{N(I,J)} D(I,K)D(J,K) - \frac{\sum_{K=1}^{N(I,J)} D(I,K) \sum_{K=1}^{N(I,J)} D(J,K)}{N(I,J)}}{\left[\sum_{K=1}^{N(I,J)} (D(I,K))^2 - \frac{\sum_{K=1}^{N(I,J)} D(I,K)^2}{N(I,J)}\right]^{\frac{1}{2}} \left[\sum_{K=1}^{N(I,J)} (D(J,K))^2 - \frac{\sum_{K=1}^{N(I,J)} D(I,K)^{2}}{N(I,J)}\right]^{\frac{1}{2}}}$$

Ranges and Percentiles

Let M(I) be the largest data value of the Ith variable, m(I) be the smallest data value of the Ith variable.

1. Range: $R(I) = M(I) - m(I)$

2. Midrange: $Mr(I) = \frac{M(I)+m(I)}{2}$

3. The percentiles are computed as follows: Let P be the percentile in question. If P*N(I) is an integer, the P(I)=(D(I,P*N(I))+D(1,Q))/2, where Q is the next integer value between P*N(I) and the observation index of the median. If P*N(I) is not an integer, the P(I)=D(I,N(I)*P+Q) where

$$Q = \begin{cases} 1 \text{ if } P \leq 50 \\ -1 \text{ if } P > 50 \end{cases}$$

The median refers to the 50th percentile.

Tukey's Middlemeans

- Midmean: Mm (I) =1/N Σ all observations between (and including, if applicable) 25th and 75th percentile.
- Trimean: Tm(I) =¼ (25th percentile + 2(median) + 75th percentile).
- Midspread: Ms(I) = 75th percentile - 25th percentile.

User Instructions

Press: The special function key labeled Stats.

1. If there is only one variable in the data set, go to step 5.
2. When "Summary statistics on all variables?" is displayed:
   a. Press: YES if summary statistics are desired for all of the variables in the data set.
   b. Go to step 5.

   or

   a. Press: NO if summary statistics are desired for only one variable or a set of successive variables in the data set.
3. When "Number of first variable=?" is displayed:
   a. Enter the number of the first variable for which summary statistics are desired.
   b. Press: CONT
4. When "Number of last variable=?" is displayed:
   a. Enter the number of the last variable for which summary statistics are desired. Summary statistics will be calculated for all variables between and including the first and last specified.
   b. Press: CONT
5. If the data set contains no subfiles, go to step 9.

6. When the subfile structure options are printed and "Option number=?" is displayed:

   a. Enter 1 if you desire to ignore the subfile structure, that is, to lump all the observations together and calculate summary statistics on the data set as a whole.

   b. Press: CONT

   c. Go to step 9.

   or

   a. Enter 2 if you desire to calculate summary statistics for only one subfile or for each of a set of adjacent subfiles; for example, for subfiles 3 through 5. The statistics will be calculated by subfile, that is, calculated as though each subfile were a separate data set.

   b. Press: CONT

   c. Go to step 7.

   or

   a. Enter 3 if you desire to calculate summary statistics for all of the subfiles. The statistics will be calculated as though each subfile were a separate data set.

   b. Press: CONT

   c. Go to step 9.

7. When "Number of first subfile=?" is displayed:

   a. Enter the number of the first subfile for which summary statistics will be calculated.

   b. Press: CONT

8. When "Number of last subfile=?" is displayed:

   a. Enter the number of the last subfile for which summary statistics will be calculated. Statistics will be calculated for all subfiles between and including the first and last subfiles specified as though they were separate data sets.

   b. Press: CONT

9. At this point a heading and basic statistics will be output. If no hard-copy printer has been specified, the program will pause occasionally to allow examination of the output and "Press 'CONT' when ready." will be displayed. To resume execution, press: CONT.

10. When "Confidence coefficient for confidence interval on the mean . . ." is displayed:

    a. Enter the confidence coefficient or confidence level to be used in constructing a confidence interval on the mean; for example, enter 95 for a 95% confidence on the mean.

    b. Press: CONT

11. At this point several more statistics will be output. If the output is to the screen, it will be necessary to press 'CONT' to resume program execution after examining the output.

12. If summary statistics are being computed for a single variable, go to step 15.

13. When "Correlation matrix?" is displayed:

    a. Press: NO if correlations are not desired.

    b. Go to step 15.

    or

    a. Press: YES to obtain a matrix of correlations among the variables.

14. At this point the correlation matrix will be computed and output. If output is to the screen, it will be necessary to press 'CONT' to resume program execution after examining the correlations.
15. At this point several order statistics are computed and output.
16. When "More order statistics (Tukey's middlemeans & percentiles)?" is displayed:

    a. Press: NO if no further order statistics are desired.

    b. Go to step 21.

    or

    a. Press: YES if Tukey's middlemeans (see Methods and Formulae), the 25th, 50th, and 75th percentiles, are desired.

17. At this point the data matrix is sorted by variable. Then, several order statistics are computed and output.
18. When "Other percentiles?" is displayed:

    a. Press: NO if no further percentiles are desired.

    b. Go to step 21.

    or

    a. Press: YES if additional percentiles are desired.

19. When "Press 'NO' when finished - Percentile=?" is displayed:

    a. Enter the percentile which is desired; for example, enter 90 to obtain the 90th percentile. The number entered must be between 1 and 99, inclusive.

    b. Press: CONT

    or

    a. Press: NO if no more percentiles are desired.

    b. Go to step 21.

20. At this point, the desired percentile is computed for each variable under consideration and the results are output. Go to step 19.
21. If summary statistics are being computed for more than one subfile, steps 9-20 are repeated for each subfile.

22. If the additional order statistics were computed for the data set (or any subfile), the original data matrix is re-loaded at this point.
23. When "SELECT ANY KEY?" is displayed:

    a. Press: Any of the keys identified on the BASIC STATISTICS AND DATA MANIPULATION overlay.
    b. Go to the start of the user instructions.

Example

When the basic statistics key (Stats) is pressed, basic statistics and order statistics are printed, first for the entire data set, and then for individual subfiles.

```
*******************************************************************************
*                          SUMMARY STATISTICS                                 *
*                            ON DATA SET:                                     *
*                               SAMPLE                                        *
*******************************************************************************
```

(SUBFILES IGNORED)

BASIC STATISTICS

VARIABLE	# OBSERVATIONS	# MISS. VALUES	SUM	MEAN
Temp(C)	20	0	366.50000	18.32500
Production	20	0	232643.00000	11632.15000
Days	20	0	425.00000	21.25000
Payroll	20	0	3751.00000	187.55000
Water Use	20	0	73590.00000	3679.50000

VARIABLE	VARIANCE	STANDARD DEV.	COEF OF SKEWNESS	COEF OF KURTOSIS
Temp(C)	51.93987	7.20693	-.51175	-.87415
Production	9253846.44947	3042.01355	-.64356	-.42042
Days	2.19737	1.48235	.55329	.23443
Payroll	421.62895	20.53361	-1.37170	.95821
Water Use	242672.26316	492.61777	.99654	.11191

VARIABLE	COEF VARIATION	STANDARD ERROR OF THE MEAN	95 % CONFIDENCE INTERVAL ON MEAN LOWER LIMIT	UPPER LIMIT
Temp(C)	39.32841	1.61152	14.95123	21.69877
Production	26.15177	680.21491	10208.09757	13056.20243
Days	6.97578	.33146	20.55607	21.94393
Payroll	10.94834	4.59145	177.93764	197.16236
Water Use	13.38817	110.15268	3448.89172	3910.10828

CORRELATION MATRIX

	Production	Days	Payroll	Water Use
Temp(C)	-.1165685	.2535951	-.1443165	.1428234
Production		.1245049	.8729479	.6347921
Days			-.0272339	-.1524027
Payroll				.4291423

ORDER STATISTICS

VARIABLE	MAXIMUM	MINIMUM	RANGE	MIDRANGE
Temp(C)	27.20000	4.20000	23.00000	15.70000
Production	16716.00000	5736.00000	10980.00000	11226.00000
Days	25.00000	19.00000	6.00000	22.00000
Payroll	211.00000	134.00000	77.00000	172.50000
Water Use	4936.00000	3110.00000	1826.00000	4023.00000

TUKEY'S HINGES

VARIABLE	MEDIAN	25-th %-ile	75-th %-ile
Temp(C)	18.50000	12.90000	25.25000
Production	12482.00000	10041.50000	13215.50000
Days	21.00000	20.00000	22.00000
Payroll	194.50000	181.50000	201.50000
Water Use	3484.50000	3340.00000	4050.50000

TUKEY'S MIDDLEMEANS

VARIABLE	MIDMEAN	TRIMEAN	MIDSPREAD
Temp(C)	19.29000	18.78750	12.35000
Production	12303.30000	12055.25000	3174.00000
Days	21.20000	21.00000	2.00000
Payroll	193.50000	193.00000	20.00000
Water Use	3544.00000	3589.87500	710.50000

VARIABLE	10-th percentile
Temp(C)	6.65000
Production	6256.00000
Days	19.50000
Payroll	152.00000
Water Use	3197.00000

VARIABLE	90-th percentile
Temp(C)	26.30000
Production	14837.50000
Days	23.00000
Payroll	205.00000
Water Use	4316.00000

```
*****************************************************************************
*                         SUMMARY STATISTICS                                *
*                           ON DATA SET:                                    *
*                              SAMPLE                                       *
*****************************************************************************
```

Subfile: FY'76

BASIC STATISTICS

VARIABLE	# OBSERVATIONS	# MISS. VALUES	SUM	MEAN
Temp(C)	12	0	212.60000	17.71667
Production	12	0	128130.00000	10677.50000
Days	12	0	257.00000	21.41667
Payroll	12	0	2170.00000	180.83333
Water Use	12	0	42330.00000	3527.50000

VARIABLE	VARIANCE	STANDARD DEV.	COEF OF SKEWNESS	COEF OF KURTOSIS
Temp(C)	50.45242	7.10299	-.24164	-1.10813
Production	10346097.9091 M7D.5D	3216.53508	-.47514	-1.24452
Days	2.99242	1.72986	.53169	-.47442
Payroll	563.60606	23.74039	-.78573	-.59264
Water Use	113867.00000	337.44185	.94917	.15129

VARIABLE	COEF VARIATION	STANDARD ERROR OF THE MEAN	95 % CONFIDENCE INTERVAL ON MEAN LOWER LIMIT	UPPER LIMIT
Temp(C)	40.09212	2.05046	13.20247	22.23086
Production	30.12442	928.53370	8633.28011	12721.71989
Days	8.07718	.49937	20.31728	22.51605
Payroll	13.12833	6.85326	165.74549	195.92118
Water Use	9.56603	97.41107	3313.04397	3741.95603

CORRELATION MATRIX

	Production	Days	Payroll	Water Use
Temp(C)	-.2644635	.4396054	-.1782662	-.0077412
Production		.1294077	.9312213	.6450021
Days			.0040583	.0388568
Payroll				.4892023

ORDER STATISTICS

VARIABLE	MAXIMUM	MINIMUM	RANGE	MIDRANGE
Temp(C)	27.20000	6.40000	20.80000	16.80000
Production	15022.00000	5736.00000	9286.00000	10379.00000
Days	25.00000	19.00000	6.00000	22.00000
Payroll	211.00000	134.00000	77.00000	172.50000
Water Use	4287.00000	3110.00000	1177.00000	3698.50000

		TUKEY'S HINGES	
VARIABLE	MEDIAN	25-th %-ile	75-th %-ile
Temp(C)	18.05000	12.90000	23.70000
Production	11781.50000	7341.50000	13111.00000
Days	21.50000	20.00000	22.50000
Payroll	191.50000	164.50000	196.50000
Water Use	3413.50000	3329.50000	3733.00000

	TUKEY'S MIDDLEMEANS		
VARIABLE	MIDMEAN	TRIMEAN	MIDSPREAD
Temp(C)	18.01667	18.17500	10.80000
Production	11405.50000	11003.87500	5769.50000
Days	21.16667	21.37500	2.50000
Payroll	187.16667	186.00000	32.00000
Water Use	3452.00000	3472.37500	403.50000

VARIABLE	10-th percentile
Temp(C)	6.90000
Production	6116.00000
Days	20.00000
Payroll	146.00000
Water Use	3180.00000

VARIABLE	90-th percentile
Temp(C)	25.20000
Production	13114.00000
Days	23.00000
Payroll	198.00000
Water Use	3852.00000

Subfile: FY'77

BASIC STATISTICS

VARIABLE	# OBSERVATIONS	# MISS. VALUES	SUM	MEAN
Temp(C)	8	0	153.90000	19.23750
Production	8	0	104513.00000	13064.12500
Days	8	0	168.00000	21.00000
Payroll	8	0	1581.00000	197.62500
Water Use	8	0	31260.00000	3907.50000

VARIABLE	VARIANCE	STANDARD DEV.	COEF OF SKEWNESS	COEF OF KURTOSIS
Temp(C)	60.11125	7.75314	-.91030	-.30986
Production	4953615.26714	2225.67187	-.06985	-.38572
Days	1.14286	1.06904	-.75000	-.50000
Payroll	65.41071	8.08769	-.76833	-.75334
Water Use	380730.85714	617.03392	.33651	-1.17044

VARIABLE	COEF VARIATION	STANDARD ERROR OF THE MEAN	95 % CONFIDENCE INTERVAL ON MEAN LOWER LIMIT	UPPER LIMIT
Temp(C)	40.30225	2.74115	12.75397	25.72103
Production	17.03652	786.89384	11202.91630	14925.33370
Days	5.09069	.37796	20.10602	21.89398
Payroll	4.09244	2.85943	190.86170	204.38830
Water Use	15.79102	218.15443	3391.50799	4423.49201

CORRELATION MATRIX

	Production	Days	Payroll	Water Use
Temp(C)	.0040190	-.0844548	-.4503797	.2109508
Production		.4515051	.4876946	.6262209
Days			.2643635	-.3473772
Payroll				.2501814

ORDER STATISTICS

VARIABLE	MAXIMUM	MINIMUM	RANGE	MIDRANGE
Temp(C)	26.30000	4.20000	22.10000	15.25000
Production	16716.00000	9315.00000	7401.00000	13015.50000
Days	22.00000	19.00000	3.00000	20.50000
Payroll	205.00000	183.00000	22.00000	194.00000
Water Use	4936.00000	3214.00000	1722.00000	4075.00000

		TUKEY'S HINGES	
VARIABLE	MEDIAN	25-th %-ile	75-th %-ile
Temp(C)	20.70000	15.30000	25.70000
Production	13109.00000	11777.50000	14354.50000
Days	21.00000	20.50000	22.00000
Payroll	200.00000	192.50000	204.00000
Water Use	3914.50000	3340.00000	4300.50000

	TUKEY'S MIDDLEMEANS		
VARIABLE	MIDMEAN	TRIMEAN	MIDSPREAD
Temp(C)	21.22500	20.60000	10.40000
Production	13132.75000	13087.50000	2577.00000
Days	21.25000	21.12500	1.50000
Payroll	199.75000	199.12500	11.50000
Water Use	3860.25000	3867.37500	960.50000

VARIABLE	10-th percentile
Temp(C)	4.20000
Production	9315.00000
Days	19.00000
Payroll	183.00000
Water Use	3214.00000
VARIABLE	90-th percentile
Temp(C)	26.10000
Production	14653.00000
Days	22.00000
Payroll	205.00000
Water Use	4345.00000

One Sample Tests

Object of Program

This program allows you to run a series of tests and plots on one variable (or subfile of one variable) from the data matrix defined by the Data Manipulation program. Each test will automatically sort or restore the data to its original form as needed.

Special Considerations

If your system configuration does not contain Graphics, then Serial Plot and Histogram may not be run. Since Serial Correlation is in the same file as these programs, ignore any message appearing on the CRT. When the program is loaded in, just press 'RUN'.

Methods and Formulas

In general, whenever a population is estimated, it is estimated by the maximum likelihood estimator.

Let N denote the same size, X_i denote the ith sample value, (i=1,2,...,N).

The sample mean: $\bar{X} = \sum_{i=1}^{N} X_i / N.$

The sample variance: $S^2 = (\sum_{i=1}^{N} X_i^2 - N \cdot \bar{X}^2)/(N-1).$

The sample standard deviation: $S = \sqrt{S^2}$.

Serial Correlation with lag k: $[\sum_{i=1}^{N-k} (X_i - \bar{X})(X_{i+k} - \bar{X})]/[\sum_{i=1}^{N} X_i^2 - N \cdot \bar{X}^2].$

Standard error of the mean: $S / \sqrt{N}$.

Coefficient of variation: $(S/\bar{X}) \cdot 100.$

Skewness: $[\sum_{i=1}^{N} X_i^3/N - 3\bar{X} \cdot \sum_{i=1}^{N} X_i^2/N + 2\bar{X}^3] / S^3$.

Kurtosis: $[\sum_{i=1}^{N} X_i^4/N - 4\bar{X} \cdot \sum_{i=1}^{N} X_i^3/N + 6\bar{X}^2 \cdot \sum_{i=1}^{N} X_i^2/N - 3\bar{X}^4] / S^4$.

A$(1-\alpha)$ x 100% confidence interval on the mean:

$$[\bar{X} - t_{N-1,\alpha/2} \cdot S/\sqrt{N} ; \bar{X} + t_{N-1,\alpha/2} \cdot S/\sqrt{N}]$$

where $t_{N-1,\alpha/2}$ is the value of the student's t-distribution with N−1 degrees of freedom such that

$P[T > t_{N-1,\alpha/2}] = \alpha/2$, where $t_{n-1,\alpha/2}$ is approximated by an asymptotic expansion.

A(1−∝)x100% confidence interval on the variance

$$= [(N - 1)S^2/\chi^2_{N-1,\ \alpha/2},\ (N - 1)S^2/\chi^2_{N-1,\ 1-\alpha/2}].$$

Where $\chi^2_{N-1,(\ )}$ is the value of the chi-square distribution with N−1 degrees of freedom such that $P[X > \chi^2_{N-1,\alpha/2}] = \alpha/2$ and $P[X > \chi^2_{N-1,1-\alpha/2}] = 1 - \alpha/2$.

$\chi^2_{N-1,\alpha/2}$ and $\chi^2_{N-1,1-\alpha/2}$ are approximated.

In the X^2−G.O.F. and K.S.−G.O.F. keys, the cumulative normal probabilities are computed by Simpson's Rule. The exponential and uniform cumulative probabilities are computed using their cumulative density function.

NOTE:

1. **Serial Correlation** allows you to check for randomness of the sample by computing the serial correlation with lag=0,1,2,3..., up to one-half the sample size.
2. **Serial Plot** produces a plot of the observed values against the observation number.
3. **Ranks** orders the data and then prints each distinct data point with its rank. Ties are assigned their average ranks.
4. **Histogram** plots the relative frequency for each cell, then prints the cell statistics.
5. **t-test** computes a one or two tailed student's t-test of the hypothesis:

   H0: μ=user specified mean.

   The computed t-value and corresponding one or two tailed probability are printed.
6. **Kolmogorov-Smirnov Goodness-of-fit test** performs a fit for a normal, exponential or uniform distribution as selected by the user.
7. **Chi-Square Goodness-of-fit test** computes a fit for a normal, exponential or uniform distribution as you select.
8. **Shapiro-Wilk test** performs a test for normality for a sample of size 3 to 50, inclusive.
9. **Runs test** counts the number of runs, N, above and below the median and then compares it to the statistic Wp for significance.

   $$Wp = N+1 + Xp\sqrt{\frac{N^2N}{2N-1}}$$

   where Xp is the pth quantile of a standard normal random variable.

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User Instructions

One Sample Start-Up

1. When "IS THE ONE SAMPLE TESTS OVERLAY IN PLACE?" is displayed:
   a. Put the overlay in place.
   b. Press: YES
2. When "TESTS ON WHICH VARIABLE NUMBER?" is displayed:
   a. Enter: The variable number
   b. Press: CONT
3. When "SUBFILE NUMBER? (0=IGNORE SUBFILES)" is displayed:
   a. Enter: The subfile number, or 0 to use all the observations of the variable.
   b. Press: CONT
4. "ONE SAMPLE TESTS" and the chosen variable and subfile names will be printed.
5. When "SELECT KEY" is displayed:
   a. Press: Any special function key defined on the overlay.
   b. If a One Sample Test is pressed, go to step 1 of that test.
   c. If the "Data Manipulation" key is pressed, go to step 6.
6. The following menu will be displayed:
   1. ONE SAMPLE TESTS
   2. MULTIPLE SAMPLE TESTS
   3. STATISTICAL DISTRIBUTIONS
   4. DATA MANIPULATION
7. When "CHOOSE A NUMBER AND PRESS CONT" is displayed:
   a. To pick another set of data for a one sample test:
      1. Enter: 1
      2. Press: CONT
      3. Go to step 1.

b. To perform multiple sample tests:

1. Enter: 2
2. Press: CONT
3. Go to step 1 of Multiple Sample Start-up.

c. To perform statistical distributions:

1. Enter: 3
2. Press: CONT
3. Go to step 1 of Distributions Start-up.

NOTE

The current data set in use has been stored in the data scratch file.)

---

d. To define another data base, or to manipulate the current one:

1. Enter: 4
2. Press: CONT
3. Go to step 4 of User Instructions in START.

1. "SERIAL CORRELATION" will be printed.
2. When "CORRELATION LAG=?" is displayed:
   a. Enter: The desired correlation lag. (0,1,2,...; up to one-half the sample size)
3. If an incorrect correlation lag is entered:
   a. "LAG=(incorrect correlation lag) IS OUTSIDE OF BOUNDS (0,½ the sample size) OR ISN'T AN INTEGER" is printed.
   b. Go to step 2.
4. The correlation value will be printed.
5. When "ENTER ANOTHER LAG?" is displayed:
   a. To enter another correlation lag:
      1. Press: YES
      2. Go to step 2.
   b. To continue with another test:
      1. Press: NO
      2. Got to step 5 of One Sample Start-up.

1. When "PLOTTER SELECT CODE=?" is displayed:

    a. Enter: The plotter select code.

    b. Press: CONT

    c. If plotter is set to CRT (select code 13), go to step 5.

2. When "USING HP-IB?" is displayed:
(See Appendix for explanation of HP-IB.)

    a. If using an HP-IB interface cable:

        1. Press: YES

        2. Go to step 3.

    b. If not using an HP-IB interface cable:

        1. Press: NO

        2. Go to step 4.

3. When "HP-IB ADDRESS=?" is displayed:

    a. Enter: HP-IB address of the plotter.

    b. Press: CONT

4. When "PLOTTER ID=?" is displayed:

    a. Enter: Plotter identifier string (9872A for the 9872 Plotter, INCREMENTAL for an incremental plotter)

    b. Press: CONT

5. The serial plot will then be plotted.

    a. Press: CONT

6. If "DO YOU WANT A HARD COPY?" is displayed:

    a. To have the plot dumped to the thermal line printer:

        1. Press: YES

    b. If no hard-copy is desired:

        1. Press: NO

7. If "ARE YOU USING PERFORATED PAPER?" is displayed:

   a. If you have perforated paper:

      1. Press: YES

   b. If you don't have perforated paper:

      1. Press: NO

8. To continue the program:

   a. Press: CONT

   b. Go to step 5 of One Sample Start-up.

1. "RANKED DATA" will be printed.
2. A table of all distinct data points and their ranks will be printed.
3. Go to step 5 of One Sample Start-up.

1. When "PLOTTER SELECT CODE=?" is displayed:

   a. Enter: The plotter select code.

   b. Press: CONT

   c. If plotter is set to CRT (select code 13) go to step 5.

2. When "USING HP-IB?" is displayed:
   (See Appendix for explanation of HP-IB)

   a. If using an HP-IB interface cable:

      1. Press: YES
      2. Go to step 3.

   b. If not using an HP-IB interface cable:

      1. Press: NO
      2. Go to step 4.

3. When "HP-IB ADDRESS=?" is displayed:

   a. Enter: HP-IB address of the plotter

   b. Press: CONT

4. When "PLOTTER ID=?" is displayed:

   a. Enter: Plotter identifier string (9872A for the 9872 Plotter, INCREMENTAL for an incremental plotter)

   b. Press: CONT

5. "HISTOGRAM", along with the minimum and maximum values of the data set will be printed.
6. When "OFFSET=?" is displayed:

   a. Enter: The desired lower bound for the first cell.

   b. Press: CONT

   c. "OFFSET = (entered offset)" will be printed.

7. When "NUMBER OF CELLS=?" is displayed:

   a. Enter: The desired number of cells.

   b. Press: CONT

   c. "NUMBER OF CELLS = (entered # of cells)" will be printed.

8. "OPTIMUM CELL WIDTH = (computed cell width)" will be printed.
9. When "CELL WIDTH=?" is displayed:
   a. Enter: The desired cell width.
   b. Press: CONT
   c. "CELL WIDTH = (the entered cell width)" will be printed.
10. If any observations are below the offset or above the last cell's upper bound, a message will be printed.
11. When "OFFSET AND CELL WIDTH OK?" is displayed:
    a. If these values are acceptable:
       1. Press: YES
       2. Go to step 12.
    b. If these values are incorrect:
       1. Press: NO
       2. Go to step 6.
12. The histogram will then be plotted.
13. To continue the program:
    a. Press: CONT
14. If "DO YOU WANT A HARD COPY?" is displayed:
    a. To receive a hard-copy of the histogram on the thermal line printer:
       1. Press: YES
    b. If a hard-copy is not desired:
       1. Press: NO
15. If "ARE YOU USING PERFORATED PAPER?" is displayed:
    a. If you have perforated paper:
       1. Press: YES
    b. If you don't have perforated paper:
       1. Press: NO
16. The cell numbers with their corresponding lower bounds and relative frequencies will be printed.
17. Go to step 5 of One Sample Start-up.

1. "ONE-SAMPLE t TEST" will be printed.
2. When "1 OR 2 TAIL TEST" is displayed:
   a. Enter: 1 or 2
   b. Press: CONT
   c. The type of test will be printed.
3. When "H0: MU=(Sample mean) OR =" is displayed:
   a. Enter: The mean value to be used for H0.
   b. Press: CONT
4. The null hypothesis, sample size, sample mean, standard deviation, standard error of the mean, computed t-value and degrees of freedom will be printed.
5. If computing a one tail test:
   a. "P(t> (computed t-value)) = (computed probability)" will be printed.
   b. Go to step 5 of One Sample Start-up.
6. If computing a two tail test:
   a. "1-P(−(computed t-value)< t < (computed t-value)) = (computed probability)" will be printed.
   b. Go to step 5 of One Sample Start-up.

1. "KOLMOGOROV-SMIRNOV GOODNESS-OF-FIT TEST" will be printed.
2. The following codes will be displayed:
   "GOODNESS-OF-FIT (GOF) CODES:"
   "1-NORMAL; 2=EXPONENTIAL; 3=UNIFORM"
3. When "SEE ABOVE: GOF CODE=" is displayed:
   Select one of the three distributions as the assumed form of the underlying distribution:
   a. For a normal GOF:
      1. Enter: 1
      2. Press: CONT
      3. Go to step 10.
   b. For an exponential GOF:
      1. Enter: 2
      2. Press: CONT
      3. Go to step 7.
   c. For a uniform GOF:
      1. Enter: 3
      2. Press: CONT
      3. Go to step 4.
4. When "LOWER & UPPER LIMIT ON UNIFORM=?" is displayed:
   a. Enter: The lower and upper limits separated by a comma.
   b. Press: CONT
5. "TEST ON UNIFORM (lower limit, upper limit)" will be printed.
6. Go to step 13.
7. When "MEAN = (sample mean) OR=?" is displayed:
   a. Enter: Hypothesized mean.
      (The hypothesized mean for exponential GOF must be greater than zero.)
   b. Press: CONT
8. "MEAN = (hypothesized mean)" will be printed.
9. Go to step 13.

10. When "MEAN = (sample mean) OR=?" is displayed:

    a. Enter: Hypothesized mean.

    b. Press: CONT

11. When "VARIANCE = (sample variance) OR=?" is displayed:

    a. Enter: Hypothesized variance.

    b. Press: CONT

12. "MEAN = (hypothesized mean) VARIANCE = (hypothesized variance)" will be printed.

13. "N = (sample size), KOLMOGOROV-SMIRNOV STATISTICS: DN = (computed value), SQR(N)*DN−KN = (computed value)" will be printed.

14. When "ANOTHER GOF CODE?" is displayed:

    a. To enter another code:

        1. Press: YES

        2. Go to step 2.

    b. To continue the program:

        1. Press: NO

        2. Go to step 5 of One Sample Start-up.

1. "CHI-SQUARE GOODNESS-OF-FIT TEST" will be printed.
2. The following codes will be displayed:
   "GOODNESS-OF-FIT (GOF) CODES:
   1 = NORMAL; 2 = EXPONENTIAL; 3 = UNIFORM"
3. When "SEE ABOVE: GOF CODE=?" is displayed:
   a. For a normal GOF:
      1. Enter: 1
      2. Press: CONT
      3. Go to step 7.
   b. For an exponential GOF:
      1. Enter: 2
      2. Press: CONT
      3. Go to step 6.
   c. For a uniform GOF:
      1. Enter: 3
      2. Press: CONT
      3. Go to step 4.
4. When "LOWER & UPPER LIMIT ON UNIF.=?" is displayed:
   a. Enter: The lower and upper limits, separated by a comma.
   b. Press: CONT
5. "TEST ON UNIFORM (lower limit, upper limit)" will be printed. Go to step 7.
6. If the sample mean is negative or zero, a message is displayed. Go to step 3.
7. When "OFFSET=?" is displayed:
   a. Enter: The lower limit of the first cell.
   b. Press: CONT

NOTE:
The offset must not be less than zero for GOF code = 2 and must not be less than the lower limit of the uniform distribution.

---

8. "OFFSET = (offset)" will be printed.
9. If the offset is greater than the maximum sample value:
   a. Go to step 8.
10. When "# OF CELLS = ?" is displayed:
    a. Enter: The number of desired cells.
    b. Press: CONT

NOTE:
Maximum number of cells allowed = 50.

---

11. "OPTIMUM CELL WIDTH = (optimum computed width)" will be printed.
12. When "CELL WIDTH = (optimum cell width) OR = ?" is displayed:
    a. Enter: The desired cell width.
    b. Press: CONT
13. "YOUR CELL WIDTH = (desired cell width)" is printed.
14. If there are observations either too large or too small for the specified offset, number of cells, and cell width, messages will be printed.
15. When "OFFSET AND CELL WIDTH OK?" is displayed:
    a. If the offset and cell width are acceptable:
       1. Press: YES
       2. Go to step 16.
    b. If the offset and cell width are not correct:
       1. Press: NO
       2. Go to step 7.
16. The cell numbers, lower limits, observed number of observations, and calculated expected number of observations for every cell will be printed.
17. The chi-square value and number of degrees of freedom will be printed.

18. When "ANOTHER GOF CODE?" is displayed:

    a. To run another chi-square test:

        1. Press: YES
        2. Go to step 2.

    b. To continue the program:

        1. Press: NO
        2. Go to step 5 of One Sample Start-up.

1. "SHAPIRO-WILK NORMALITY TEST" will be printed.

NOTE

If the sample size is less than 3 or greater than 50, a message is printed stating that this program will not work and to try a chi-square goodness of fit test for N>50. Go to step 5 of One Sample Start-up.

2. "W STATISTIC FOR NORMALITY (N=(sample size)) = (computed value)" is printed.
3. "% POINTS FOR W (SMALL VALUE SIGNIFICANT) .01, .02, .05, .1, .5" is printed, along with their corresponding W values.
4. Go to step 5 of One Sample Start-up.

1. "RUNS TEST" will be printed.
2. When "TEST FOR TOO FEW RUNS?" is displayed:

    a. To test for too few runs about the median:

        1. Press: YES

    b. To continue the program:

        1. Press: NO
        2. Go to step 6.

3. The following significance levels will be displayed:

    SIGNIFICANCE LEVELS

1.	.01
2.	.025
3.	.05

4. When "CHOOSE A NUMBER AND PRESS CONT" is displayed:

    a. Enter: The number corresponding to the desired significance level.
    b. Press: CONT

5. A message indicating if the test is significant will be printed.
6. When "TEST FOR TOO MANY RUNS?" is displayed:

    a. To test for too many runs about the median:

        1. Press: YES

    b. To continue the program:

        1. Press: NO
        2. Go to step 10.

7. The following significance levels will be displayed:

    SIGNIFICANCE LEVELS

1.	.01
2.	.025
3.	.05

8. When "CHOOSE A NUMBER AND PRESS CONT" is displayed:

   a. Enter: The number corresponding to the desired significance level.

   b. Press: CONT

9. A message indicating if the test is significant will be printed.

10. Go to step 5 of One Sample Start-up.

Examples

One Hundred Failure – Time Data

One hundred observations of the time until failure of an electronic circuit were obtained from a life testing experiment. The coded data values are shown below. The serial correlations with lag 1 and lag 2 were quite small indicating apparent "independence" of the observation. Also, a serial plot of the data shows no particular patterns. The Runs test further confirms the randomness of the data.

This type of data is assumed to come from an exponential random variable with parameter m mean = 1.0. The histogram of the data indicates that this assumption might be valid. If the data really is exponential with parameter $\lambda = 1$, then the sample mean and standard deviation also should be about 1. From the output we see that x = 1.0856 and s = .9301 which do not differ from 1 by a great deal. This is confirmed by the one sample t test.

Both the Chi-square Goodness of Fit Test and the Kolmogorov-Smirnov Goodness of Fit Test indicate that we cannot reject the hypothesis that the data came from an exponentially distributed population with parameter $\lambda = 1$. The χ^2 test yields a test statistc of 9.248 with 8 degrees of freedom, which is not significant even at the $\alpha = .10$ level. The K-S test statistic DN = .09907, is not significant at $\alpha = .20$ level. However, both tests (χ^2 and K-S) indicate that the data is not normally distributed.

Since the sample size for this example was too large to perform a Shapiro Wilk Normality test, half of the observations were selected to give you an idea of the output.

TIME DATA

Data file name:
Number of observations: 100
Number of variables: 1

Variables names:
1. X1

TIME DATA

VARIABLE # 1

I	OBS(I)	OBS(I+1)	OBS(I+2)	OBS(I+3)	OBS(I+4)
1	2.00790	2.45450	2.55760	.50250	1.71430
6	1.71430	2.52460	.84390	2.99900	.32220
11	.18180	3.38780	1.71490	.16020	.10360
16	.53530	1.18870	.01480	.03510	.21580
21	.84770	1.85770	1.08500	3.25370	1.73570
26	1.03880	1.72300	1.72300	1.95580	.89840
31	.14220	.12790	1.49950	.11010	3.37350
36	.60190	1.90800	.52140	.29580	.49730
41	1.63010	.05740	1.08360	.57650	2.25210
46	2.72780	.83400	1.14640	.02070	.23900
51	3.84480	1.29530	.81290	.85020	.97390
56	.43280	.83970	1.08490	.95980	.51170
61	.89530	2.51070	.32380	1.06270	3.21960
66	1.20550	.39400	.29730	1.27110	.98670
71	2.31500	.48060	1.34410	.78670	2.28790
76	.12190	.54020	3.11250	.17480	.06320
81	.65310	.54450	.01050	.18050	.46430
86	.55340	.99490	.28950	1.26600	.15090
91	1.51270	1.53900	.77450	.14300	.44900
96	.43340	.16540	1.76060	.40100	.43230

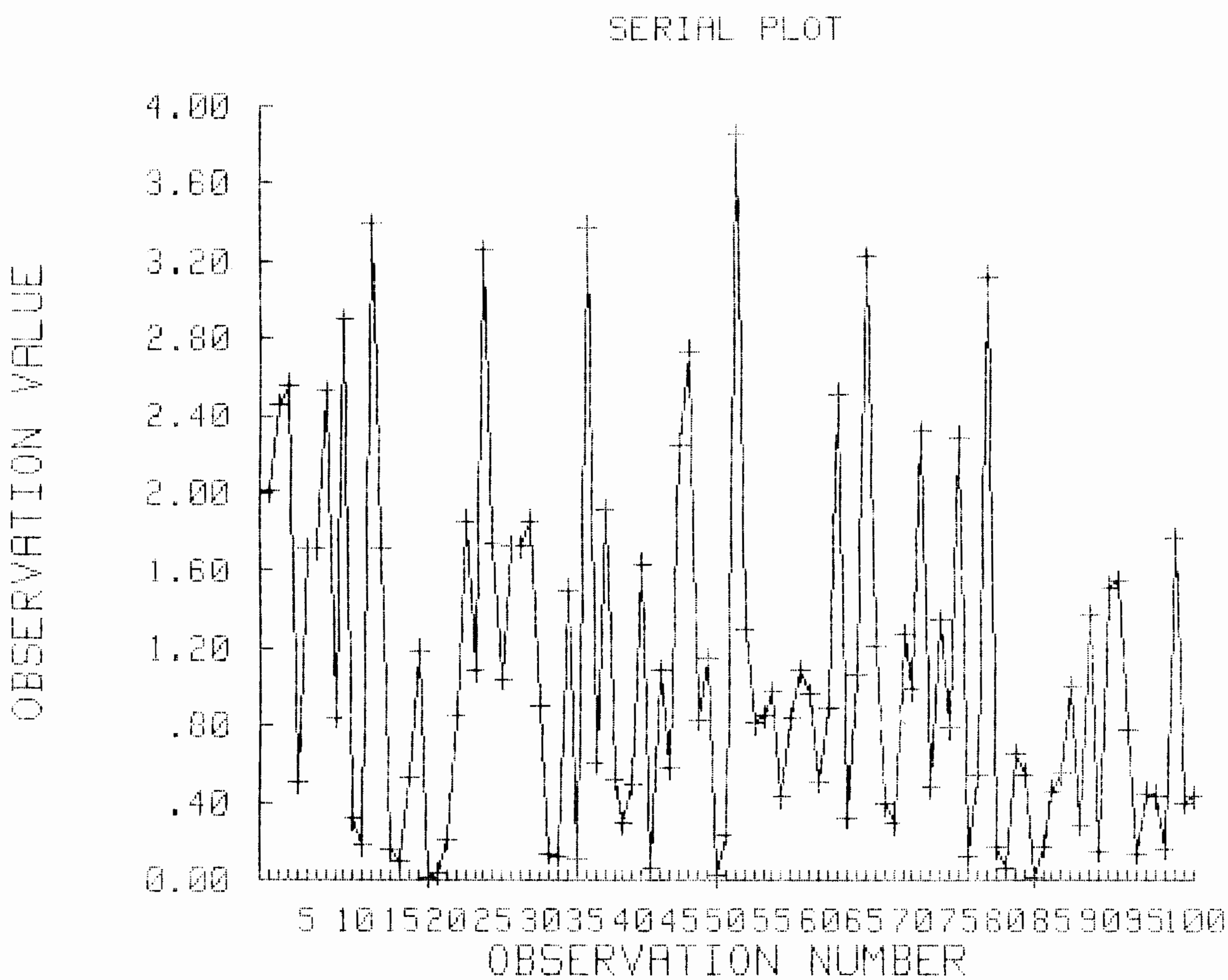
ONE SAMPLE TESTS

VARIABLE --X1

SERIAL CORRELATION

SERIAL CORRELATION WITH LAG = 1 IS .01605

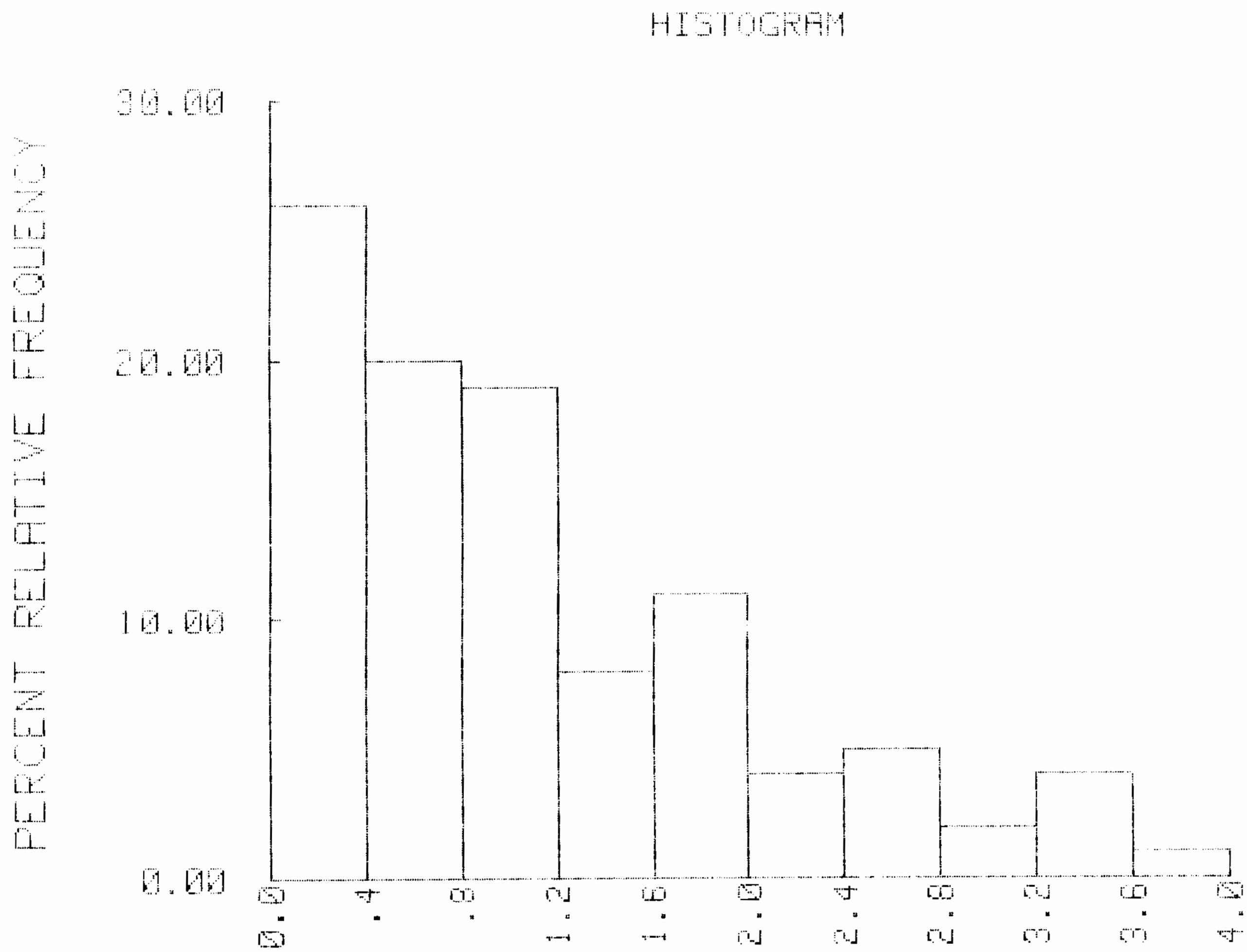
SERIAL CORRELATION WITH LAG = 2 IS -.01235



HISTOGRAM

MINIMUM VALUE = .0105
MAXIMUM VALUE = 3.8448

OFFSET = 0.0000
NUMBER OF CELLS = 10
OPTIMUM CELL WIDTH = .3845
CELL WIDTH = .4000



CELL #	LOWER BOUND	RELATIVE FREQUENCY
1	0.000	26
2	.400	20
3	.800	19
4	1.200	8
5	1.600	11
6	2.000	4
7	2.400	5
8	2.800	2
9	3.200	4
10	3.600	1

CHI-SQUARE GOODNESS-OF-FIT TEST
GOODNESS-OF-FIT CODE = 2

OFFSET = 0.0000
# OF CELLS = 10.0000
OPTIMUM CELL WIDTH = .3845

YOUR CELL WIDTH = .4000

CELL #	LOWER LIMIT	OBSERVED # OF OBS.	EXPECTED # OF OBS.
1	0.0000	26	30.82
2	.4000	20	21.32
3	.8000	19	14.75
4	1.2000	8	10.20
5	1.6000	11	7.06
6	2.0000	4	4.88
7	2.4000	5	3.38
8	2.8000	2	2.34
9	3.2000	4	1.62
10	3.6000	1	1.12

CHI-SQUARE GOODNESS-OF-FIT FOR EXPONENTIAL DISTRIBUTION

CHI-SQUARE VALUE = 9.248; DEGREES OF FREEDOM = 8

KOLMOGOROV-SMIRNOV GOODNESS-OF-FIT TEST

MEAN= 1.0000

N= 100, KOLMOGOROV-SMIRNOV STATISTICS: DN = .09907
SQR(N)*DN - KN = .99

RUNS TEST

OF RUNS IS NOT SIGNIFICANT AT THE .05 SIGNIFICANCE LEVEL FOR TOO FEW RUNS
OF RUNS IS NOT SIGNIFICANT AT THE .05 SIGNIFICANCE LEVEL FOR TOO MANY RUNS

RANKED DATA:
****

(RANK	DISTINCT DATA POINT)	(RANK	DISTINCT DATA POINT)	(RANK	DISTINCT DATA POINT)
(1.00	.0105)	(2.00	.0148)	(3.00	.0207)
(4.00	.0351)	(5.00	.0574)	(6.00	.0632)
(7.00	.1036)	(8.00	.1101)	(9.00	.1219)
(10.00	.1279)	(11.00	.1422)	(12.00	.1430)
(13.00	.1509)	(14.00	.1602)	(15.00	.1654)
(16.00	.1748)	(17.00	.1805)	(18.00	.1818)
(19.00	.2158)	(20.00	.2390)	(21.00	.2895)
(22.00	.2958)	(23.00	.2973)	(24.00	.3222)
(25.00	.3238)	(26.00	.3940)	(27.00	.4010)
(28.00	.4323)	(29.00	.4328)	(30.00	.4334)
(31.00	.4490)	(32.00	.4643)	(33.00	.4806)
(34.00	.4973)	(35.00	.5025)	(36.00	.5117)
(37.00	.5214)	(38.00	.5353)	(39.00	.5402)
(40.00	.5445)	(41.00	.5534)	(42.00	.5765)
(43.00	.6019)	(44.00	.6531)	(45.00	.7745)
(46.00	.7867)	(47.00	.8129)	(48.00	.8340)
(49.00	.8397)	(50.00	.8439)	(51.00	.8477)
(52.00	.8502)	(53.00	.8953)	(54.00	.8984)
(55.00	.9598)	(56.00	.9739)	(57.00	.9867)
(58.00	.9949)	(59.00	1.0388)	(60.00	1.0627)
(61.00	1.0836)	(62.00	1.0849)	(63.00	1.0850)
(64.00	1.1464)	(65.00	1.1887)	(66.00	1.2055)
(67.00	1.2711)	(68.00	1.2953)	(69.00	1.3441)
(70.00	1.3660)	(71.00	1.4995)	(72.00	1.5127)
(73.00	1.5390)	(74.00	1.6301)	(75.50	1.7143)
(77.00	1.7149)	(78.50	1.7230)	(80.00	1.7357)
(81.00	1.7606)	(82.00	1.8558)	(83.00	1.8577)
(84.00	1.9080)	(85.00	2.0079)	(86.00	2.2521)
(87.00	2.2879)	(88.00	2.3150)	(89.00	2.4545)
(90.00	2.5107)	(91.00	2.5248)	(92.00	2.5576)
(93.00	2.7278)	(94.00	2.8990)	(95.00	3.1125)
(96.00	3.2196)	(97.00	3.2537)	(98.00	3.3735)
(99.00	3.3878)	(100.00	3.8448)		

ONE-SAMPLE t-TEST

2 TAIL TEST

H0: MU= 1.0000

N=	100
MEAN=	1.0856
STD DEV =	.9301
STD ERROR OF MEAN=	.0930
T=	.9204
DF=	99

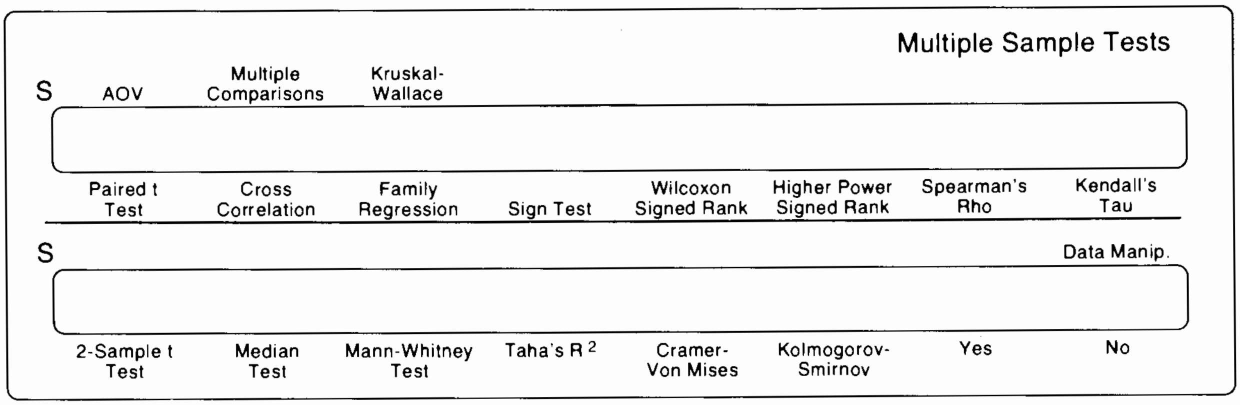
1 − P(−.9204 < t < .9204) = .3596

B. Since the sample size for this example was too large to perform a Shapiro Wilk Normality test, half of the observations were selected to give you an idea of the output.

```
ONE SAMPLE TESTS

VARIABLE --X1
SUBFILE  --ONE


SHAPIRO-WILK NORMALITY TEST

W STATISTIC FOR NORMALITY (N= 50 ) =        .904821834706
                         % POINTS FOR W (SMALL VALUE SIGNIFICANT)
                               .01       .02       .05       .1       .5
CORRESPONDING W VALUES:   .93       .938      .947      .955     .974
```

Multiple Sample Tests

Introduction



Object of Program

This program allows you to run a series of tests on more than one variable or subfile from the data matrix defined by the Data Manipulation program. (The only exception is the Multiple Comparisons program which will be discussed later.) Each test will automatically sort, rank or restore the data to its raw form as needed.

Special Considerations

In the Multiple Sample Start-up program, you may select which kind of test to run and which data under the following guidelines:

1. For Paired Sample tests, two variables or the same subfile of two variables must be used.
2. For Two Independent Sample tests, two different subfiles of the same variable must be used.
3. For ⩾3 Sample tests, three or more different subfiles of the same variable must be used.

If you specify data for one type of test (for example, Paired Samples), you will not be allowed to run a test of a different kind (for example, ⩾3 Samples) without returning to the Multiple Sample Start-up. You would then redefine the segment of the data matrix to be tested.

References

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Boardman, T.J., and Moffitt, D.R., "Graphical Monte Carlo Type I Error Rates for Multiple Comparison Procedures", Biometrics, 27: September 1971.

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Dixon, Wilfred and Massey, Frank, Introduction to Statistical Analysis, McGraw-Hill, New York, 1969, pp. 119-123.

Draper, N.R. and Smith, H., Applied Regression Analysis, John Wiley & Sons, New York, 1966, pp. 7-20.

Mielke, P.W. (1967), "Note on Some Squared Rank Tests with Existing Ties". Technometrics, 9:312.

Mielke, P.W. (1972), "Asymptotic Behavior of Two-Sample Tests Based on Powers of Ranks for Detecting Scales and Location Alternatives".

Mosteller, F. and Robert E.K. Rourke (1973), Sturdy Statistics. Addison-Wesley Publishing Co., Reading, Mass.

Siegel, S. (1956), Nonparametric Statistics. McGraw-Hill, New York.

Snedecor, George and Cochran, William, Statistical Methods, Iowa State University Press, Ames, Iowa; 1971, pp. 91-119.

Multiple Sample Start-up

User Instructions

1. When "IS THE MULTIPLE SAMPLE OVERLAY IN PLACE?" is displayed:

   a. Put the multiple sample overlay in place.

   b. Press: YES

NOTE

If you have just come from the Multiple Sample Start-up, go to step 4 for two paired sample tests, or go to step 10 for two independent sample tests, or go to step 14 for multiple sample tests.)

---

2. The following menu will be displayed:

   1. PAIRED SAMPLES
   2. TWO INDEPENDENT SAMPLES
   3. MULTIPLE SAMPLES

3. When "CHOOSE A NUMBER AND PRESS CONT" is displayed:

   a. To run two paired sample tests:

      1. Enter: 1
      2. Press: CONT
      3. Go to step 4.

   b. To run two independent sample tests:

      1. Enter: 2
      2. Press: CONT
      3. Go to step 10.

   c. To run multiple sample tests:

      1. Enter: 3
      2. Press: CONT
      3. Go to step 14.

4. If the data being tested contains only one variable:

   a. "ONLY ONE VARIABLE HAS BEEN DEFINED, SO PAIRED SAMPLE TESTS MAY NOT BE RUN." is displayed.

   b. Go to step 2.

5. When "VARIABLE NUMBER FOR X=?" is displayed:

   a. Enter: The variable number for the X sample.

   b. Press: CONT

6. When "VARIABLE NUMBER FOR Y=?" is displayed:

   a. Enter: The variable number for the Y sample.

   b. Press: CONT

7. When "WHAT SUBFILE NUMBER? (0 = IGNORE SUBFILE) is displayed:

   a. To use subfile boundaries on the chosen variables:

      1. Enter: The subfile number.
      2. Press: CONT

   b. To compare all observations:

      1. Enter: 0
      2. Press: CONT

8. "PAIRED SAMPLE TESTS", and the chosen variable and subfile names will be printed.
9. Go to step 18.
10. If the data being used contains one or no subfiles:

    a. "IMPROPER NUMBER OF SUBFILES AVAILABLE. TWO INDEPENDENT SAMPLE TESTS MAY NOT BE RUN." will be displayed.

    b. Go to step 2.

11. When "VARIABLE NUMBER=?" is displayed:

    a. Enter: The variable number.

    b. Press: CONT

    c. "TWO INDEPENDENT SAMPLE TESTS" will be printed.

    d. The variable name will be printed.

12. When "SUBFILE NUMBER FOR THE 'X' DATA?" is displayed:

    a. Enter: The subfile number for sample X.

    b. Press: CONT

    c. The subfile name will be printed.

13. When "SUBFILE NUMBER FOR THE 'Y' DATA?" is displayed:

    a. Enter: The subfile number for sample Y.

    b. Press: CONT

    c. The subfile name will be printed.

    d. Go to step 18.

14. If the data being used contains less than three subfiles:

    a. "IMPROPER NUMBER OF SUBFILES AVAILABLE. MULTIPLE SAMPLE TESTS MAY NOT BE RUN." is displayed.

    b. Go to step 2.

15. When "NUMBER OF TREATMENTS=?" is displayed:

    a. Enter: The number of samples.

    b. Press: CONT

16. When "VARIABLE NUMBER=?" is displayed:

    a. Enter: The variable number.

    b. Press: CONT

    c. "MULTIPLE SAMPLE TESTS" and the variable name will be printed.

17. When "SUBFILE NUMBER FOR TREATMENT # (treatment number)=?" is displayed:

    a. Enter: The desired subfile number.

    b. Press: CONT

    c. The subfile name will be printed.

    d. Repeat: Step 17 for each treatment.

18. When "SELECT KEY" is displayed:

    a. Press: Any special function key defined on the overlay, corresponding to the type of data selected.

    b. If a particular test is selected, go to step 1 of that test.

    c. If the "Data Manipulation" key is pressed, go to step 19.

NOTE

If the type of data does not correspond to the kind of test chosen, a message will be displayed.

---

19. The following menu will be displayed:

    1. ONE SAMPLE TESTS
    2. MULTIPLE SAMPLE TESTS
    3. STATISTICAL DISTRIBUTIONS
    4. DATA MANIPULATION

20. When "CHOOSE A NUMBER AND PRESS CONT" is displayed:

    a. To choose another set of samples for paired, independent or multiple sample tests:

        1. Enter: 2
        2. Press: CONT
        3. Go to step 2.

    b. To perform one sample test:

        1. Enter: 1
        2. Press: CONT
        3. Go to step 1 of the One Sample Start-up.

    c. To perform statistical distributions:

        1. Enter: 3
        2. Press: CONT
        3. Go to step 1 of Distributions Start-up.

    d. To define another data base, or manipulate the current one:

        1. Enter: 4
        2. Press: CONT
        3. Go to step 4 of User Instructions in START.

Example

Paired Sample Tests:

```
IS THE MULTIPLE SAMPLE OVERLAY IN PLACE?
 YES
VARIABLE NUMBER FOR X =?
1
VARIABLE NUMBER FOR Y =?
2
PAIRED SAMPLE TESTS

VARIABLE FOR X --    X1
VARIABLE FOR Y --    X2
WHAT SUBFILE NUMBER? (0=IGNORE SUBFILES)
0
SELECT KEY
```

Two Independent Sample Tests:

```
IS THE MULTIPLE SAMPLE OVERLAY IN PLACE?
 YES
VARIABLE NUMBER =?
1
TWO INDEPENDENT SAMPLE TESTS

VARIABLE --            X1
SUBFILE NUMBER FOR THE 'X' DATA?
1
X SUBFILE --           SUB1
SUBFILE NUMBER FOR THE 'Y' DATA?
3
Y SUBFILE --           SUB3
```

≥3 Sample Tests:

```
IS THE MULTIPLE SAMPLE OVERLAY IN PLACE?
 YES
NUMBER OF TREATMENTS =?
3
VARIABLE NUMBER =?
1
MULTIPLE SAMPLE TESTS

VARIABLE --           X1
SUBFILE NUMBER FOR TREATMENT # 1  =
?
1
SUBFILE --            SUB1
SUBFILE NUMBER FOR TREATMENT # 2  =
?
2
SUBFILE --            SUB2
SUBFILE NUMBER FOR TREATMENT # 3  =
?
3
SUBFILE --            SUB3
SELECT KEY
```

Two Paired Sample Tests

Methods and Formulas

1. **Paired-t test** is performed on the differences between two paired samples. The null hypothesis is that the population average difference between the two samples is D (user specified) or that U_d=D. A large t value indicates rejection of the null hypothesis.
2. **Cross Correlation** provides a correlation between two paired samples with a lag between them.
3. **Family Regression** provides four different regression models. All of the models are solved (except quadratic) by "linearizing" the model to the form f(y)="b"+"a" g(x) and solving by ordinary linear least squares. The AOV table which is printed out for each model is in units of the transformed y's (f(y)). R^2, the squared multiple correlation coefficient is expressed in units of the transformed y's. The following models are provided:

Linear $y=ax+b$

Quadratic $y=ax^2+bx+c$

Exponential $y=a\exp(bx)$

Power $y=ax_b$

4. **Sign Test** determines the number of positive differences (T) and a normalized T, based on T being distributed binomially under the hypothesis. Points where $X_i=Y_i$ are excluded from the analysis, reducing the total number of pairs (N). The normalized T is given by

$$\frac{T - \frac{N}{2}}{\sqrt{\frac{N}{4}}}$$

5. **Wilcoxon Signed Rank** ranks the N differences, $|X_i - Y_i|$ from greatest to smallest. T, the test statistic, is given by the sum of the ranks of the positive differences. Pairs for which $X_i = Y_i$ are excluded from the analysis. Two standard normal deviates based on the conditional distribution of the existing ties are output: 1) with correction for continuity, 1/2, and 2) without the correction for continuity. The standard normal deviates are derived from:

$$\frac{T - \mu_T}{\sigma_T} \quad \text{where}$$

$$\mu_T = \frac{N(N+1)}{4}, \quad \text{or} \quad \mu_T = \frac{N(N+1)}{4} - \frac{1}{2}$$

and $\sigma T = (2n+1)\mu T/6$

6. **Higher Power Signed Rank** ranks the N differences, $X_i - Y_i$, from smallest to greatest. T, the test statistic, is given by the sum of the ranks of the positive differences raised to the specified power (2, 3, 4, or 5). Note that if the power specified were 1, this test is the Wilcoxon Signed Rank test, and if the power were 0, this test is the Sign test.

   Using higher powers of the ranks can lead to a more powerful test when it is desired to weigh larger values more heavily. This would be true in highly skewed distributions.

   In addition to T, the output consists of a standard normal deviate based on the existing ties as given by:

$$\frac{T - \mu_T}{\sigma_T} \quad \text{where} \quad \mu_T = \frac{1}{2} \sum_{j=1}^{k} \sum_{S_j+1}^{S_j} I^r \quad \text{and} \quad \sigma_T^2 = \frac{1}{4} \sum_{j=1}^{k} \frac{1}{t_j} \left(\sum_{I=S_{j-1}+1}^{S_j} I^r \right)^2$$

   and t_j = the number of distinct observations at the jth data point and $s_j = \sum_{i=1}^{j} t_j$

7. **Spearman's Rho** obtains from the N pairs, the measure of rank correlation, or degree of association between two samples X and Y, given by ρ, where

$$d = \sum_{i=1}^{N} \left(R_{X_i} - R_{Y_i} \right)^2 , \quad \text{and} \quad \rho = 1 - \frac{6d}{N(N^2-1)}$$

   (R_{x_i} is the rank of X_i among the X's and R_{Y_i} is the rank of Y_i among the Y's.)

8. **Kendall's Tau** determines from the N pairs, the number of concordant pairs, P_c, and the number of discordant pairs, P_d, for the permutations of N pairs taken two pairs at a time. A pair $((X_i, Y_i)$ and $(X_j, Y_j))$ is concordant if $X_i - X_j > 0$ and $Y_i - Y_j > 0$ or $X_i - X_j < 0$ and $Y_i - Y_j < 0$. If $X_i = X_j$ or $Y_i = Y_j$, the pair is disregarded, and the pair is discordant otherwise. The test statistic is given by

$$\tau = \frac{P_c - P_d}{N(N-1)/2}$$

User Instructions

1. "PAIRED-t-TEST" will be printed.
2. When "1 or 2 TAILED?" is displayed:

   a. For a one tail test:

      1. Enter: 1
      2. Press: CONT
      3. Go to step 3.

   b. For a two tailed test:

      1. Enter: 2
      2. Press: CONT
      3. Go to step 6.

3. When "H0:MU(X)−MU(Y)=?" is displayed:

   a. Enter: The hypothesized difference between the means of the two populations, noting the sign.

   b. Press: CONT

4. The following hypotheses will be printed:

   "H0: MU(X) − MU(Y) = (hypothesized difference)
   H1: MU(X) − MU(Y) > (<) (hypothesized difference)"

5. Go to step 8.
6. When "H0: MU(X) − MU(Y) =" is displayed:

   a. Enter: The hypothesized difference.

   b. Press: CONT

7. The following hypotheses will be printed:

   "H0: MU(X) − MU(Y) = (hypothesized difference)
   H1: MU(X) − MU(Y)= (hypothesized difference)"

8. When "LEVEL OF SIGNIFICANCE" is displayed:

   a. Enter: A number between .005 and .3.

   b. Press: CONT

9. The calculated t value, the degrees of freedom and calculated probability will be printed.
10. A message to either accept or reject H0 at the chosen level of significance will be printed.
11. When "ANOTHER TEST?" is displayed:

    a. To perform another paired-t test:

        1. Press: YES
        2. Go to step 2.

    b. To another paired sample test:

        1. Press: NO
        2. Go to step 18 of Multiple Sample Start-up.

1. "CROSS CORRELATION" will be printed.
2. When "LAG ON X OR Y" is displayed:

   a. For a lag on X:

      1. Enter: X
      2. Press: CONT

   b. For a lag on Y:

      1. Enter: Y
      2. Press: CONT

3. When "LAG ON (X or Y) = ?" is displayed:

   a. Enter: The lag desired for the correlation.
   b. Press: CONT

4. "Lag on (X or Y) # (lag) COEFF. = (calculated coefficient)" will be printed.
5. When "ANOTHER CROSS CORRELATION?" is displayed:

   a. To run another cross correlation:

      1. Press: YES
      2. Go to step 2.

   b. To run another paired sample test:

      1. Press: NO
      2. Go to step 18 of Multiple Sample Start-up.

1. "FAMILY REGRESSION / AOV" will be printed.
2. The following regression codes will be displayed:

REGRESSION CODES

0:	FINISHED
1:	Y=A+BX
2:	Y=A+BX+CX∧2
3:	Y=Ae∧BX
4:	Y=AX∧B

3. When "REGRESSION CODE=?" is displayed:

   a. To select another paired sample test:

      1. Enter: 0
      2. Press: CONT
      3. Go to step 18 of Multiple Sample Start-up.

   b. To choose a linear model:

      1. Enter: 1
      2. Press: CONT
      3. "AOV: LINEAR REG OF Y ON X CODE 1" will be printed.

   c. To choose a quadratic model:

      1. Enter: 2
      2. Press: CONT
      3. "AOV: QUADRATIC CODE 2" will be printed.

   d. To choose an exponential model:

      1. Enter: 3
      2. Press: CONT
      3. "AOV: EXPONENTIAL CODE 3" will be printed.

   e. To choose a power model:

      1. Enter: 4
      2. Press: CONT
      3. "AOV: POWER CODE 4" will be printed.

4. The regression will be performed and the AOV table will be printed.
5. When "EVALUATE Y AT X?" is displayed:
    a. To evaluate Y at X:
        1. Press: YES
        2. Go to step 6.
    b. To choose another regression code:
        1. Press: NO
        2. Go to step 2.
6. When "AT ALL X(I)'S?" is displayed:
    a. To estimate Y at all X values:
        1. Press: YES
        2. Go to step 7.
    b. To estimate Y at selected X values:
        1. Press: NO
        2. Go to step 9.
7. A listing of all X(I) vs. Y is printed with Y(I) and RES(I)=Y −Y(I).
8. To continue the program:
    a. Press: CONT
    b. Go to step 2.
9. When "YHAT EVAL. AT X=?" is displayed:
    a. Enter: X value
    b. Press: CONT
    c. X and Y are printed.
10. When "ANOTHER YHAT?" is displayed:
    a. To evaluate another X value:
        1. Press: YES
        2. Go to step 9.

b. To choose another regression code:

1. Press: NO

2. To continue the program:
   a. Press: CONT
   b. Go to step 2.

1. "SIGN TEST" will be printed.
2. The following output is printed:
   N=(number of pairs)
   NUMBER OF POSITIVE DIFFERENCES = (THE (#) POINTS WHERE X(I)=Y(I) ARE EXCLUDED FROM THE TEST) YIELDS AN APPROX. STD. NOR. DEV.=
3. Go to step 18 of Multiple Sample Start-up.

1. "WILCOXON SIGNED RANK" will be printed.
2. The following is then printed:

   N=(# of differences)

   SUM OF POSITIVE RANKS=(#)

   USING RANKS OF X(I)−Y(I) AND EXCLUDING THE (#) POINTS WHERE X(I)=Y(I)

   YIELDS APPROXIMATE STANDARD NORMAL DEVIATES

   1) WITHOUT CORRELATION FOR CONTINUITY:
      A) NOT COMPENSATING FOR TIED DIFFERENCES: (#)
      B) CONDITIONAL ON THE EXISTING TIED DIFFERENCES: (#)

   2) WITH CORRECTION FOR CONTINUITY:
      A) NOT COMPENSATING FOR TIED DIFFERENCES (#)
      B) CONDITIONAL ON THE EXISTING TIED DIFFERENCES: (#)

3. Go to step 18 of Multiple Sample Start-up.

1. "HIGHER POWERED SIGNED RANKS" will be printed.
2. When "POWER OF THE RANK" is displayed:
   a. Enter: The desired power (2 through 5).
   b. Press: CONT
3. The sum of the positive ranks and the standard normal deviate will be printed.
4. Go to step 18 of Multiple Sample Start-up.

1. "SPEARMAN's RHO" will be printed.
2. The sum of the squared rank differences and rho will be printed.
3. Go to step 18 of Multiple Sample Start-up.

1. "KENDALL'S TAU" will be printed
2. The number of concordant and discordant pairs and tau will be printed.
3. Go to step 18 of Multiple Sample Start-up.

Examples

Pig Weight Changes

176 pigs were paired on the basis of sex, age, and initial weight, and fed daily one of two iron compounds to supplement that which they lacked due to confinement in pens. It was desired to determine if there was any difference in pig weight due to the two different compounds as applied over a one month period. From the Paired-t test and the correlation coefficient we see the difference is not significant.

```
                                        PIGS

     Data file name:
     Number of observations: 88
     Number of variables: 2

     Variables names:
        1. CMPD#1
        2. CMPD#2
```

PIGS

OBS#	Variable # 1	Variable # 2
1	54.00000	46.00000
2	44.00000	42.00000
3	46.00000	44.00000
4	54.00000	44.00000
5	45.00000	45.00000
6	46.00000	52.00000
7	50.00000	51.00000
8	43.00000	55.00000
9	47.00000	60.00000
10	40.00000	43.00000
11	40.00000	20.00000
12	46.00000	48.00000
13	52.00000	54.00000
14	50.00000	55.00000
15	54.00000	62.00000
16	49.00000	41.00000
17	30.00000	48.00000
18	50.00000	45.00000
19	48.00000	46.00000
20	38.00000	31.00000
21	27.00000	35.00000
22	50.00000	59.00000
23	107.00000	135.00000
24	77.00000	90.00000

25	91.00000	98.00000
26	88.00000	98.00000
27	93.00000	96.00000
28	89.00000	74.00000
29	95.00000	98.00000
30	105.00000	133.00000
31	107.00000	126.00000
32	95.00000	91.00000
33	114.00000	52.00000
34	128.00000	98.00000
35	110.00000	119.00000
36	104.00000	105.00000
37	94.00000	110.00000
38	87.00000	81.00000
39	66.00000	83.00000
40	96.00000	112.00000
41	120.00000	104.00000
42	90.00000	101.00000
43	95.00000	88.00000
44	86.00000	86.00000
45	158.00000	221.00000
46	125.00000	176.00000
47	149.00000	150.00000
48	175.00000	176.00000
49	196.00000	209.00000
50	121.00000	118.00000
51	181.00000	180.00000
52	201.00000	238.00000
53	175.00000	196.00000
54	147.00000	138.00000
55	209.00000	133.00000
56	194.00000	159.00000
57	203.00000	209.00000
58	179.00000	205.00000
59	170.00000	201.00000
60	148.00000	149.00000
61	138.00000	159.00000
62	232.00000	230.00000
63	223.00000	198.00000
64	151.00000	161.00000
65	142.00000	147.00000
66	167.00000	176.00000
67	210.00000	320.00000
68	240.00000	267.00000
69	245.00000	221.00000
70	263.00000	247.00000
71	263.00000	293.00000
72	182.00000	211.00000
73	261.00000	178.00000
74	280.00000	320.00000
75	264.00000	266.00000
76	187.00000	178.00000
77	280.00000	199.00000
78	287.00000	230.00000
79	230.00000	256.00000
80	234.00000	272.00000

```
81
      238.00000       245.00000
82
      202.00000       222.00000
83
      202.00000       245.00000
84
      317.00000       243.00000
85
      293.00000       264.00000
86
      215.00000       215.00000
87
      171.00000       172.00000
88
      242.00000       233.00000



PAIRED SAMPLE TESTS

VARIABLE FOR X --   CMPD#1
VARIABLE FOR Y --   CMPD#2


PAIRED-t TEST

 1
H0 : MU(X)-MU(Y) = 0
H1 : MU(X)-MU(Y) < 0

T VALUE =                       -.736
DF =                           87

T(       .950, 87 ) =          1.663

          DO NOT REJECT H0 AT  .050 LEVEL OF SIGNFICANCE



CROSS CORRELATION

  LAG ON  Y =   1    COEFF. =  .8513

  LAG ON  Y =   2    COEFF. =  .8253

  LAG ON  Y =   3    COEFF. =  .8823

  LAG ON  Y =  22    COEFF. =  .8905
```

AOV : LINEAR REG OF Y ON X CODE 1

SOURCE	SS	DF	MS	F RATIO
REG	481475.711	1	481475.711	581.18
RES	71246.789	86	828.451	
TOTAL COR	552722.500	87		
R SQUARED =	.871			

PAIRED SAMPLE TESTS

VARIABLE FOR X -- # BOARDING
VARIABLE FOR Y -- SERV TIME

AOV : LINEAR REG OF Y ON X CODE 1

SOURCE	SS	DF	MS	F RATIO
REG	3970.237	1	3970.237	543.72
RES	211.758	29	7.302	
TOTAL COR	4181.995	30		

R SQUARED = .949

YHAT = .586 + 1.996 X
OR
YHAT = .5863300971 + 1.99576699029 X

	X(I)	YHAT	Y(I)	RES(I)
1	11.000	22.539767	22.60000	.06023
2	1.000	2.582097	3.00000	.41790
3	5.000	10.565165	7.50000	3.06517
4	13.000	26.531301	25.20000	1.33130
5	1.000	2.582097	1.80000	.78210
6	2.000	4.577864	4.70000	.12214
7	1.000	2.582097	2.00000	.58210
8	2.000	4.577864	2.50000	2.07786
9	3.000	6.573631	6.20000	.37363
10	7.000	14.556699	13.50000	1.05670
11	2.000	4.577864	8.00000	3.42214
12	8.000	16.552466	26.00000	9.44753
13	1.000	2.582097	2.80000	.21790
14	8.000	16.552466	14.10000	2.45247
15	11.000	22.539767	22.90000	.36023
16	4.000	8.569398	11.70000	3.13060
17	9.000	18.548233	19.00000	.45177
18	6.000	12.560932	13.60000	1.03907
19	8.000	16.552466	12.00000	4.55247
20	6.000	12.560932	11.60000	.96093
21	3.000	6.573631	5.20000	1.37363
22	2.000	4.577864	3.00000	1.57786
23	3.000	6.573631	9.40000	2.82637
24	7.000	14.556699	14.70000	.14330
25	10.000	20.544000	21.20000	.65600
26	1.000	2.582097	1.40000	1.18210
27	25.000	50.480505	54.20000	3.71950
28	5.000	10.565165	11.90000	1.33483
29	17.000	34.514369	33.50000	1.01437
30	19.000	38.505903	33.70000	4.80590
31	6.000	12.560932	12.40000	.16093

3. This example is included for your convenience as a check sample problem.

```
                                    ANOTHER EXAMPLE

Data file name:
Number of observations: 12
Number of variables: 2

Variables names:
   1. X
   2. Y


                                    ANOTHER EXAMPLE


     Variable # 1    Variable # 2
OBS#
   1
         86.00000        88.00000
   2
         71.00000        77.00000
   3
         77.00000        76.00000
   4
         68.00000        64.00000
   5
         91.00000        96.00000
   6
         72.00000        72.00000
   7
         77.00000        65.00000
   8
         91.00000        90.00000
   9
         70.00000        65.00000
  10
         71.00000        80.00000
  11
         88.00000        81.00000
  12
         87.00000        72.00000

PAIRED SAMPLE TESTS

VARIABLE FOR X --    X
VARIABLE FOR Y --    Y
```

Bus Passenger Service Time

The time required to service passengers boarding at a bus stop was measured together with the actual number of passengers boarding. The service time as recorded from the moment that the bus stopped and the door opened until the last passenger boarded the bus. The objective is to determine a model for predictive passengers service time, given knowledge of the number boarding at a particular stop. Let X = number boarding and Y = passenger service time, the following data was gathered during the month of May 1968 at twelve downtown locations in Louisville, Kentucky.

```
****************************************************************************
*                    DATA MANIPULATION
****************************************************************************
                                                      BUS

Data file name:
Number of observations: 31
Number of variables: 2

Variables names:
   1. # BOARDING
   2. SERV TIME


                                                      BUS


      Variable # 1    Variable # 2
OBS#
   1
         11.00000        22.60000
   2
          1.00000         3.00000
   3
          5.00000         7.50000
   4
         13.00000        25.20000
   5
          1.00000         1.80000
   6
          2.00000         4.70000
   7
          1.00000         2.00000
   8
          2.00000         2.50000
   9
          3.00000         6.20000
  10
          7.00000        13.50000
```

11	2.00000	8.00000
12	8.00000	26.00000
13	1.00000	2.80000
14	8.00000	14.10000
15	11.00000	22.90000
16	4.00000	11.70000
17	9.00000	19.00000
18	6.00000	13.60000
19	8.00000	12.00000
20	6.00000	11.60000
21	3.00000	5.20000
22	2.00000	3.00000
23	3.00000	9.40000
24	7.00000	14.70000
25	10.00000	21.20000
26	1.00000	1.40000
27	25.00000	54.20000
28	5.00000	11.90000
29	17.00000	33.50000
30	19.00000	33.70000
31	6.00000	12.40000

```
SIGN TEST

     N=    11
     NUMBER OF POSITIVE DIFFERENCES =    7
     (THE 1 POINTS WHERE X(I)=Y(I) ARE EXCLUDED FROM THE TEST)

     YIELDS AN APPROX. STD. NOR. DEV. = .9045

WILCOXON SIGNED RANK

     N=     11
     SUM OF POSITIVE RANKS = 41.5000

     (USING RANKS OF X(I)-Y(I) AND EXCLUDING THE      1
      POINTS WHERE X(I)=Y(I))

     YIELDS APPROXIMATE STANDARD NORMAL DEVIATES

           1) WITHOUT CORRECTION FOR CONTINUITY :

             A) NOT COMPENSATING FOR TIED DIFFERENCES : .7557

             B) CONDITIONAL ON THE EXISTING TIED DIFFERENCES : .7565

           2) WITH CORRECTION FOR CONTINUITY :

             A) NOT COMPENSATING FOR TIED DIFFERENCES : .7113

             B) CONDITIONAL ON THE EXISTING TIED DIFFERENCES : .7120

HIGHER POWERED SIGNED RANKS

     N=   11
     SUM OF POSITIVE RANKS SQUARED = 335.75

     (USING RANKS OF X(I)-Y(I) AND EXCLUDING THE 1
      POINTS WHERE X(I)=Y(I))

     YIELDS AN APPROX. STD. NOR. DEV. OF       .8284
     CONDITIONAL ON THE EXISTING TIES AND
     WITHOUT A CORRECTION FOR CONTINUITY

SPEARMAN'S RHO

     SUM OF SQUARED RANK DIFFERENCES = 75

     RHO = .7378

KENDALL'S TAU

     NUMBER OF CONCORDANT PAIRS = 49
     NUMBER OF DISCORDANT PAIRS = 12

          TAU = .5606
```

Two Independent Samples Tests

Methods and Formulae

1. The **Two Sample t test** is used to determine whether the means of two samples drawn from normal populations having the same variance are equal. The t statistic is computed as follows:

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{\Sigma X_i^2 - n_1 \bar{X}^2 + \Sigma Y_i^2 - n_2 \bar{Y}^2}{n_1 + n_2 - 2}}}$$

   where n_1 and n_2 are the X and Y sample sizes, respectively.

2. **Median test** provides three separate tests of the null hypothesis that both populations have the same median. First, a test statistic T is derived using the ranking function as defined:

$$\Psi(I) = \begin{cases} -1 \text{ if } 0 < \frac{I}{N+1} < \frac{1}{2} \\ 0 \text{ if } \frac{I}{N+1} = \frac{1}{2} \\ 1 \text{ if } \frac{1}{2} < \frac{I}{N+1} < 1 \end{cases}$$

   T is then the number of X's greater than the grand median minus the number of X's less than the grand median. The standard normal deviate for T based on the distribution of the existing ties is given. Secondly, a more familiar 2x2 contingency table is given with the corresponding chi-square value. In addition, if $N+M<30$, Fisher's exact probability is given, where N=sample size of X's and M=sample size of Y's.

3. **Mann-Whitney** assumes the null hypothesis that the two populations have the same distribution, and ranks the pooled sample of X and Y from smallest to largest. The test statistic, T, is then the sum of the X ranks.

   T is standardized by $\frac{T - \mu_T}{\sigma_T}$ where $\mu = \frac{N(N+M+1)}{2}$

   and

$$\sigma_T^2 = \frac{MN}{12(M+N)(M+N-1)} \left[(M+N)^3 - \sum_{j=1}^{k} t_j^3 \right]$$

Output consists of T, and the standard normal deviate of T based on the existing ties.

4. **Taha's Squared R** is similar to the Mann-Whitney test, because it ranks the pooled sample of X's and Y's, and defines (I) by (I) $= I^2$. Again, the null hypothesis is that the two populations have the same distribution. T is normalized by $\frac{T-\mu_T}{\sigma_T}$ where

$$\mu_T = \frac{N(N+M+1)\,(2(N+M)+1)}{6} \quad \text{and}$$

σT is complicated, but can be found in Mielke. (See References)

5. **Cramer-Von Mises** is another test of the hypothesis that the two populations are identical, that is, the distribution function $F(X) = G(Y)$. It calculates $\Sigma(\tilde{F}(Xi) - \tilde{G}(Yj))^2$ where Σ is over consecutive i and j. That is, over the "pooled" cumulative distribution functions. $\tilde{F}(Xi)$ and $\tilde{G}(Yj)$ are empirical cumulative distribution functions.

   The output consists of $\Sigma(\tilde{F}(Xi) - \tilde{G}(Yj))^2$ and .10, .05, and .01 significance levels.

6. **Kolmogorov-Smirnov**, as with the Cramer-Von Mises test, the null hypothesis is that the two cumulative distribution functions are equal, that is, $\tilde{F}(X) = \tilde{G}(Y)$. It calculates max $| \tilde{F}(X_i) - \tilde{G}(Y_j) |$ over consecutive i and j in the "pooled" cumulative distribution functions. The output consists of the maximum absolute deviation and the .10, .05, and .01 significance levels.

User Instructions

1. "t STAT FOR THE MEAN OF TWO SAMPLES" will be printed.
2. The number of observations, beginning and ending data points, the mean and the standard deviation for each sample are printed.
3. The degrees of freedom, t table value and t probability are printed.
4. Go to step 18 of Multiple Sample Start-up.

1. "MEDIAN TESTS" will be printed.
2. When "PRINT COMBINED RANKS?" is displayed:
   a. To have the combined ranks printed:
      1. Press: YES
   b. To continue the program:
      1. Press: NO
3. The following output will be printed:
   a. The test statistic.
   b. The standard normal deviate.
   c. The contingency table analysis.
   d. The chi-square value.
   e. The Fisher's exact probability.
4. Go to step 18 of Multiple Sample Start-up.

1. "MANN-WHITNEY TEST" will be printed.
2. When "PRINT COMBINED RANKS?" is displayed:
   a. To have the combined ranks printed:
      1. Press: YES
   b. To continue the program:
      1. Press: NO
3. The following output will be printed:
   a. The sum of the ranks of X.
   b. The standard normal deviate.
   c. The number of ties.
4. Go to step 18 of Multiple Sample Start-up.

1. "TAHA'S SQUARED RANK" is printed.
2. When "PRINT COMBINED RANKS?" is displayed:
   a. To have the combined ranks printed:
      1. Press: YES
   b. To continue the program:
      1. Press: NO
3. The following output will be printed:
   a. The sum of the X ranks squared.
   b. The standard normal deviate.
   c. The number of ties.
4. Go to step 18 of Multiple Sample Start-up.

1. "CRAMER-VON MISES" is printed.
2. The following output is printed:
   a. The sum of the squared differences.
   b. The test statistic T.
   c. The critical regions for .1, .05, and .01.
3. Go to step 18 of Multiple Sample Start-up.

1. "KOLMOGOROV-SMIRNOV" is printed.
2. The following output is printed:
   a. The maximum difference.
   b. The large sample critical regions for .1, .05, and .01.
3. Go to step 18 of Multiple Sample Start-up.

Examples

1. The following is an example of a two-sample t test.

```
                                   ANOTHER EXAMPLE

Data file name:
Number of observations: 13
Number of variables: 1

Variables names:
   1. MEANS

Subfile name:     beginning observation--number of observations
 1. X                                 1                        6
 2. Y                                 7                        7


                                   ANOTHER EXAMPLE


                                     VARIABLE # 1
    I          OBS(I)        OBS(I+1)        OBS(I+2)        OBS(I+3)        OBS(I+4)
    1         2.00000        3.00000         4.00000         2.00000         3.00000
    6         4.00000        5.00000         4.00000         2.00000         2.00000
   11         6.00000        3.00000         7.00000

TWO INDEPENDENT SAMPLE TESTS

VARIABLE --          MEANS
X SUBFILE --         X
Y SUBFILE --         Y


t STAT FOR THE MEAN OF TWO SAMPLES

SAMPLE 1
N =    6
         2.00000         3.00000  ...          6.00000

MEAN =              3.0000
STD. DEV. =         .8944

SAMPLE 2
N =    7
         5.00000         4.00000   ...         13.00000

MEAN =              4.1429
STD. DEV. =         1.9518

t=             1.3731     DF=        11
PROB (t > 1.37311 ) = .09853
```

2. A cloud seeding experiment was performed, using 16 nonseeded and 10 seeded days. The amount of rainfall, in inches, was recorded for the seeded (X) and nonseeded (Y) cases.

   Three tests to see if the median rainfall was identical were performed, none of which indicates that the two medians differ significantly.

   Taha's squared rank test was performed, since it was assumed that greater precipitation amounts are more important, and should therefore be weighed more heavily in this type of experiment.

```
                                CLOUD

Data file name:
Number of observations: 26
Number of variables: 1

Variables names:
   1. DAYS

Subfile name:     beginning observation--number of observations
 1. SEEDED                          1                          10
 2. NONSEEDED                      11                          16
```

```
                                CLOUD


                          VARIABLE # 1
   I        OBS(I)      OBS(I+1)      OBS(I+2)      OBS(I+3)      OBS(I+4)
   1        .05000        .72000        .69000        .09000        .04000
   6        .62000        .37000        .23000       1.18000        .26000
  11        .18000        .88000        .12000        .74000        .43000
  16        .10000        .65000        .06000        .09000        .41000
  21        .12000        .41000        .05000        .03000        .32000
  26        .05000
```

```
TWO INDEPENDENT SAMPLE TESTS

VARIABLE --          DAYS
X SUBFILE --         SEEDED
Y SUBFILE --         NONSEEDED
```

MEDIAN TESTS

```
                         COMBINED
   I          RANK OF X(I)          RANK OF Y(I)

   1             4.000                 12.000
   2            23.000                 25.000
   3            22.000                 10.500
   4             7.500                 24.000
   5             2.000                 19.000
   6            20.000                  9.000
   7            16.000                 21.000
   8            13.000                  6.000
   9            26.000                  7.500
  10            14.000                 17.500
  11                                   10.500
  12                                   17.500
  13                                    4.000
  14                                    1.000
  15                                   15.000
  16                                    4.000

  I)       TEST STATISTIC, T = 2.0000
         YIELDS A STD. NOR. DEV. OF   .2894
           CONDITIONAL ON THE 5 EXISTING TIES

         - - - - - - - - - - - - - - - - - - - - - - - - -

  II) CONTINGENCY TABLE ANALYSIS
                                  X               Y               TOTAL

                             *****************************
                             *              *              *
          # OF OBS. >        *      6       *      7       *      13
           GRAND MEDIAN      *              *              *
                             *****************************
                             *              *              *
          # OF OBS. <=       *      4       *      9       *      13
           GRAND MEDIAN      *              *              *
                             *****************************

          TOTAL                    10              16              26

     1) YIELDS AN APPROXIMATE CHI-SQUARE VALUE WITH 1 DF OF

          A) USING YATES' CORRECTION FOR CONTINUITY :
                             .1625

          B) WITHOUT CORRECTION FOR CONTINUITY :
                             .6500

     2) FISHER'S EXACT PROBABILITY OF THE
        EXISTING CELL FREQUENCIES OR WORSE :

                             .3441
```

```
MANN-WHITNEY TEST

    SUM OF THE RANKS OF X = 147.5

    YIELDS AN APPROX. STD. NOR. DEV. OF :            .6583
         CONDITIONAL ON THE 5 EXISTING TIES

TAHA'S SQUARED RANK

    SUM OF X RANKS SQUARED = 2786.25

    YIELDS AN APPROX. STD. NOR. DEV. OF : .7605
         CONDITIONAL ON THE  5  EXISTING TIES
```

3. An investigator is interested in whether there is a significant difference in the time required to pace himself for one mile between a near sea level location and a high-altitude location.

Forty-five low-altitude observations (Y) and forty high-altitude observations (X) were collected. It was decided to test whether the two populations from which the investigator sampled have the same distribution.

Initially, histograms of both sets of data were drawn to get a feel for the data. Both the Cramer-Von Mises and Kolmogorov-Smirnov tests were performed, neither of which indicates that there is a significant difference between low-altitude and high-altitude pacing.

```
                         ALTITUDE

Data file name:
Number of observations: 85
Number of variables: 1

Variables names:
    1. ALTITUDE

Subfile name:     beginning observation--number of observations
 1. HIGH                                 1                     40
 2. LOW                                 41                     45
```


ALTITUDE

VARIABLE # 1

I	OBS(I)	OBS(I+1)	OBS(I+2)	OBS(I+3)	OBS(I+4)
1	405.00000	387.00000	400.00000	392.00000	343.00000
6	394.00000	366.00000	389.00000	356.00000	330.00000
11	394.00000	379.00000	359.00000	357.00000	342.00000
16	367.00000	380.00000	395.00000	442.00000	358.00000
21	361.00000	361.00000	360.00000	353.00000	361.00000
26	387.00000	352.00000	385.00000	349.00000	384.00000
31	351.00000	367.00000	364.00000	363.00000	345.00000
36	348.00000	350.00000	353.00000	355.00000	353.00000
41	361.00000	362.00000	359.00000	382.00000	350.00000
46	392.00000	371.00000	398.00000	400.00000	367.00000
51	379.00000	370.00000	365.00000	362.00000	355.00000
56	376.00000	371.00000	369.00000	375.00000	366.00000
61	373.00000	360.00000	374.00000	412.00000	397.00000
66	360.00000	364.00000	377.00000	360.00000	450.00000
71	436.00000	408.00000	380.00000	414.00000	383.00000
76	386.00000	362.00000	380.00000	377.00000	360.00000
81	357.00000	393.00000	357.00000	369.00000	373.00000

```
TWO INDEPENDENT SAMPLE TESTS

VARIABLE --          ALTITUDE
X SUBFILE --         HIGH
Y SUBFILE --         LOW


CRAMER-VON MISES

          SUM OF THE SQUARED DIFFERENCES  .947

          YIELDS A TEST STATISTIC, T= .236

          CRITICAL REGION OF SIZE 0.10 IS FOR T > 0.347
                                  0.05 IS FOR T > 0.461
                                  0.01 IS FOR T > 0.743


KOLMOGOROV-SMIRNOV

          MAXIMUM DIFFERENCE, T (IN ABS. VALUE) = .256

          LARGE SAMPLE CRITICAL REGION OF SIZE 0.10 IS FOR T > .265
                                               0.05 IS FOR T > .296
                                               0.01 IS FOR T > .354
```

Multiple Samples (≥3 Samples) Tests

Methods And Formulae

1. **One-way Analysis of Variance** is used to test the hypothesis that the means of several populations are equal. The assumption is that all the populations are normal and have equal variances, although the sample sizes may be unequal.

   Suppose k is the number of populations and n_i is the number of observations in the sample from the ith population. The total variation of the data is

$$SST = \sum_{i=1}^{k} \left(\sum_{j=1}^{n_i} \left((X_{ij} - \overline{\overline{X}})^2 \right) \right)$$

   where X is the overall mean. The variation due to error, or variation within samples is

$$SSE = \sum_{i=1}^{k} \left(\sum_{j=1}^{n_i} \left((X_{ij} - \overline{X}_i)^2 \right) \right)$$

   where X_i is the mean of the ith sample. The variation between samples is

$$SSB = \sum_{i=1}^{k} \left(n_i \, (\overline{X}_i - \overline{\overline{X}})^2 \right).$$

   The error mean square is defined as

$$MSE = SSE \,/ (N-k), \qquad \text{where } N = \sum_{i=1}^{k} \left(n_i \right)$$

   and the between samples mean square is defined as MSB = SSB / (k−1).

   The F-ratio, MSB/MSE, has the F distribution with k−1 and N−k degrees of freedom. The null hypothesis that the population means are equal may be rejected if the F ratio is greater than or equal to $F_{\alpha,\, k-1,\, N-k}$, where α is the significance level of the experiment. This may be summarized in a table:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
Between samples	k−1	SSB	$MSB = \frac{SSB}{k-1}$	$\frac{MSB}{MSE}$
Error	N−k	SSE	$MSE = \frac{SSE}{N-k}$	
Total	N−1	SST		

2. **Multiple Comparisons** will perform the following tests: Least Significant Difference, Tukey's HSD, Scheffe's Test, Student-Newman-Keuls, and Duncan's Test. If this test is entered via the Multiple Sample Start-up, or from another multiple sample test, the maximum number of means to be compared is 10, the maximum number of subfiles allowed. However, you may run these tests with up to 30 means by following these instructions:

   1. Enter: LOAD"MltCmp",10

   2. Press: EXECUTE

You then input the mean values, as well as the standard error of a mean and the appropriate table values. The notation used in these tests is defined as follows:

EMS = Error mean square used in testing for significance in the analysis of

n = number of observations per mean (you may wish to use a harmonic average if sample sizes are not equal)

$S_{\bar{x}} = \sqrt{\frac{EMS}{n}}$ = standard error of a mean

ν = degrees of freedom for EMS
k = number of groups
X_i = ith group mean (unordered); i = 1,2,...k
Y_j = jth ordered group mean; j=1,2,...k
Q = Appropriate table value
$\delta = Q * S_{\bar{X}}$ = minimum significant differences

Group means are sorted and then all possible comparisons are made. Only one table value is necessary for Least Significant Difference, Tukey's HSD or Scheffe's test while k−1 table values are needed for either Student-Newman-Keuls test or Duncan's Multiple Range test. The minimum significant difference (δ) is the smallest difference there can be between two means for the means to be significantly different. Calculation of the various δ's is described below, assuming a chosen level of significance of $\propto$.

1. Least Significance Difference: $\delta = Q{*}S_{\bar{X}}$
   Q is selected from a table of the Student's t on the basis of ν, the degrees of freedom for the EMS.

2. Tukey's HSD: $\delta = Q{*}S_{\bar{X}}$
   Q is selected from a table of the Studentized range on the basis of K, the total number of means, and ν, the degrees of freedom for the EMS.

3. Scheffe's Test: $\delta = \sqrt{2(K-1)F\propto,K-1,\nu}\ S-x$, $Q = \sqrt{2(K)-1)F\propto,K-1,\nu}$
   where $F\propto,K-1,\nu$ is the $(1-\propto)$th Quantile from an F distribution with K−1 and ν degrees of freedom.

4. Student-Newman-Keuls: $\delta g = Qg{*}S_{\bar{X}}$
   Qg is selected from a table of the Studentized range on the basis of ν, the degrees of freedom for the EMS and g, the number of ordered means between and including the two means being compared.

5. Duncan's Test: $\delta g = Qg^*S_{\overline{X}}$
   Qg is selected from a table of the New Multiple Range Test on the basis of ν, the degrees of freedom for the EMS, and g, the number of ordered means between and including the two means being compared.

NOTE

When the groups have different numbers of observations, an approximate procedure for the HSD, SNK, and Duncan's test is to use $\overline{n}_h$, the harmonic mean of the sample sizes, in calculating $S_{\overline{x}}$. An exact method is available for Scheffe's test. See Bancroft for a more complete discussion.

In all cases, comparisons are made starting with the largest difference and then progressing to the smallest difference. The process should be terminated when there are no significant differences found at a given step.

Once again, if you do not have graphics capability, ignore all messages displayed on the CRT. When the program has been loaded in, press "RUN".

3. **The Kruskal-Wallis** procedure computes a test statistic that is a function of the ranks of the combined samples' observations. First, let N denote the total number of observations,

$$N = \sum_{i=1}^{k} n_i.$$

Then the observations are ranked and a summation of ranks for each sample is performed.

$$R_i = \sum_{j=1}^{n_i} R(X_{ij}), i = 1,2,\ldots,k.$$

The null hypothesis is that the k populations have identical means, although the populations need not be normal. The T statistic is then defined as follows:

$$T = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1).$$

The chi-square distribution with K−1 degreees of freedom is found, and the probability that the chi-square value is greater than T is computed.

User Instructions

1. "ONE WAY AOV" is printed.
2. The treatments and their sample size, mean, variance, standard deviation, and standard error is printed along with the AOV table.
3. Go to step 18 of Multiple Sample Start-up.

1. If the data set has been selected through the Multiple Sample Start-up:

   a. Go to step 5.

2. When "NUMBER OF SAMPLE MEANS?" is displayed:

   a. Enter: The number of means.

   b. Press: CONT

3. When "NUMBER OF OBSERVATIONS IN SAMPLE # (#)" is displayed:

   a. Enter: The number of observations for that sample.

   b. Press: CONT

4. When "MEAN # (#) = " is displayed:

   a. Enter: The mean for that sample.

   b. Press: CONT

5. NOTE: If the number of observations in the samples are not equal, the harmonic mean of the sample sizes will be used in all calculations.
6. The following menu will be displayed:

   1. LEAST SIGNIFICANT DIFFERENCE
   2. TUKEY'S HSD
   3. SCHEFFE'S TEST
   4. STUDENT-NEWMAN-KEULS
   5. DUNCAN'S TEST
   6. SELECT KEY

7. When "CHOOSE A NUMBER AND PRESS CONT" is displayed:

   a. Enter: The number corresponding to the desired test.

   b. Press: CONT

   or

   a. Enter: 6 to select another multiple sample test.

   b. Go to step 18 of Multiple Sample Start-up.

8. If the One-Way AOV test was run immediately before this test:

   a. Go to step 11.

9. When "ERROR MEAN SQUARE=" is displayed:

   a. Enter: The error mean square.
   b. Press: CONT

10. When "DEGREES OF FREEDOM=" is displayed:

   a. Enter: The degrees of freedom.
   b. Press: CONT

11. When "WHAT CONFIDENCE LEVEL? (.99, .95, etc.)" is displayed:

   a. Enter: The level of confidence.
   b. Press: CONT

12. Go to the following steps, depending on which test was chosen:

   a. LEAST SIGNIFICANT DIFFERENCE – step 13.
   b. TUKEY'S HSD – step 14.
   c. SCHEFFE'S TEST – step 15.
   d. STUDENT-NEWMAN-KEULS – step 16.
   e. DUNCAN'S TEST – step 17.

13. When "TABLE VALUE FROM STUDENT'S T" is displayed:

   a. Enter: The correct table value.
   b. Press: CONT
   c. Go to step 18.

14. When "TABLE VALUE FROM STUDENTIZED RANGE" is displayed:

   a. Enter: The correct table value.
   b. Press: CONT
   c. Go to step 18.

15. When "TABLE VALUE FROM F DISTRIBUTION" is displayed:

   a. Enter: The correct table value.
   b. Press: CONT
   c. Go to step 18.

16. When "INPUT TABLE VALUE FROM THE STUDENTIZED RANGE FOR (#) MEANS" is displayed:

    a. Enter: The correct table value.

    b. Press: CONT

    c. Repeat step 16 for # of means−1 times.

    d. Go to step 25.

17. When "INPUT TABLE VALUE FROM THE NEW MULTIPLE RANGE TEST FOR (#) MEANS" is displayed:

    a. Enter: The correct table value.

    b. Press: CONT

    c. Repeat: Step 17 for # of means−1 times.

    d. Go to step 25.

18. When "PLOTTER SELECT CODE=?" is displayed:

    a. Enter: The plotter select code.

    b. Press: CONT

    c. If plotter is set to CRT (select code 13) go to step 22.

19. When "ARE YOU USING HP-IB?" is displayed:
    (See Appendix for explanation of HP-IB)

    a. If using an HP-IB interface cable:

        1. Press: YES

        2. Go to step 20.

    b. If not using an HP-IB interface cable:

        1. Press: NO

        2. Go to step 21.

20. When "SELECT CODE FOR HP-IB=?" is displayed:

    a. Enter: The select code.

    b. Press: CONT

21. When "PLOTTER ID=?" is displayed:

    a. Enter: The plotter identifier string (The plotter ID for the 9872 Plotter = 9872A; INCREMENTAL for an incremental plotter).

    b. Press: CONT

22. The difference graph will be plotted.
23. Press: CONT to continue the program.
24. If the plotter select code was 13:
    a. When "DO YOU WANT A HARD COPY?" is displayed:
        1. To have the graph dumped on the thermal line printer:
            a. Press: YES
            b. The graph will be dumped.
25. If "ARE YOU USING PERFORATED PAPER?" is displayed:
    a. If you are using perforated paper:
        1. Press: YES
    b. If you aren't using perforated paper:
        1. Press: NO
26. The following output will be printed:
    a. The name of the test.
    b. The error mean square.
    c. The degrees of freedom.
    d. The level of confidence.
    e. The input table values.
    f. The difference value.
    g. Two pictorial representations of how the sample means are related. (See "*" below for explanation.)
27. When "PRESS CONT TO CONTINUE PROGRAM" is displayed:
    a. Press: CONT
    b. Go to step 6.

* – The first table shows the samples ranked. Those continuously underlined are not significantly different. The second table lists the sample numbers. Any two means not containing the same letter are significantly different.

1. "KRUSKAL-WALLIS TEST" will be printed.
2. The following output will be printed:

   a. The chi-square table value.
   b. The degrees of freedom.
   c. The chi-square probability.

3. Go to step 18 of Multiple Sample Start-up.

Examples

1. The following example was run to determine the effect of the addition of different sugars on length (in ocular units) of pea sections grown in tissue culture with auxin present. The first sample contains the control results, while the other samples contain:
   a. 2% glucose added
   b. 2% fructose added
   c. 1% glucose and 1% fructose added, and
   d. 2% sucrose added

   After running the one way AOV, a large F value was calculated, indicating there was some difference. To determine which samples were different, two multiple comparison tests were run. In both the Least Significant Difference and in the Duncan's test, all samples differed significantly from the control sample. The Kruskal-Wallis test further supports this conclusion.

```
**************************************************************************************
*                              DATA MANIPULATION                                     *
**************************************************************************************
                            TISSUE CULTURE GROWTH

Data file name:
Number of observations: 50
Number of variables: 1

Variables names:
   1. LENGTH

Subfile name:     beginning observation--number of observations
 1. CONTROL                          1                    10
 2. 2% GLUCOSE                      11                    10
 3. 2% FRUCT.                       21                    10
 4. 1%GLU+1FRU                      31                    10
 5. 2%SUCROSE                       41                    10

                            TISSUE CULTURE GROWTH

                                VARIABLE # 1
```

I	OBS(I)	OBS(I+1)	OBS(I+2)	OBS(I+3)	OBS(I+4)
1	75.00000	67.00000	70.00000	75.00000	65.00000
6	71.00000	67.00000	67.00000	76.00000	68.00000
11	57.00000	58.00000	60.00000	59.00000	62.00000
16	60.00000	60.00000	57.00000	59.00000	61.00000
21	58.00000	61.00000	56.00000	58.00000	57.00000
26	56.00000	61.00000	60.00000	57.00000	58.00000
31	58.00000	59.00000	58.00000	61.00000	57.00000
36	56.00000	58.00000	57.00000	57.00000	59.00000
41	62.00000	66.00000	65.00000	63.00000	64.00000
46	62.00000	65.00000	65.00000	62.00000	67.00000

```
MULTIPLE SAMPLE TESTS

VARIABLE --          LENGTH
SUBFILE --           CONTROL
SUBFILE --           2% GLUCOSE
SUBFILE --           2% FRUCT.
SUBFILE --           1%GLU+1FRU
SUBFILE --           2%SUCROSE
```

ONE WAY AOV

TRT # 1				
	75.00000	67.00000	70.00000	
	75.00000	65.00000	71.00000	67.00000
	67.00000	76.00000	68.00000	
TRT # 2				
	57.00000	58.00000	60.00000	
	59.00000	62.00000	60.00000	60.00000
	57.00000	59.00000	61.00000	
TRT # 3				
	58.00000	61.00000	56.00000	
	58.00000	57.00000	56.00000	61.00000
	60.00000	57.00000	58.00000	
TRT # 4				
	58.00000	59.00000	58.00000	
	61.00000	57.00000	56.00000	58.00000
	57.00000	57.00000	59.00000	
TRT # 5				
	62.00000	66.00000	65.00000	
	63.00000	64.00000	62.00000	65.00000
	65.00000	62.00000	67.00000	

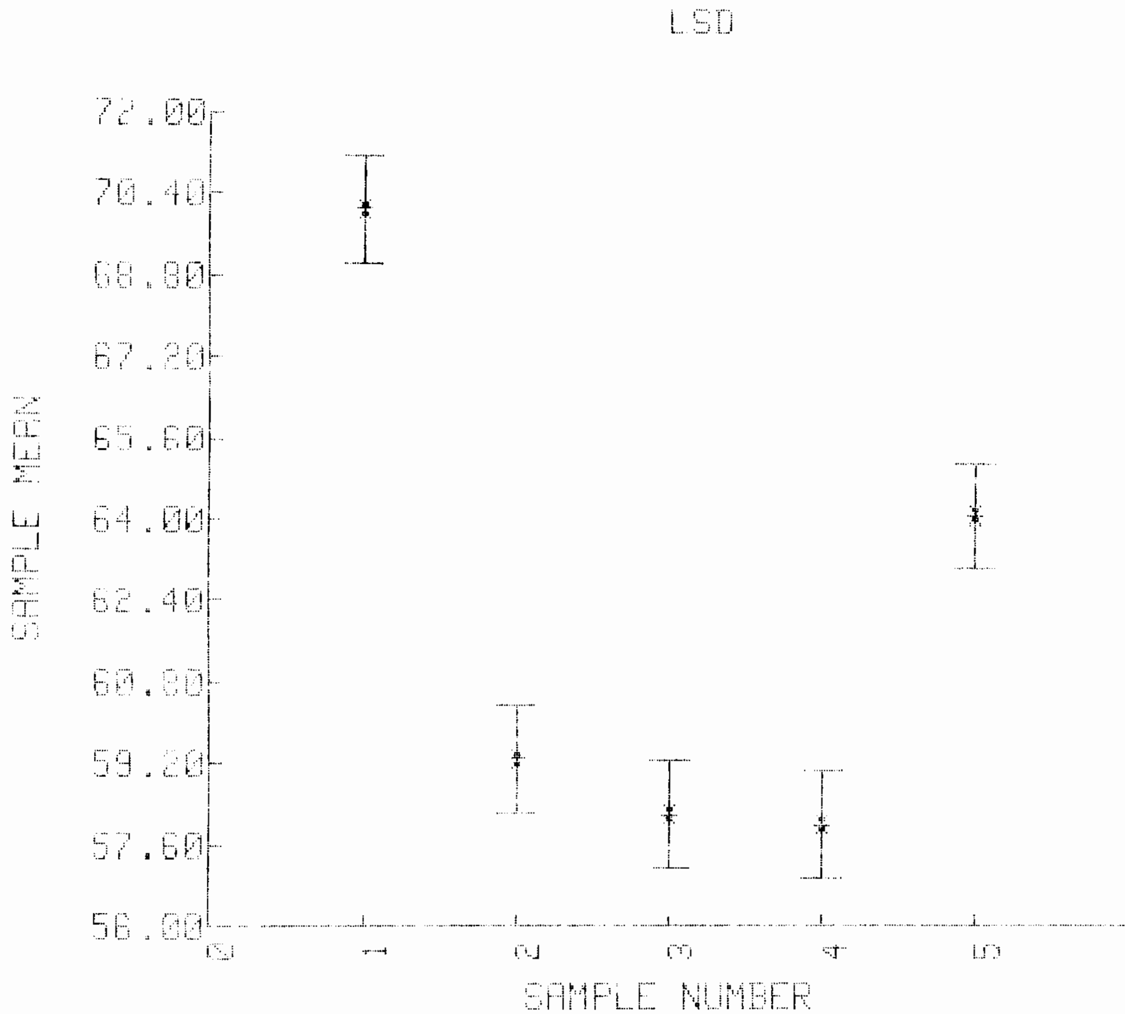
TRT.#	N	MEAN	VARIANCE	STD DEV	STD ERROR
1	10	70.1000	15.8778	3.9847	1.2601
2	10	59.3000	2.6778	1.6364	.5175
3	10	58.2000	3.5111	1.8738	.5925
4	10	58.0000	2.0000	1.4142	.4472
5	10	64.1000	3.2111	1.7920	.5667

AOV

SOURCE	DF	SS	MS	F
TOTAL	49	1322.8200		
TRTS	4	1077.3200	269.3300	49.3680
ERROR	45	245.5000	5.4556	

PROB (F > 49.3680) = .0000

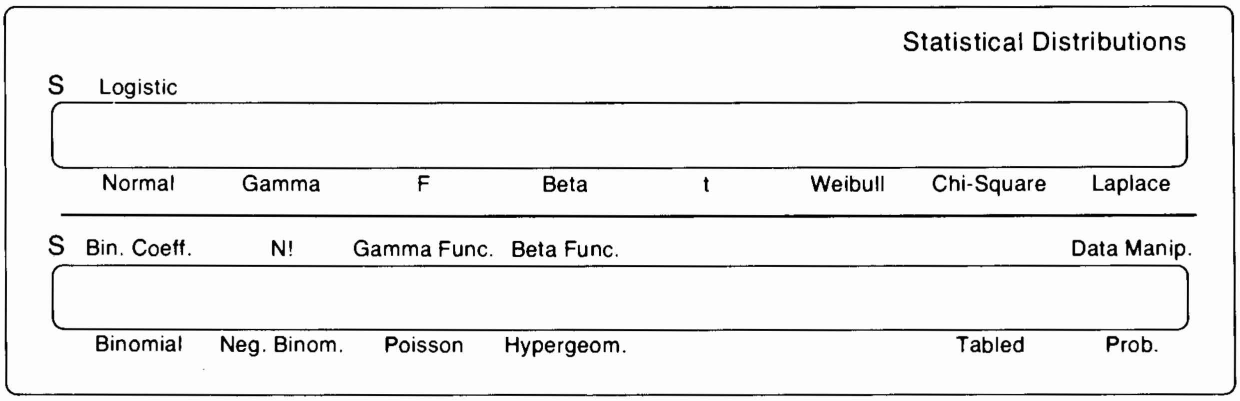
BARTLETT'S TEST
DF = 4.0000 ,CHI-SQUARE = 13.9386
PROB (CHI-SQUARE > 13.9386) = .0075




```
LSD

ERROR MEAN SQUARE = 5.45555555556
DEGREES OF FREEDOM = 45
CONFIDENCE LEVEL = .95
TABLE VALUE FROM STUDENT'S T =     2.0200, LSD =     2.1100

SAMPLES RANKED
  4  3  2  5  1
A -------
B             -
C                 -

MEANS
 1 -C
 2 -A
 3 -A
 4 -A
 5 -B
```

```
DUNCAN'S TEST

ERROR MEAN SQUARE = 5.45555555556
DEGREES OF FREEDOM = 45
LEVEL OF CONFIDENCE = .95
NUMBER OF MEANS = 5, TABLE VALUE =     3.170 , DIFFERENCE =     2.341
NUMBER OF MEANS = 4, TABLE VALUE =     3.100 , DIFFERENCE =     2.290
NUMBER OF MEANS = 3, TABLE VALUE =     3.010 , DIFFERENCE =     2.223
NUMBER OF MEANS = 2, TABLE VALUE =     2.860 , DIFFERENCE =     2.112

SAMPLES RANKED
   4   3   2   5   1
A  ---------
B              -
C                  -

MEANS
 1 -C
 2 -A
 3 -A
 4 -A
 5 -B


KRUSKAL-WALLIS TEST

CHI-SQUARE =     19.0551         DF =  2
P(CHI-SQUARE >  38.110117647 ) =  1.06344991475E-07
```

Statistical Distributions



Object of Program

This program allows you to run a series of continuous and discrete statistical distributions. Both tabled values and right-tailed probabilities are available for the continuous distribution. The discrete distributions calculate right-tail probabilities, single term probabilities and an approximate value for a specified right-tailed probability.

Additionally, this program will calculate n factorial, the complete gamma function, the complete beta function and binomial coefficients.

This program may be entered via Data Manipulation, any One Sample test, or any Multiple Sample test. You may also load the program directly by following these instructions:

1. Insert second program cartridge.
2. Enter: LOAD "Distri",10
3. Press: EXECUTE

Methods and Formulae

Continuous

The continuous distributions included in this program are:

1. Normal (Gaussian)
2. Two-parameter gamma
3. Central F
4. Beta

5. Student's T
6. Weibull
7. Chi-squared
8. Laplace (double exponential, bilateral exponential, extreme value distribution, or Poisson's first law of error)
9. Logistic (autocatalytic function, growth curve)

For the central F, beta, T, chi-square and gamma distributions, the algorithms generally converge most rapidly for small or large right tail probabilities. For moderate tails, the time increases as the right tail approaches .5. For the beta distribution, both parameters should be greater than 10^{-3}. If the parameters are smaller than this, the time required for convergence is excessive.

For the chi-square, it is recommended that the degrees of freedom be less than 500.

For the logistic, Laplace and Weibull it is necessary that the right-tailed probabilities, p, satisfy $1 - 10^{-95} > p > 10^{-95}$.

For the incomplete gamma, it is recommended that the ratio A/B be less than 250.

Some special terms used are:

1. **Right-tailed probability**. Given that X is a random variable and "a" is an observable value of X, then the right-tailed probability associated wih "a" is PR(X>a).
2. **Tabled values**. Given that X is a random variable and P is a right-tailed probability, then the tabled value associated with P is that value "a" such that PR(X>a) = P.

To specify the distributions, the respective density functions that are evaluated will be written down. Let f(x) be a density then

1. Normal (standard)

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \qquad -\infty < x < \infty$$

2. Two parameter gamma, parameters A,B

$$f(x) = \frac{1}{\Gamma(A)\, B^A} * x^{A-1} * e^{-x/B} \qquad x > 0$$

3. Central F with N degrees of freedom in the numerator and D in the denominator

$$f(x) = \frac{\Gamma((N+D)/2)\ (N/D)^{N/2}}{\Gamma(N/2)\ \Gamma(D/2)} \quad \frac{x^{N/2 - 1}}{\left(1 + \frac{Nx}{D}\right)^{(N+D)/2}}$$

4. Beta with parameters A and B

$$f(x) = \frac{\Gamma(A + B)}{\Gamma(A)\,\Gamma(B)} * (1-x)^{B-1} * x^{A-1}$$

$$0 \leq x \leq 1$$

5. Student's t with N degrees of freedom

$$f(x) = \frac{\Gamma((N + 1)/2)}{\sqrt{N\pi}\;\Gamma(N/2)} * \frac{1}{(1 + x^2/N)^{(N+1)/2}}$$

$$-\infty < x < \infty$$

6. Weibull with parameters A,B

$$f(x) = -ABx^{B-1}_{*} \exp[-Ax^{B}]$$

$$x > 0$$

7. Chi-square with N degrees of freedom

$$f(x) = \frac{1}{\Gamma(N/2)\, 2^{N/2}} * x^{N/2-1} \quad e^{-x/2}$$

8. Laplace with parameters A,B

$$f(x) = \frac{Bx \exp(-(A + Bx))}{[1 + \exp(-(A + Bx))]^2}$$

$$B > 0 \text{ and } -\infty < x < \infty$$

9. Logistic with parameters A and B

$$f(x) = \frac{1}{2B} \exp\{-|x - A|/B\}$$

$$B > 0 \text{ and } -\infty < x < \infty$$

Discrete

The discrete distributions included in this program are:

1. Binomial
2. Negative Binomial
3. Poisson
4. Hypergeometric
5. Gamma Function
6. Beta Function
7. Single Term Binomial
8. Single Term Negative Binomial
9. Single Term Poisson
10. Single Term Hypergeometric

Other routines of this program are N factorial and Binomial Coefficients.

Some special terms used are:

1. **Tabled value**. Let X be a binomial, hypergeometric or Poisson random variable. Given all appropriate parameters and p, a desired right-tailed probability, then the tabled value is defined to be x such that $P(X>x) = p$.
2. **Single term probability**. Given that X is one of the above three distributions and x is the counter domain of X, then the single term probability is defined to be $P(X = x)$.

All tabled values are normal approximations. It should be noted that if a right-tailed probability p is desired, it is an unlikely coincidence that there will exist an element x in the counter domain such that $P(X>x) = p$ where x is one of the distributions in (2) above. Thus, after getting the normal approximation to the tabled value, values in the counter domain near the approximation should be checked to see which value is best for the particular application.

The distributions are defined as follows:

1. Hypergeometric

Let N = number of items in a lot
M = sample size
K = number of defective items in the sample
X = number of defective items in the lot
then P (exactly x defectives are in the sample)

$$= P(X = x) = \frac{\binom{K}{x}\binom{N-K}{M-x}}{\binom{N}{M}} \qquad x = 0,1,\ldots,M$$

and

$$P = P(X \geq x) = \sum_{i=x}^{\min(M,K)} P(X=i)$$

2. Binomial

Let N = number of trials
p = probability of success at each trial
X = number of successes

$$P(X = R) = \binom{N}{R} p^{R} (1 - p)^{N-R} \qquad R = 0,1,\ldots,N$$

and

$$PO = P(X \geq R) = \sum_{i=R}^{N} \binom{N}{i} p^{i} (1-p)^{N-i}$$

3. Poisson
Let m = rate parameter or mean = lambda
X = number of occurrences

$$P = P(X \geq N) = e^{-m} \sum_{i=N}^{\infty} \frac{m^{i}}{i!}$$

4. Negative Binomial

For a sequence of Bernoulli trials with probability p of success,

let R = number of failures before the Nth success then

$$P(X = R) = \binom{N + R - 1}{R} p^{N} \quad (1 - p)^{R} \qquad R = 0,1,2 \ldots$$

and

A = number of failures before the Nth success then

$$P(X \geq A) = \sum_{i=A}^{\infty} \binom{N + i - 1}{i} p^{N}(1 - p)^{i} ; A = 0,1,2$$

5. N! and Γ (x) and complete beta function.

An asymptotic Stirling's approximation is used to calculate N! and Γ (x) and complete beta function.

NOTE

For right-tailed probabilities, the exact probabilities are calculated. Thus, there is no need to use a continuity correction. There is no restriction that the parameters be integers, so if for some reason a continuity correction is desired, one may be used.

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User Instructions

Distributions Start-Up

1. When "IS THE SECOND PROGRAM TAPE IN PLACE?" is displayed:

   a. Put the distributions tape in place.

   b. Enter: YES

   c. Press: CONT

NOTE

If the distributions tape is being used in a storage device other than the first program tape, set the standard mass storage device to that storage device first. See Appendix.

2. When "IS THE STATISTICAL DISTRIBUTIONS OVERLAY IN PLACE (Y/N)?" is displayed:

   a. Put the overlay in place.

   b. Enter: YES

   c. Press: CONT

3. When "SELECT KEY" is displayed:

   a. To perform a distribution:

      1. Press: "Tabled" for a tabled value, or "Prob" for a probability (when applicable).
      2. Press: The corresponding special function key.
      3. Go to step 1 of that distribution.

NOTE

The program will continue to perform this distribution until another is chosen.

   b. To return to the Data Manipulation program:

      1. Press: The Data Manipulation special function key.
      2. When "IS THE FIRST PROGRAM TAPE IN PLACE (Y/N)?", is displayed:
         a. Put the first program tape in place.
         b. Enter: YES
         c. Press: CONT

NOTE

If the standard mass storage device was changed to use the distributions tape, reset it to where the first program tape is. See Appendix.

---

3. Go to step 4 of User Instructions in START.

Normal

1. If a table value has been selected:
   a. "NORMAL TABLED" is printed.
   b. When "ENTER PROB" is displayed:
      1. Enter: The probability.
      2. Press: CONT
   c. The table value will be printed.
   d. This step will be repeated until another distribution is chosen.
2. If a probability has been selected:
   a. "NORMAL PROB" is printed.
   b. When "ENTER TABLE VALUE" is displayed:
      1. Enter: The table value.
      2. Press: CONT
   c. The probability will be printed.
   d. This step will be repeated until another distribution is chosen.

1. If a table value has been chosen:
   a. "GAMMA TABLED" is printed.
   b. When "ENTER PROB" is displayed:
      1. Enter: The probability.
      2. Press: CONT
   c. When "PARAMETERS A,B" is displayed:
      1. Enter: The parameters A and B.
      2. Press: CONT
   d. The table value will be printed.
   e. This step will be repeated until another distribution is chosen.
2. If a probability has been chosen:
   a. "GAMMA PROB" is printed.
   b. When "ENTER TABLE VALUE" is displayed:
      1. Enter: The table value.
      2. Press: CONT
   c. When "PARAMETERS A,B" is displayed:
      1. Enter: The parameters A and B.
      2. Press: CONT
   d. The probability will be printed.
   e. This step will be repeated until another distribution is chosen.

1. If a table value has been chosen:

   a. "F TABLED" is printed.

   b. When "ENTER PROB" is displayed:

      1. Enter: The probability.
      2. Press: CONT

   c. When "PARAMETERS N,D" is displayed:

      1. Enter: N = numerator, degrees of freedom, D = denominator, degrees of freedom.
      2. Press: CONT

   d. The table value will be printed.

   e. This step will be repeated until another distribution is chosen.

2. If a probability has been chosen:

   a. "F PROB" is printed.

   b. When "ENTER TABLE VALUE" is displayed:

      1. Enter: The table value.
      2. Press: CONT

   c. When "PARAMETERS N,D" is displayed:

      1. Enter: N = numerator, degrees of freedom, D = denominator, degrees of freedom.
      2. Press: CONT

   d. The probability will be printed.

   e. This step will be repeated until another distribution is chosen.

1. If a table value has been chosen:
   a. "BETA TABLED" is printed.
   b. When "ENTER PROB" is displayed:
      1. Enter: The probability.
      2. Press: CONT
   c. When "PARAMETERS A,B" is displayed:
      1. Enter: The parameters A and B.
      2. Press: CONT
   d. The table value will be printed.
   e. This step will be repeated until another distribution is chosen.
2. If a probability has been chosen:
   a. "BETA PROB" is printed.
   b. When "ENTER TABLE VALUE" is displayed:
      1. Enter: The table value.
      2. Press: CONT
   c. When "PARAMETERS A,B" is displayed:
      1. Enter: The parameters A and B.
      2. Press: CONT
   d. The probability will be printed.
   e. This step will be repeated until another distribution is chosen.

1. If a table value has been chosen:

    a. "T TABLED" is printed.

    b. When "ENTER PROB" is displayed:

        1. Enter: The probability.
        2. Press: CONT

    c. When "DEGREES OF FREEDOM" is displayed:

        1. Enter: The number of degrees of freedom.
        2. Press: CONT

    d. The table value will be printed.

    e. This step will be repeated until another distribution is chosen.

2. If a probability has been chosen:

    a. "T PROB" is printed.

    b. When "ENTER TABLE VALUE" is displayed:

        1. Enter: The table value.
        2. Press: CONT

    c. When "DEGREES OF FREEDOM" is displayed:

        1. Enter: The number of degrees of freedom.
        2. Press: CONT

    d. The probability will be printed.

    e. This step will be repeated until another distribution is chosen.

1. If a table value has been chosen:

   a. "WEIBULL TABLED" is printed.

   b. When "ENTER PROB" is displayed:

      1. Enter: The probability.
      2. Press: CONT

   c. When "PARAMETERS A,B" is displayed:

      1. Enter: The parameters A and B.
      2. Press: CONT

   d. The table value will be printed.

   e. This step will be repeated until another distribution is chosen.

2. If a probability has been chosen:

   a. "WEIBULL PROB" is printed.

   b. When "ENTER TABLE VALUE" is displayed:

      1. Enter: The table value.
      2. Press: CONT

   c. When "PARAMETERS A,B" is displayed:

      1. Enter: The parameters A and B.
      2. Press: CONT

   d. The probability will be printed.

   e. This step will be repeated until another distribution is chosen.

1. If a table value has been chosen:
    a. "CHI-SQUARE TABLED" is printed.
    b. When "ENTER PROB" is displayed:
        1. Enter: The probability.
        2. Press: CONT
    c. When "DEGREES OF FREEDOM" is displayed:
        1. Enter: The number of degrees of freedom.
        2. Press: CONT
    d. The table value will be printed.
    e. This step will be repeated until another distribution is chosen.
2. If a probability has been chosen:
    a. "CHI-SQUARE PROB" is printed.
    b. When "ENTER TABLE VALUE" is displayed:
        1. Enter: The table value.
        2. Press CONT
    c. When "DEGREES OF FREEDOM" is displayed:
        1. Enter: The number of degrees of freedom.
        2. Press: CONT
    d. The probability will be printed.
    e. This step will be repeated until another distribution is chosen.

1. If a table value has been chosen:

   a. "LAPLACE TABLED" is printed.

   b. When "ENTER PROB" is displayed:

      1. Enter: The probability.
      2. Press: CONT

   c. When "PARAMETERS A,B" is displayed:

      1. Enter: The parameters A and B.
      2. Press: CONT

   d. The table value will be printed.

   e. This step will be repeated until another distribution is chosen.

2. If a probability has been chosen.

   a. "LAPLACE PROB" is printed.

   b. When "ENTER TABLE VALUE" is displayed:

      1. Enter: The table value.
      2. Press: CONT

   c. When "PARAMETERS A,B" is displayed:

      1. Enter: The parameters A and B.
      2. Press CONT

   d. The probability will be printed.

   e. This step will be repeated until another distribution is chosen.

1. If a table value has been chosen:
   a. "LOGISTIC TABLED" is printed.
   b. When "ENTER PROB" is displayed:
      1. Enter: The probability.
      2. Press: CONT
   c. When "PARAMETERS A,B" is displayed:
      1. Enter: The parameters A and B.
      2. Press: CONT
   d. The table value will be printed.
   e. This step will be repeated until another distribution is chosen.
2. If a probability has been chosen:
   a. "LOGISTIC PROB" is printed.
   b. When "ENTER TABLE VALUE" is displayed:
      1. Enter: The table value.
      2. Press: CONT
   c. When "PARAMETERS A,B" is displayed:
      1. Enter: The parameters A and B.
      2. Press: CONT
   d. The probability will be printed.
   e. This step will be repeated until another distribution is chosen.

1. If a table value has been chosen:

   a. "TABLED BINOMIAL" is printed.

   b. When "N,P,P0" is displayed:

      1. Enter: The number of trials, the probability of success at each trial and P(X>R).
      2. Press: CONT

   c. The table value will be printed.

   d. This step will be repeated until another distribution is chosen.

2. If a probability is desired:

   a. When "RIGHT TAIL (0) OR SINGLE TERM (1)" is displayed:

      1. To obtain a right tail probability:

         a. Enter: 0
         b. Press: CONT
         c. Go to step 3.

      2. To obtain a single term probability:

         a. Enter: 1
         b. Press: CONT
         c. Go to step 4.

3. The following sequence will occur:

   a. "RIGHT TAIL BINOMIAL" is printed.

   b. When "N,R,P" is displayed:

      1. Enter: The number of trials, R = 0,1,...N, and the probability of success at each trial.

   c. The probability will be printed.

   d. This step will be repeated until another distribution is chosen.

4. The following sequence will occur:

   a. "SINGLE TERM BINOMIAL" is printed.

   b. When "N,R,P" is displayed:

      1. Enter: Parameters N and R -- NR and the probability.

      2. Press: CONT

   c. The single term binomial will be printed.

   d. This step will be repeated until another distribution is chosen.

Negative Binomial

1. When "RIGHT TAIL (0) OR SINGLE TERM (1)" is displayed:

   a. To obtain a right tail probability:

      1. Enter: 0
      2. Press: CONT
      3. Go to step 2.

   b. To obtain a single term probability:

      1. Enter: 0
      2. Press: CONT
      3. Go to step 3.

2. The following sequence will occur:

   a. "NEGATIVE BINOMIAL" is printed.
   b. When "N,A,P" is displayed:

      1. Enter: The Nth success, the number of failures before the Nth success and the probability of success.
      2. Press: CONT

   c. The negative binomial will be printed.
   d. This step will be repeated until another distribution is chosen.

3. The following sequence will occur:

   a. "SINGLE TERM NEGATIVE BINOMIAL" is printed.
   b. When "N,R,P" is displayed:

      1. Enter: The parameters N and R -- N R and the probability.
      2. Press: CONT

   c. The single term negative binomial will be printed.
   d. This step will be repeated until another distribution is chosen.

1. If a tabled value has been chosen:

   a. "TABLED POISSON" is printed.

   b. When "LAMBDA,P" is displayed:

      1. Enter: The rate parameter and the probability.

      2. Press: CONT

   c. The table value will be printed.

   d. This step will be repeated until another distribution is chosen.

2. If a probability is desired:

   a. When "RIGHT TAIL (0) OR SINGLE TERM (1)" is displayed:

      1. To obtain a right tail probability:

         a. Enter: 0
         b. Press: CONT
         c. Go to step 3.

      2. To obtain a single term probability:

         a. Enter: 1
         b. Press: CONT
         c. Go to step 4.

3. The following sequence will occur:

   a. "RIGHT TAIL POISSON" is printed.

   b. When "LAMBDA,N" is displayed:

      1. Enter: The rate parameter and N = 0,1,...

      2. Press: CONT

   c. The probability will be printed.

   d. This step will be repeated until another distribution is chosen.

4. The following sequence will occur:

   a. "SINGLE TERM POISSON" is printed.

   b. When "LAMBDA,N" is displayed:

      1. Enter: The parameters Lambda and N.

      2. Press: CONT

   c. The single term poisson will be printed.

   d. This step will be repeated until another distribution is chosen.

Hypergeometric

1. If a tabled value has been chosen:

   a. "TABLED HYPERGEOMETRIC" is printed.

   b. When "N,M,K,P" is displayed:

      1. Enter: Number of items in a lot, sample size, number of defective items and probability.

      2. Press: CONT

   c. The tabled value will be printed.

   d. This step will be repeated until another distribution is chosen.

2. If a probability is desired:

   a. When "RIGHT TAIL (0) OR SINGLE TERM (1)" is displayed:

      1. To obtain a right tail probability:

         a. Enter: 0
         b. Press: CONT
         c. Go to step 3.

      2. To obtain a single term probability:

         a. Enter: 0
         b. Press: CONT
         c. Go to step 4.

3. The following sequence will occur:

   a. "RIGHT TAIL HYPERGEOMETRIC" is printed.

   b. When "N,M,K,X" is displayed:

      1. Enter: Number of items in a lot, sample size, number of defective items in the lot, number of defective items in the sample.

      2. Press: CONT

   c. The probability will be printed.

   d. This step will be repeated until another distribution is chosen.

4. The following sequence will occur:

   a. "SINGLE TERM HYPERGEOMETRIC" is printed.

   b. When "N,M,K,X" is displayed:

      1. Enter: The parameters N,M,K and X.
      2. Press: CONT

   c. The single term hypergeometric will be printed.

   d. This step will be repeated until another distribution is chosen.

Binomial Coefficients

1. "BINOMIAL COEFFICIENTS" is printed.
2. When "N,R" is displayed:
   a. Enter: The parameters N and R -- NR
   b. Press: CONT
3. The binomial coefficient will be printed.
4. Binomial Coefficients will be repeated until another distribution is chosen.

N Factorial

1. "N FACTORIAL" is printed.
2. When "N" is displayed:
   a. Enter: A number.
   b. Press: CONT
3. The number entered and its factorial will be printed.
4. N factorial will be repeated until another distribution is chosen.

Gamma Function

1. "GAMMA FUNCTION" is printed.
2. When "X" is displayed:

   a. Enter: X

   b. Press: CONT

3. The gamma function will be printed.
4. Gamma function will be repeated until another distribution is chosen.

Beta Function

1. "BETA FUNCTION" is printed.
2. When "A,B" is displayed:
   a. Enter: The parameters A and B.
   b. Press: CONT
3. The beta function will be printed.
4. Beta function will be repeated until another distribution is chosen.

Examples

```
NORMAL TABLED
P=  .05 ,  Z=  1.6448534742
P=  .25 ,  Z=  .67448952474
P=  .001 ,  Z=  3.0902525790 2

NORMAL PROB
P(Z>  1.2 ) = .11506973158 6
P(Z>  5 ) = 2.87104999481E-07
P(Z>  .56 ) = .28773968204 3

GAMMA TABLED
A =  12 ,  B =  4 ,  P =  .05 ,  GAMMA=  72.8300278 2
A =  12 ,  B =  3 ,  P =  .05 ,  GAMMA=  54.622520865
A =  12 ,  B =  3 ,  P =  .00001 ,  GAMMA=  98.3713621075

GAMMA PROB
A =  50 ,  B =  4 ,  P(G>  65 ) = .99999999999
A =  12 ,  B =  3 ,  P(G>  5 ) = .99999979253
A =  12 ,  B =  3 ,  P(G>  75 ) = 1.41595597435E-03

F TABLED
N =  1 ,  D =  4 ,  P =  .05 ,  F =  7.7086474224 6
N =  1 ,  D =  4 ,  P =  .95 ,  F =  4.45269198504E-03
N =  4 ,  D =  26 ,  P =  .0001 ,  F =  9.0736219342 8

F PROB
N =  2 ,  D =  2 ,  P(F>  19 ) = .050000000006
N =  100 ,  D =  100 ,  P(F>  1 ) = .49999999963
N =  4 ,  D =  1250 ,  P(F>  1.2 ) = .30902526863

BETA TABLED
A =  1 ,  B =  1 ,  P =  .05 ,  X =  .95000000001
A =  3.6 ,  B =  1.2 ,  P =  .0001 ,  X =  .99986379277
A =  2.3 ,  B =  12 ,  P =  .9999 ,  X =  2.23290066002E-03

BETA PROB
A =  1 ,  B =  1 ,  P(X > .5 ) = .49999999994
A =  1.2 ,  B =  3.6 ,  P(X > .7 ) = .01803678837
A =  27.123 ,  B =  25 ,  P(X > .4 ) = .95985398092

T TABLED
DEGREES OF FREEDOM =  1 ,  P =  .05 ,  T =  6.31375151546
DEGREES OF FREEDOM =  60 ,  P =  .01 ,  T =  2.39011947 22
DEGREES OF FREEDOM =  4 ,  P =  .999 ,  T = -7.17318222 01

T PROB
DEGREES OF FREEDOM =  2 ,  P(T>  12 ) = 3.43646683907E-03
DEGREES OF FREEDOM =  25 ,  P(T>  2.01 ) = 2.76695014408E-02
DEGREES OF FREEDOM =  25 ,  P(T>  1.96 ) = 3.06162829054E-02
```

```
WEIBULL TABLED
A =  2 ,  B =  2 ,  P =  .05 ,  X =  1.22387341534
A =  2 ,  B =  6 ,  P =  .99 ,  X =  .413864742303
A =  2 ,  B =  8 ,  P =  .5 ,  X =  .875940149694

WEIBULL PROB
A = 2 ,  B =  2 ,  P(X>  1.25 ) = 4.39369336225E-02
A = 2 ,  B =  6 ,  P(X>  .5 ) = .969233234447
A = 2 ,  B =  8 ,  P(X>  .77 ) = .781025672568

CHI-SQUARE TABLED
DEGREES OF FREEDOM =  90 ,  P =  .01 ,  CHI-SQUARE =  124.116224103
DEGREES OF FREEDOM =  2 ,  P =  .05 ,  CHI-SQUARE =  5.99146285594
DEGREES OF FREEDOM =  1250 ,  P =  .01 ,  CHI-SQUARE =  1369.24623136

CHI-SQUARE PROB
DEGREES OF FREEDOM =  11 ,  P(X>  19 ) = 6.10935302602E-02
DEGREES OF FREEDOM =  100 ,  P(X>  120 ) = 8.44038890692E-02
DEGREES OF FREEDOM =  250 ,  P(X>  275 ) = .132952812313

LAPLACE TABLED
A =  5 ,  B =  9 ,  P =  .3 ,  X =  9.59743061393
A =  12 ,  B =  3 ,  P =  .99 ,  X =  .2639309837
A =  12 ,  B =  6 ,  P =  .27 ,  X =  15.5971168366

LAPLACE PROB
A =  5 ,  B =  9 ,  P(X>  10 ) = .28687671037
A =  12 ,  B =  3 ,  P(X>  4 ) = .96525827409
A =  12 ,  B =  6 ,  P(X>  31.2 ) = .02038110199


LOGISTIC TABLED
A =  2 ,  B =  4 ,  P =  .05 ,  X =  .236109744793
A =  2 ,  B =  4 ,  P =  .99 ,  X = -1.64877996253
A =  5 ,  B =  4 ,  P =  .01 ,  X = -.101220037467

LOGISTIC PROB
A =  2 ,  B =  2 ,  P(X>  .01 ) = .117118990873
A =  3 ,  B =  6 ,  P(X>  10 ) = 4.35960999955E-28
A =  1 ,  B =  3 ,  P(X>  5 ) = 1.12535162049E-07

N FACTORIAL
N=  12 ,  N!=  479001600
N=  6 ,  N!=  720
N=  650 ,  LOG(N!)=  1547.90787065

TABLED BINOMIAL
N =  10 ,  P =  .5 ,  P0 =  .05 ,  X =  8.10377220242
N =  12 ,  P =  .4 ,  P0 =  .01 ,  X =  9.25098347572
N =  50 ,  P =  .4 ,  P0 =  .05 ,  X =  26.2045790795

RIGHT TAIL BINOMIAL
N =  15 ,  R =  12 ,  P =  .5 ,  P =  1.75781250006E-02
N =  50 ,  R =  10 ,  P =  .4 ,  P =  .999242270342
N =  50 ,  R =  40 ,  P =  .4 ,  P =  8.92848366285E-09
```

```
NEGATIVE BINOMIAL
N =  60 ,  A =  50 ,  P =           .400000,  P0 =          .998943
N =  60 ,  A =  80 ,  P =           .400000,  P0 =          .751167
N =  10 ,  A =  20 ,  P =           .500000,  P0 =          .030714

TABLED POISSON
LAMBDA = 25 ,  P =  .05 ,  X =  34.2338506679
LAMBDA = 69 ,  P =  .01 ,  X =  89.3389777982
LAMBDA = 12 ,  P =  .5 ,  X =  13.0073083049

RIGHT TAIL POISSON
LAMBDA =  4 ,  N =  2 ,  P =  .90842180554
LAMBDA =  25 ,  N =  6 ,  P =  .99999860288
LAMBDA =  25 ,  N =  44 ,  P =  .00036895261

TABLED HYPERGEOMETRIC
N=  125 ,  M=  25 ,  K=  12 ,  P=  .05 ,  X=  5.07827132989
N=  560 ,  M=  260 ,  K=  50 ,  P=  .01 ,  X=  31.5563866907
N=  60 ,  M=  23 ,  K=  10 ,  P=  .2 ,  X=  5.52690765957

RIGHT TAIL HYPERGEOMETRIC
N =  125 ,  M =  25 ,  K =  12 ,  X =  4 ,  P =  .19620961689
N =  125 ,  M =  25 ,  K =  12 ,  X =  3 ,  P =  .44457408868
N =  500 ,  M =  260 ,  K =  100 ,  X =  50 ,  P =  .71221599618

GAMMA FUNCTION
GAMMA( 25 ) = 6.20448401632E+23
GAMMA( 3 ) = 1.99999999964
LOG(GAMMA( 89 ) =  134.268303272

BINOMIAL COEFFICIENTS
N=  25 ,  R=  12 ,  COEFF =  5200300
N=  3 ,  R=  2 ,  COEFF =  3
N=  890 ,  R=  450 ,  LOG(COEFF) = 266.319447525

SINGLE TERM BINOMIAL
N =  25 ,  R =  12 ,  P =  .3 ,  PROB =  2.67767568652E-02
N =  1500 ,  R =  1200 ,  P =  .6 ,  PROB =  6.21144339691E-62
N =  125 ,  R =  12 ,  P =  .8 ,  PROB =  1.25710385560E-64

SINGLE TERM NEGATIVE BINOMIAL
N =  25 ,  R =  10 ,  P =  .25 ,  P0 =  6.55856386075E-09
N =  24 ,  R =  12 ,  P =  .36 ,  P0 =  8.84750796208E-05
N =  18 ,  R =  13 ,  P =  .25 ,  P0 =  4.14026199156E-05

SINGLE TERM POISSON
N =  2 ,  LAMBDA =  4 ,  P =  .14652511136
N =  5 ,  LAMBDA =  8 ,  P =  9.16036615987E-02
N =  25 ,  LAMBDA =  8 ,  P =  8.17049071498E-07

SINGLE TERM HYPERGEOMETRIC
N =  125 ,  M =  25 ,  K =  12 ,  X =  4 ,  P =  .193630884463
N =  125 ,  M =  25 ,  K =  12 ,  X =  3 ,  P =  .248364471983
N =  125 ,  M =  25 ,  K =  12 ,  X =  2 ,  P =  .294797829893
```

Quantiles of the Spearman Test Statistic[a]

n	p = .900	.950	.975	.990	.995	.999
4	.8000	.8000				
5	.7000	.8000	.9000	.9000		
6	.6000	.7714	.8286	.8857	.9429	
7	.5357	.6786	.7450	.8571	.8929	.9643
8	.5000	.6190	.7143	.8095	.8571	.9286
9	.4667	.5833	.6833	.7667	.8167	.9000
10	.4424	.5515	.6364	.7333	.7818	.8667
11	.4182	.5273	.6091	.7000	.7455	.8364
12	.3986	.4965	.5804	.6713	.7273	.8182
13	.3791	.4780	.5549	.6429	.6978	.7912
14	.3626	.4593	.5341	.6220	.6747	.7670
15	.3500	.4429	.5179	.6000	.6536	.7464
16	.3382	.4265	.5000	.5824	.6324	.7265
17	.3260	.4118	.4853	.5637	.6152	.7083
18	.3148	.3994	.4716	.5480	.5975	.6904
19	.3070	.3895	.4579	.5333	.5825	.6737
20	.2977	.3789	.4451	.5203	.5684	.6586
21	.2909	.3688	.4351	.5078	.5545	.6455
22	.2829	.3597	.4241	.4963	.5426	.6318
23	.2767	.3518	.4150	.4852	.5306	.6186
24	.2704	.3435	.4061	.4748	.5200	.6070
25	.2646	.3362	.3977	.4654	.5100	.5962
26	.2588	.3299	.3894	.4564	.5002	.5856
27	.2540	.3236	.3822	.4481	.4915	.5757
28	.2490	.3175	.3749	.4401	.4828	.5660
29	.2443	.3113	.3685	.4320	.4744	.5567
30	.2400	.3059	.3620	.4251	.4665	.5479

[a] The entries in this table are selected quantiles w_p of the Spearman rank correlation coefficient ρ when used as a test statistic. The lower quantiles may be obtained from the equation

$$w_p = -w_{1-p}$$

The critical region corresponds to values of ρ smaller than (or greater than) but not including the appropriate quantile. Note that the median of ρ is 0.

Quantiles of the Wilcoxon Signed Ranks Test Statistic[a]

	$w_{.005}$	$w_{.01}$	$w_{.025}$	$w_{.05}$	$w_{.10}$	$w_{.20}$	$w_{.30}$	$w_{.40}$	$w_{.50}$	$\frac{n(n+1)}{2}$
$n = 4$	0	0	0	0	1	3	3	4	5	10
5	0	0	0	1	3	4	5	6	7.5	15
6	0	0	1	3	4	6	8	9	10.5	21
7	0	1	3	4	6	9	11	12	14	28
8	1	2	4	6	9	12	14	16	18	36
9	2	4	6	9	11	15	18	20	22.5	45
10	4	6	9	11	15	19	22	25	27.5	55
11	6	8	11	14	18	23	27	30	33	66
12	8	10	14	18	22	28	32	36	39	78
13	10	13	18	22	27	33	38	42	45.5	91
14	13	16	22	26	32	39	44	48	52.5	105
15	16	20	26	31	37	45	51	55	60	120
16	20	24	30	36	43	51	58	63	68	136
17	24	28	35	42	49	58	65	71	76.5	153
18	28	33	41	48	56	66	73	80	85.5	171
19	33	38	47	54	63	74	82	89	95	190
20	38	44	53	61	70	82	91	98	105	210

[a] The entries in this table are quantiles w_p of the Wilcoxon signed ranks test statistic T, for selected values of $p \leq .50$. Quantiles w_p for $p > .50$ may be computed from the equation

$$w_p = n(n+1)/2 - w_{1-p}$$

where $n(n+1)/2$ is given in the right hand column in the table. Note that $P(T < w_p) \leq p$ and $P(T > w_p) \leq 1 - p$ if H_0 is true. Critical regions correspond to values of T less than (or greater than) but not including the appropriate quantile.

Quantiles of the Kolmogorov Test Statistic[a]

One-Sided Test	$p = .90$	.95	.975	.99	.995		$p = .90$	.95	.975	.99	.995
Two-Sided Test	$p = .80$	.90	.95	.98	.99		$p = .80$	.90	.95	.98	.99
$n = 1$	.900	.950	.975	.990	.995	$n = 21$	.226	.259	.287	.321	.344
2	.684	.776	.842	.900	.929	22	.221	.253	.281	.314	.337
3	.565	.636	.708	.785	.829	23	.216	.247	.275	.307	.330
4	.493	.565	.624	.689	.734	24	.212	.242	.269	.301	.323
5	.447	.509	.563	.627	.669	25	.208	.238	.264	.295	.317
6	.410	.468	.519	.577	.617	26	.204	.233	.259	.290	.311
7	.381	.436	.483	.538	.576	27	.200	.229	.254	.284	.305
8	.358	.410	.454	.507	.542	28	.197	.225	.250	.279	.300
9	.339	.387	.430	.480	.513	29	.193	.221	.246	.275	.295
10	.323	.369	.409	.457	.489	30	.190	.218	.242	.270	.290
11	.308	.352	.391	.437	.468	31	.187	.214	.238	.266	.285
12	.296	.338	.375	.419	.449	32	.184	.211	.234	.262	.281
13	.285	.325	.361	.404	.432	33	.182	.208	.231	.258	.277
14	.275	.314	.349	.390	.418	34	.179	.205	.227	.254	.273
15	.266	.304	.338	.377	.404	35	.177	.202	.224	.251	.269
16	.258	.295	.327	.366	.392	36	.174	.199	.221	.247	.265
17	.250	.286	.318	.355	.381	37	.172	.196	.218	.244	.262
18	.244	.279	.309	.346	.371	38	.170	.194	.215	.241	.258
19	.237	.271	.301	.337	.361	39	.168	.191	.213	.238	.255
20	.232	.265	.294	.329	.352	40	.165	.189	.210	.235	.252
						Approximation for $n > 40$	$\frac{1.07}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.52}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

[a] The entries in this table are selected quantiles w_p of the Kolmogorov test statistics T_1, T_1^+, and T_1^- as defined by (6.1.1) for two-sided tests and by (6.1.2) and (6.1.3) for one-sided tests. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile given in this table. These quantiles are exact for $n \leq 20$ in the two-tailed test. The other quantiles are approximations which are equal to the exact quantiles in most cases.

Quantiles of the Mann-Whitney Test Statistic

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	.001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.005	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
2	.01	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	2	2
	.025	0	0	0	0	0	0	1	1	1	1	2	2	2	2	2	3	3	3	3
	.05	0	0	0	1	1	1	2	2	2	2	3	3	4	4	4	4	5	5	5
	.10	0	1	1	2	2	2	3	3	4	4	5	5	5	6	6	7	7	8	8
	.001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
	.005	0	0	0	0	0	0	0	1	1	1	2	2	2	3	3	3	3	4	4
3	.01	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	5	5	5	6
	.025	0	0	0	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9
	.05	0	1	1	2	3	3	4	5	5	6	6	7	8	8	9	10	10	11	12
	.10	1	2	2	3	4	5	6	6	7	8	9	10	11	11	12	13	14	15	16
	.001	0	0	0	0	0	0	0	0	1	1	1	2	2	2	3	3	4	4	4
	.005	0	0	0	0	1	1	2	2	3	3	4	4	5	6	6	7	7	8	9
4	.01	0	0	0	1	2	2	3	4	4	5	6	6	7	9	8	9	10	10	11
	.025	0	0	1	2	3	4	5	5	6	7	8	9	10	11	12	12	13	14	15
	.05	0	1	2	3	4	5	6	7	8	9	10	11	12	13	15	16	17	18	19
	.10	1	2	4	5	6	7	8	10	11	12	13	14	16	17	18	19	21	22	23
	.001	0	0	0	0	0	0	1	2	2	3	3	4	4	5	6	6	7	8	8
	.005	0	0	0	1	2	2	3	4	5	6	7	8	8	9	10	11	12	13	14
5	.01	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	.025	0	1	2	3	4	6	7	8	9	10	12	13	14	15	16	18	19	20	21
	.05	1	2	3	5	6	7	9	10	12	13	14	16	17	19	20	21	23	24	26
	.10	2	3	5	6	8	9	11	13	14	16	18	19	21	23	24	26	28	29	31
	.001	0	0	0	0	0	0	2	3	4	5	5	6	7	8	9	10	11	12	13
	.005	0	0	1	2	3	4	5	6	7	8	10	11	12	13	14	16	17	18	19
6	.01	0	0	2	3	4	5	7	8	9	10	12	13	14	16	17	19	20	21	23
	.025	0	2	3	4	6	7	9	11	12	14	15	17	18	20	22	23	25	26	28
	.05	1	3	4	6	8	9	11	13	15	17	18	20	22	24	26	27	29	31	33
	.10	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	35	37	39
	.001	0	0	0	0	1	2	3	4	6	7	8	9	10	11	12	14	15	16	17
	.005	0	0	1	2	4	5	7	8	10	11	13	14	16	17	19	20	22	23	25
7	.01	0	1	2	4	5	7	8	10	12	13	15	17	18	20	22	24	25	27	29
	.025	0	2	4	6	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35
	.05	1	3	5	7	9	12	14	16	18	20	22	25	27	29	31	34	36	38	40
	.10	2	5	7	9	12	14	17	19	22	24	27	29	32	34	37	39	42	44	47
	.001	0	0	0	1	2	3	5	6	7	9	10	12	13	15	16	18	19	21	22
	.005	0	0	2	3	5	7	8	10	12	14	16	18	19	21	23	25	27	29	31
8	.01	0	1	3	5	7	8	10	12	14	16	18	21	23	25	27	29	31	33	35
	.025	1	3	5	7	9	11	14	16	18	20	23	25	27	30	32	35	37	39	42
	.05	2	4	6	9	11	14	16	19	21	24	27	29	32	34	37	40	42	45	48
	.10	3	6	8	11	14	17	20	23	25	28	31	34	37	40	43	46	49	52	55
	.001	0	0	0	2	3	4	6	8	9	11	13	15	16	18	20	22	24	26	27
	.005	0	1	2	4	6	8	10	12	14	17	19	21	23	25	28	30	32	34	37
9	.01	0	2	4	6	8	10	12	15	17	19	22	24	27	29	32	34	37	39	41
	.025	1	3	5	8	11	13	16	18	21	24	27	29	32	35	38	40	43	46	49
	.05	2	5	7	10	13	16	19	22	25	28	31	34	37	40	43	46	49	52	55
	.10	3	6	10	13	16	19	23	26	29	32	36	39	42	46	49	53	56	59	63
	.001	0	0	1	2	4	6	7	9	11	13	15	18	20	22	24	26	28	30	33
	.005	0	1	3	5	7	10	12	14	17	19	22	25	27	30	32	35	38	40	43
10	.01	0	2	4	7	9	12	14	17	20	23	25	28	31	34	37	39	42	45	48
	.025	1	4	6	9	12	15	18	21	24	27	30	34	37	40	43	46	49	53	56
	.05	2	5	8	12	15	18	21	25	28	32	35	38	42	45	49	52	56	59	63
	.10	4	7	11	14	18	22	25	29	33	37	40	44	48	52	55	59	63	67	71
	.001	0	0	1	3	5	7	9	11	13	16	18	21	23	25	28	30	33	35	38
	.005	0	1	3	6	8	11	14	17	19	22	25	28	31	34	37	40	43	46	49
11	.01	0	2	5	8	10	13	16	19	23	26	29	32	35	38	42	45	48	51	54
	.025	1	4	7	10	14	17	20	24	27	31	34	38	41	45	48	52	56	59	63
	.05	2	6	9	13	17	20	24	28	32	35	39	43	47	51	55	58	62	66	70
	.10	4	8	12	16	20	24	28	32	37	41	45	49	53	58	62	66	70	74	79

Quantiles of the Mann-Whitney Test Statistic (continued)

n	*p*	*m* = 2	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*
	.001	0	0	1	3	5	8	10	13	15	18	21	24	26	29	32	35	38	41	43
	.005	0	2	4	7	10	13	16	19	22	25	28	32	35	38	42	45	48	52	55
12	.01	0	3	6	9	12	15	18	22	25	29	32	36	39	43	47	50	54	57	61
	.025	2	5	8	12	15	19	23	27	30	34	38	42	46	50	54	58	62	66	70
	.05	3	6	10	14	18	22	27	31	35	39	43	48	52	56	61	65	69	73	78
	.10	5	9	13	18	22	27	31	36	40	45	50	54	59	64	68	73	78	82	87
	.001	0	0	2	4	6	9	12	15	18	21	24	27	30	33	36	39	43	46	49
	.005	0	2	4	8	11	14	18	21	25	28	32	35	39	43	46	50	54	58	61
13	.01	1	3	6	10	13	17	21	24	28	32	36	40	44	48	52	56	60	64	68
	.025	2	5	9	13	17	21	25	29	34	38	42	46	51	55	60	64	68	73	77
	.05	3	7	11	16	20	25	29	34	38	43	48	52	57	62	66	71	76	81	85
	.10	5	10	14	19	24	29	34	39	44	49	54	59	64	69	75	80	85	90	95
	.001	0	0	2	4	7	10	13	16	20	23	26	30	33	37	40	44	47	51	55
	.005	0	2	5	8	12	16	19	23	27	31	35	39	43	47	51	55	59	64	68
14	.01	1	3	7	11	14	18	23	27	31	35	39	44	48	52	57	61	66	70	74
	.025	2	6	10	14	18	23	27	32	37	41	46	51	56	60	65	70	75	79	84
	.05	4	8	12	17	22	27	32	37	42	47	52	57	62	67	72	78	83	88	93
	.10	5	11	16	21	26	32	37	42	48	53	59	64	70	75	81	86	92	98	103
	.001	0	0	2	5	8	11	15	18	22	25	29	33	37	41	44	48	52	56	60
	.005	0	3	6	9	13	17	21	25	30	34	38	43	47	52	56	61	65	70	74
15	.01	1	4	8	12	16	20	25	29	34	38	43	48	52	57	62	67	71	76	81
	.025	2	6	11	15	20	25	30	35	40	45	50	55	60	65	71	76	81	86	91
	.05	4	8	13	19	24	29	34	40	45	51	56	62	67	73	78	84	89	95	101
	.10	6	11	17	23	28	34	40	46	52	58	64	69	75	81	87	93	99	105	111
	.001	0	0	3	6	9	12	16	20	24	28	32	36	40	44	49	53	57	61	66
	.005	0	3	6	10	14	19	23	28	32	37	42	46	51	56	61	66	71	75	80
16	.01	1	4	8	13	17	22	27	32	37	42	47	52	57	62	67	72	77	83	88
	.025	2	7	12	16	22	27	32	38	43	48	54	60	65	71	76	82	87	93	99
	.05	4	9	15	20	26	31	37	43	49	55	61	66	72	78	84	90	96	102	108
	.10	6	12	18	24	30	37	43	49	55	62	68	75	81	87	94	100	107	113	120
	.001	0	1	3	6	10	14	18	22	26	30	35	39	44	48	53	58	62	67	71
	.005	0	3	7	11	16	20	25	30	35	40	45	50	55	61	66	71	76	82	87
17	.01	1	5	9	14	19	24	29	34	39	45	50	56	61	67	72	78	83	89	94
	.025	3	7	12	18	23	29	35	40	46	52	58	64	70	76	82	88	94	100	106
	.05	4	10	16	21	27	34	40	46	52	58	65	71	78	84	90	97	103	110	116
	.10	7	13	19	26	32	39	46	53	59	66	73	80	86	93	100	107	114	121	128
	.001	0	1	4	7	11	15	19	24	28	33	38	43	47	52	57	62	67	72	77
	.005	0	3	7	12	17	22	27	32	38	43	48	54	59	65	71	76	82	88	93
18	.01	1	5	10	15	20	25	31	37	42	48	54	60	66	71	77	83	89	95	101
	.025	3	8	13	19	25	31	37	43	49	56	62	68	75	81	87	94	100	107	113
	.05	5	10	17	23	29	36	42	49	56	62	69	76	83	89	96	103	110	117	124
	.10	7	14	21	28	35	42	49	56	63	70	78	85	92	99	107	114	121	129	136
	.001	0	1	4	8	12	16	21	26	30	35	41	46	51	56	61	67	72	78	83
	.005	1	4	8	13	18	23	29	34	40	46	52	58	64	70	75	82	88	94	100
19	.01	2	5	10	16	21	27	33	39	45	51	57	64	70	76	83	89	95	102	108
	.025	3	8	14	20	26	33	39	46	53	59	66	73	79	86	93	100	107	114	120
	.05	5	11	18	24	31	38	45	52	59	66	73	81	88	95	102	110	117	124	131
	.10	8	15	22	29	37	44	52	59	67	74	82	90	98	105	113	121	129	136	144
	.001	0	1	4	8	13	17	22	27	33	38	43	49	55	60	66	71	77	83	89
	.005	1	4	9	14	19	25	31	37	43	49	55	61	68	74	80	87	93	100	106
20	.01	2	6	11	17	23	29	35	41	48	54	61	68	74	81	88	94	101	108	115
	.025	3	9	15	21	28	35	42	49	56	63	70	77	84	91	99	106	113	120	128
	.05	5	12	19	26	33	40	48	55	63	70	78	85	93	101	108	116	124	131	139
	.10	8	16	23	31	39	47	55	63	71	79	87	95	103	111	120	128	136	144	152

Percentage Points of the Duncan New Multiple Range Test

n_2 \ p	2	3	4	5	6	7	8	9	10	12	14	16	18	20	50	100
1	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0
2	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09
3	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50
4	3.93	4.01	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02
5	3.64	3.74	3.79	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83
6	3.46	3.58	3.64	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68
7	3.35	3.47	3.54	3.58	3.60	3.61	3.61	3.61	3.61	3.61	3.61	3.61	3.61	3.61	3.61	3.61
8	3.26	3.39	3.47	3.52	3.55	3.56	3.56	3.56	3.56	3.56	3.56	3.56	3.56	3.56	3.56	3.56
9	3.20	3.34	3.41	3.47	3.50	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52
10	3.15	3.30	3.37	3.43	3.46	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.48	3.48	3.48
11	3.11	3.27	3.35	3.39	3.43	3.44	3.45	3.46	3.46	3.46	3.46	3.46	3.47	3.48	3.48	3.48
12	3.08	3.23	3.33	3.36	3.40	3.42	3.44	3.44	3.46	3.46	3.46	3.46	3.47	3.48	3.48	3.48
13	3.06	3.21	3.30	3.35	3.38	3.41	3.42	3.44	3,45	3.45	3.46	3.46	3.47	3.47	3.47	3.47
14	3.03	3.18	3.27	3.33	3.37	3.39	3.41	3.42	3.44	3.45	3.46	3.46	3.47	3.47	3.47	3.47
15	3.01	3.16	3.25	3.31	3.36	3.38	3.40	3.42	3.43	3.44	3.45	3.46	3.47	3.47	3.47	3.47
16	3.00	3.15	3.23	3.30	3.34	3.37	3.39	3.41	3.43	3.44	3.45	3.46	3.47	3.47	3.47	3.47
17	2.98	3.13	3.22	3.28	3.33	3.36	3.38	3.40	3.42	3.44	3.45	3.46	3.47	3.47	3.47	3.47
18	2.97	3.12	3.21	3.27	3.32	3.35	3.37	3.39	3.41	3.43	3.45	3.46	3.47	3.47	3.47	3.47
19	2.96	3.11	3.19	3.26	3.31	3.35	3.37	3.39	3.41	3.43	3.44	3.46	3.47	3.47	3.47	3.47
20	2.95	3.10	3.18	3.25	3.30	3.34	3.36	3.38	3.40	3.43	3.44	3.46	3.46	3.47	3.47	3.47
22	2.93	3.08	3.17	3.24	3.29	3.32	3.35	3.37	3.39	3.42	3.44	3.45	3.46	3.47	3.47	3.47
24	2.92	3.07	3.15	3.22	3.28	3.31	3.34	3.37	3.38	3.41	3.44	3.45	3.46	3.47	3.47	3.47
26	2.91	3.06	3.14	3.21	3.27	3.30	3.34	3.36	3.38	3.41	3.43	3.45	3.46	3.47	3.47	3.47
28	2.90	3.04	3.13	3.20	3.26	3.30	3.33	3.35	3.37	3.40	3.43	3.45	3.46	3.47	3.47	3.47
30	2.89	3.04	3.12	3.20	3.25	3.29	3.32	3.35	3.37	3.40	3.43	3.44	3.46	3.47	3.47	3.47
40	2.86	3.01	3.10	3.17	3.22	3.27	3.30	3.33	3.35	3.39	3.42	3.44	3.46	3.47	3.47	3.47
60	2.83	2.98	3.08	3.14	3.20	3.24	3.28	3.31	3.33	3.37	3.40	3.43	3.45	3.47	3.48	3.48
100	2.80	2.95	3.05	3.12	3.18	3.22	3.26	3.29	3.32	3.36	3.40	3.42	3.45	3.47	3.53	3.53
∞	2.77	2.92	3.02	3.09	3.15	3.19	3.23	3.26	3.29	3.34	3.38	3.41	3.44	3.47	3.61	3.67

*Using special protection levels based on degrees of freedom.

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Percentage Points of the Studentized Range, $q=(x_n-x_1)/s_\nu$.

Upper 10 % points

ν \ n	2	3	4	5	6	7	8	9	10
1	8·93	13·44	16·36	18·49	20·15	21·51	22·64	23·62	24·48
2	4·13	5·73	6·77	7·54	8·14	8·63	9·05	9·41	9·72
3	3·33	4·47	5·20	5·74	6·16	6·51	6·81	7·06	7·29
4	3·01	3·98	4·59	5·03	5·39	5·68	5·93	6·14	6·33
5	2·85	3·72	4·26	4·66	4·98	5·24	5·46	5·65	5·82
6	2·75	3·56	4·07	4·44	4·73	4·97	5·17	5·34	5·50
7	2·68	3·45	3·93	4·28	4·55	4·78	4·97	5·14	5·28
8	2·63	3·37	3·83	4·17	4·43	4·65	4·83	4·99	5·13
9	2·59	3·32	3·76	4·08	4·34	4·54	4·72	4·87	5·01
10	2·56	3·27	3·70	4·02	4·26	4·47	4·64	4·78	4·91
11	2·54	3·23	3·66	3·96	4·20	4·40	4·57	4·71	4·84
12	2·52	3·20	3·62	3·92	4·16	4·35	4·51	4·65	4·78
13	2·50	3·18	3·59	3·88	4·12	4·30	4·46	4·60	4·72
14	2·49	3·16	3·56	3·85	4·08	4·27	4·42	4·56	4·68
15	2·48	3·14	3·54	3·83	4·05	4·23	4·39	4·52	4·64
16	2·47	3·12	3·52	3·80	4·03	4·21	4·36	4·49	4·61
17	2·46	3·11	3·50	3·78	4·00	4·18	4·33	4·46	4·58
18	2·45	3·10	3·49	3·77	3·98	4·16	4·31	4·44	4·55
19	2·45	3·09	3·47	3·75	3·97	4·14	4·29	4·42	4·53
20	2·44	3·08	3·46	3·74	3·95	4·12	4·27	4·40	4·51
24	2·42	3·05	3·42	3·69	3·90	4·07	4·21	4·34	4·44
30	2·40	3·02	3·39	3·65	3·85	4·02	4·16	4·28	4·38
40	2·38	2·99	3·35	3·60	3·80	3·96	4·10	4·21	4·32
60	2·36	2·96	3·31	3·56	3·75	3·91	4·04	4·16	4·25
120	2·34	2·93	3·28	3·52	3·71	3·86	3·99	4·10	4·19
∞	2·33	2·90	3·24	3·48	3·66	3·81	3·93	4·04	4·13

ν \ n	11	12	13	14	15	16	17	18	19	20
1	25·24	25·92	26·54	27·10	27·62	28·10	28·54	28·96	29·35	29·71
2	10·01	10·26	10·49	10·70	10·89	11·07	11·24	11·39	11·54	11·68
3	7·49	7·67	7·83	7·98	8·12	8·25	8·37	8·48	8·58	8·68
4	6·49	6·65	6·78	6·91	7·02	7·13	7·23	7·33	7·41	7·50
5	5·97	6·10	6·22	6·34	6·44	6·54	6·63	6·71	6·79	6·86
6	5·64	5·76	5·87	5·98	6·07	6·16	6·25	6·32	6·40	6·47
7	5·41	5·53	5·64	5·74	5·83	5·91	5·99	6·06	6·13	6·19
8	5·25	5·36	5·46	5·56	5·64	5·72	5·80	5·87	5·93	6·00
9	5·13	5·23	5·33	5·42	5·51	5·58	5·66	5·72	5·79	5·85
10	5·03	5·13	5·23	5·32	5·40	5·47	5·54	5·61	5·67	5·73
11	4·95	5·05	5·15	5·23	5·31	5·38	5·45	5·51	5·57	5·63
12	4·89	4·99	5·08	5·16	5·24	5·31	5·37	5·44	5·49	5·55
13	4·83	4·93	5·02	5·10	5·18	5·25	5·31	5·37	5·43	5·48
14	4·79	4·88	4·97	5·05	5·12	5·19	5·26	5·32	5·37	5·43
15	4·75	4·84	4·93	5·01	5·08	5·15	5·21	5·27	5·32	5·38
16	4·71	4·81	4·89	4·97	5·04	5·11	5·17	5·23	5·28	5·33
17	4·68	4·77	4·86	4·93	5·01	5·07	5·13	5·19	5·24	5·30
18	4·65	4·75	4·83	4·90	4·98	5·04	5·10	5·16	5·21	5·26
19	4·63	4·72	4·80	4·88	4·95	5·01	5·07	5·13	5·18	5·23
20	4·61	4·70	4·78	4·85	4·92	4·99	5·05	5·10	5·16	5·20
24	4·54	4·63	4·71	4·78	4·85	4·91	4·97	5·02	5·07	5·12
30	4·47	4·56	4·64	4·71	4·77	4·83	4·89	4·94	4·99	5·03
40	4·41	4·49	4·56	4·63	4·69	4·75	4·81	4·86	4·90	4·95
60	4·34	4·42	4·49	4·56	4·62	4·67	4·73	4·78	4·82	4·86
120	4·28	4·35	4·42	4·48	4·54	4·60	4·65	4·69	4·74	4·78
∞	4·21	4·28	4·35	4·41	4·47	4·52	4·57	4·61	4·65	4·69

n: size of sample from which range obtained. ν: degrees of freedom of independent s_ν.

Percentage Points of the Studentized Range, $q=(x_n-x_1)/s_\nu$. (continued)

Upper 5% points

ν \ n	2	3	4	5	6	7	8	9	10
1	17·97	26·98	32·82	37·08	40·41	43·12	45·40	47·36	49·07
2	6·08	8·33	9·80	10·88	11·74	12·44	13·03	13·54	13·99
3	4·50	5·91	6·82	7·50	8·04	8·48	8·85	9·18	9·46
4	3·93	5·04	5·76	6·29	6·71	7·05	7·35	7·60	7·83
5	3·64	4·60	5·22	5·67	6·03	6·33	6·58	6·80	6·99
6	3·46	4·34	4·90	5·30	5·63	5·90	6·12	6·32	6·49
7	3·34	4·16	4·68	5·06	5·36	5·61	5·82	6·00	6·16
8	3·26	4·04	4·53	4·89	5·17	5·40	5·60	5·77	5·92
9	3·20	3·95	4·41	4·76	5·02	5·24	5·43	5·59	5·74
10	3·15	3·88	4·33	4·65	4·91	5·12	5·30	5·46	5·60
11	3·11	3·82	4·26	4·57	4·82	5·03	5·20	5·35	5·49
12	3·08	3·77	4·20	4·51	4·75	4·95	5·12	5·27	5·39
13	3·06	3·73	4·15	4·45	4·69	4·88	5·05	5·19	5·32
14	3·03	3·70	4·11	4·41	4·64	4·83	4·99	5·13	5·25
15	3·01	3·67	4·08	4·37	4·59	4·78	4·94	5·08	5·20
16	3·00	3·65	4·05	4·33	4·56	4·74	4·90	5·03	5·15
17	2·98	3·63	4·02	4·30	4·52	4·70	4·86	4·99	5·11
18	2·97	3·61	4·00	4·28	4·49	4·67	4·82	4·96	5·07
19	2·96	3·59	3·98	4·25	4·47	4·65	4·79	4·92	5·04
20	2·95	3·58	3·96	4·23	4·45	4·62	4·77	4·90	5·01
24	2·92	3·53	3·90	4·17	4·37	4·54	4·68	4·81	4·92
30	2·89	3·49	3·85	4·10	4·30	4·46	4·60	4·72	4·82
40	2·86	3·44	3·79	4·04	4·23	4·39	4·52	4·63	4·73
60	2·83	3·40	3·74	3·98	4·16	4·31	4·44	4·55	4·65
120	2·80	3·36	3·68	3·92	4·10	4·24	4·36	4·47	4·56
∞	2·77	3·31	3·63	3·86	4·03	4·17	4·29	4·39	4·47

ν \ n	11	12	13	14	15	16	17	18	19	20
1	50·59	51·96	53·20	54·33	55·36	56·32	57·22	58·04	58·83	59·56
2	14·39	14·75	15·08	15·38	15·65	15·91	16·14	16·37	16·57	16·77
3	9·72	9·95	10·15	10·35	10·52	10·69	10·84	10·98	11·11	11·24
4	8·03	8·21	8·37	8·52	8·66	8·79	8·91	9·03	9·13	9·23
5	7·17	7·32	7·47	7·60	7·72	7·83	7·93	8·03	8·12	8·21
6	6·65	6·79	6·92	7·03	7·14	7·24	7·34	7·43	7·51	7·59
7	6·30	6·43	6·55	6·66	6·76	6·85	6·94	7·02	7·10	7·17
8	6·05	6·18	6·29	6·39	6·48	6·57	6·65	6·73	6·80	6·87
9	5·87	5·98	6·09	6·19	6·28	6·36	6·44	6·51	6·58	6·64
10	5·72	5·83	5·93	6·03	6·11	6·19	6·27	6·34	6·40	6·47
11	5·61	5·71	5·81	5·90	5·98	6·06	6·13	6·20	6·27	6·33
12	5·51	5·61	5·71	5·80	5·88	5·95	6·02	6·09	6·15	6·21
13	5·43	5·53	5·63	5·71	5·79	5·86	5·93	5·99	6·05	6·11
14	5·36	5·46	5·55	5·64	5·71	5·79	5·85	5·91	5·97	6·03
15	5·31	5·40	5·49	5·57	5·65	5·72	5·78	5·85	5·90	5·96
16	5·26	5·35	5·44	5·52	5·59	5·66	5·73	5·79	5·84	5·90
17	5·21	5·31	5·39	5·47	5·54	5·61	5·67	5·73	5·79	5·84
18	5·17	5·27	5·35	5·43	5·50	5·57	5·63	5·69	5·74	5·79
19	5·14	5·23	5·31	5·39	5·46	5·53	5·59	5·65	5·70	5·75
20	5·11	5·20	5·28	5·36	5·43	5·49	5·55	5·61	5·66	5·71
24	5·01	5·10	5·18	5·25	5·32	5·38	5·44	5·49	5·55	5·59
30	4·92	5·00	5·08	5·15	5·21	5·27	5·33	5·38	5·43	5·47
40	4·82	4·90	4·98	5·04	5·11	5·16	5·22	5·27	5·31	5·36
60	4·73	4·81	4·88	4·94	5·00	5·06	5·11	5·15	5·20	5·24
120	4·64	4·71	4·78	4·84	4·90	4·95	5·00	5·04	5·09	5·13
∞	4·55	4·62	4·68	4·74	4·80	4·85	4·89	4·93	4·97	5·01

n: size of sample from which range obtained. ν: degrees of freedom of independent s_ν.

Percentage Points of the Studentized Range, $q=(x_n-x_1)/s_\nu$. (continued)

Upper 1% points

ν \ n	2	3	4	5	6	7	8	9	10
1	90·03	135·0	164·3	185·6	202·2	215·8	227·2	237·0	245·6
2	14·04	19·02	22·29	24·72	26·63	28·20	29·53	30·68	31·69
3	8·26	10·62	12·17	13·33	14·24	15·00	15·64	16·20	16·69
4	6·51	8·12	9·17	9·96	10·58	11·10	11·55	11·93	12·27
5	5·70	6·98	7·80	8·42	8·91	9·32	9·67	9·97	10·24
6	5·24	6·33	7·03	7·56	7·97	8·32	8·61	8·87	9·10
7	4·95	5·92	6·54	7·01	7·37	7·68	7·94	8·17	8·37
8	4·75	5·64	6·20	6·62	6·96	7·24	7·47	7·68	7·86
9	4·60	5·43	5·96	6·35	6·66	6·91	7·13	7·33	7·49
10	4·48	5·27	5·77	6·14	6·43	6·67	6·87	7·05	7·21
11	4·39	5·15	5·62	5·97	6·25	6·48	6·67	6·84	6·99
12	4·32	5·05	5·50	5·84	6·10	6·32	6·51	6·67	6·81
13	4·26	4·96	5·40	5·73	5·98	6·19	6·37	6·53	6·67
14	4·21	4·89	5·32	5·63	5·88	6·08	6·26	6·41	6·54
15	4·17	4·84	5·25	5·56	5·80	5·99	6·16	6·31	6·44
16	4·13	4·79	5·19	5·49	5·72	5·92	6·08	6·22	6·35
17	4·10	4·74	5·14	5·43	5·66	5·85	6·01	6·15	6·27
18	4·07	4·70	5·09	5·38	5·60	5·79	5·94	6·08	6·20
19	4·05	4·67	5·05	5·33	5·55	5·73	5·89	6·02	6·14
20	4·02	4·64	5·02	5·29	5·51	5·69	5·84	5·97	6·09
24	3·96	4·55	4·91	5·17	5·37	5·54	5·69	5·81	5·92
30	3·89	4·45	4·80	5·05	5·24	5·40	5·54	5·65	5·76
40	3·82	4·37	4·70	4·93	5·11	5·26	5·39	5·50	5·60
60	3·76	4·28	4·59	4·82	4·99	5·13	5·25	5·36	5·45
120	3·70	4·20	4·50	4·71	4·87	5·01	5·12	5·21	5·30
∞	3·64	4·12	4·40	4·60	4·76	4·88	4·99	5·08	5·16

ν \ n	11	12	13	14	15	16	17	18	19	20
1	253·2	260·0	266·2	271·8	277·0	281·8	286·3	290·4	294·3	298·0
2	32·59	33·40	34·13	34·81	35·43	36·00	36·53	37·03	37·50	37·95
3	17·13	17·53	17·89	18·22	18·52	18·81	19·07	19·32	19·55	19·77
4	12·57	12·84	13·09	13·32	13·53	13·73	13·91	14·08	14·24	14·40
5	10·48	10·70	10·89	11·08	11·24	11·40	11·55	11·68	11·81	11·93
6	9·30	9·48	9·65	9·81	9·95	10·08	10·21	10·32	10·43	10·54
7	8·55	8·71	8·86	9·00	9·12	9·24	9·35	9·46	9·55	9·65
8	8·03	8·18	8·31	8·44	8·55	8·66	8·76	8·85	8·94	9·03
9	7·65	7·78	7·91	8·03	8·13	8·23	8·33	8·41	8·49	8·57
10	7·36	7·49	7·60	7·71	7·81	7·91	7·99	8·08	8·15	8·23
11	7·13	7·25	7·36	7·46	7·56	7·65	7·73	7·81	7·88	7·95
12	6·94	7·06	7·17	7·26	7·36	7·44	7·52	7·59	7·66	7·73
13	6·79	6·90	7·01	7·10	7·19	7·27	7·35	7·42	7·48	7·55
14	6·66	6·77	6·87	6·96	7·05	7·13	7·20	7·27	7·33	7·39
15	6·55	6·66	6·76	6·84	6·93	7·00	7·07	7·14	7·20	7·26
16	6·46	6·56	6·66	6·74	6·82	6·90	6·97	7·03	7·09	7·15
17	6·38	6·48	6·57	6·66	6·73	6·81	6·87	6·94	7·00	7·05
18	6·31	6·41	6·50	6·58	6·65	6·73	6·79	6·85	6·91	6·97
19	6·25	6·34	6·43	6·51	6·58	6·65	6·72	6·78	6·84	6·89
20	6·19	6·28	6·37	6·45	6·52	6·59	6·65	6·71	6·77	6·82
24	6·02	6·11	6·19	6·26	6·33	6·39	6·45	6·51	6·56	6·61
30	5·85	5·93	6·01	6·08	6·14	6·20	6·26	6·31	6·36	6·41
40	5·69	5·76	5·83	5·90	5·96	6·02	6·07	6·12	6·16	6·21
60	5·53	5·60	5·67	5·73	5·78	5·84	5·89	5·93	5·97	6·01
120	5·37	5·44	5·50	5·56	5·61	5·66	5·71	5·75	5·79	5·83
∞	5·23	5·29	5·35	5·40	5·45	5·49	5·54	5·57	5·61	5·65

The Normal Probability Function

The integral $P(X)$ and ordinate $Z(X)$ in terms of the standardized deviate X

X	$P(X)$	δ +	δ^2 −	$Z(X)$	δ −	δ^2 −
·00	·5000000	39894	0	·3989423	199	399
·01	·5039894	39890	4	·3989223	598	399
·02	·5079783	39882	8	·3988625	997	399
·03	·5119665	39870	12	·3987628	1395	398
·04	·5159534	39854	16	·3986233	1793	398
·05	·5199388	39834	20	·3984439	2191	397
·06	·5239222	39810	24	·3982248	2588	397
·07	·5279032	39782	28	·3979661	2984	396
·08	·5318814	39750	32	·3976677	3379	395
·09	·5358564	39714	36	·3973298	3773	394
·10	·5398278	39675	40	·3969525	4166	393
·11	·5437953	39631	44	·3965360	4558	392
·12	·5477584	39584	48	·3960802	4948	390
·13	·5517168	39532	51	·3955854	5337	389
·14	·5556700	39477	55	·3950517	5724	387
·15	·5596177	39418	59	·3944793	6110	386
·16	·5635595	39355	63	·3938684	6493	384
·17	·5674949	39288	67	·3932190	6875	382
·18	·5714237	39217	71	·3925315	7255	380
·19	·5753454	39143	74	·3918060	7633	378
·20	·5792597	39065	78	·3910427	8008	375
·21	·5831662	38983	82	·3902419	8381	373
·22	·5870644	38897	86	·3894038	8752	371
·23	·5909541	38808	89	·3885286	9120	368
·24	·5948349	38715	93	·3876166	9485	365
·25	·5987063	38618	97	·3866681	9847	362
·26	·6025681	38518	100	·3856834	10207	360
·27	·6064199	38414	104	·3846627	10564	357
·28	·6102612	38306	107	·3836063	10917	354
·29	·6140919	38195	111	·3825146	11268	350
·30	·6179114	38081	114	·3813878	11615	347
·31	·6217195	37963	118	·3802264	11958	344
·32	·6255158	37842	121	·3790305	12298	340
·33	·6293000	37717	125	·3778007	12635	337
·34	·6330717	37589	128	·3765372	12968	333
·35	·6368307	37458	131	·3752403	13297	329
·36	·6405764	37323	135	·3739106	13623	325
·37	·6443088	37185	138	·3725483	13944	322
·38	·6480273	37044	141	·3711539	14262	318
·39	·6517317	36900	144	·3697277	14575	313
·40	·6554217	36753	147	·3682701	14885	309
·41	·6590970	36602	150	·3667817	15190	305
·42	·6627573	36449	153	·3652627	15491	301
·43	·6664022	36293	156	·3637136	15787	296
·44	·6700314	36133	159	·3621349	16079	292
·45	·6736448	35971	162	·3605270	16367	288
·46	·6772419	35806	165	·3588903	16650	283
·47	·6808225	35638	168	·3572253	16928	278
·48	·6843863	35467	171	·3555325	17202	274
·49	·6879331	35294	173	·3538124	17470	269
·50	·6914625		176	·3520653		264

X	$P(X)$	δ +	δ^2 −
·50	·6914625	35118	176
·51	·6949743	34939	179
·52	·6984682	34758	181
·53	·7019440	34574	184
·54	·7054015	34388	186
·55	·7088403	34200	189
·56	·7122603	34009	191
·57	·7156612	33815	193
·58	·7190427	33620	196
·59	·7224047	33422	198
·60	·7257469	33222	200
·61	·7290691	33020	202
·62	·7323711	32816	204
·63	·7356527	32610	206
·64	·7389137	32402	208
·65	·7421539	32192	210
·66	·7453731	31980	212
·67	·7485711	31767	214
·68	·7517478	31551	215
·69	·7549029	31334	217
·70	·7580363	31116	219
·71	·7611479	30896	220
·72	·7642375	30674	222
·73	·7673049	30451	223
·74	·7703500	30226	225
·75	·7733726	30001	226
·76	·7763727	29773	227
·77	·7793501	29545	228
·78	·7823046	29316	230
·79	·7852361	29085	231
·80	·7881446	28853	232
·81	·7910299	28620	233
·82	·7938919	28387	234
·83	·7967306	28152	235
·84	·7995458	27917	235
·85	·8023375	27680	236
·86	·8051055	27443	237
·87	·8078498	27205	238
·88	·8105703	26967	238
·89	·8132671	26728	239
·90	·8159399	26489	239
·91	·8185887	26249	240
·92	·8212136	26008	240
·93	·8238145	25768	241
·94	·8263912	25527	241
·95	·8289439	25285	241
·96	·8314724	25044	242
·97	·8339768	24802	242
·98	·8364569	24560	242
·99	·8389129	24318	242
1·00	·8413447		242

$$Z(X) = e^{-\frac{1}{2}X^2}/\sqrt{(2\pi)}, \quad P(X) = 1 - Q(X) = \int_{-\infty}^{X} Z(u)\, du.$$

The Normal Probability Function (continued)

Z(X)	δ −	δ² −
·3520653	17734	264
·3502919	17994	259
·3484925	18248	254
·3466677	18497	249
·3448180	18741	244
·3429439	18981	239
·3410458	19215	234
·3391243	19444	229
·3371799	19667	224
·3352132	19886	219
·3332246	20099	213
·3312147	20307	208
·3291840	20510	203
·3271330	20707	197
·3250623	20899	192
·3229724	21086	187
·3208638	21267	181
·3187371	21442	176
·3165929	21613	170
·3144317	21777	165
·3122539	21936	159
·3100603	22090	154
·3078513	22239	148
·3056274	22381	143
·3033893	22519	137
·3011374	22650	132
·2988724	22777	126
·2965948	22897	121
·2943050	23013	115
·2920038	23122	110
·2896916	23227	104
·2873689	23325	99
·2850364	23419	93
·2826945	23507	88
·2803438	23589	83
·2779849	23666	77
·2756182	23738	72
·2732444	23805	66
·2708640	23866	61
·2684774	23922	56
·2660852	23972	51
·2636880	24017	45
·2612863	24058	40
·2588805	24093	35
·2564713	24122	30
·2540591	24147	25
·2516443	24167	20
·2492277	24182	15
·2468095	24191	10
·2443904	24196	5
·2419707	—	0

X	P(X)	δ +	δ² −	Z(X)	δ −	δ² +
1·00	·8413447	24076	242	·2419707	24196	0
1·01	·8437524	23834	242	·2395511	24191	5
1·02	·8461358	23592	242	·2371320	24182	10
1·03	·8484950	23351	242	·2347138	24168	14
1·04	·8508300	23109	242	·2322970	24149	19
1·05	·8531409	22868	241	·2298821	24125	24
1·06	·8554277	22626	241	·2274696	24097	28
1·07	·8576903	22386	241	·2250599	24064	33
1·08	·8599289	22145	240	·2226535	24027	37
1·09	·8621434	21905	240	·2202508	23986	41
1·10	·8643339	21665	240	·2178522	23940	46
1·11	·8665005	21426	239	·2154582	23890	50
1·12	·8686431	21188	239	·2130691	23836	54
1·13	·8707619	20950	238	·2106856	23778	58
1·14	·8728568	20712	237	·2083078	23715	62
1·15	·8749281	20475	237	·2059363	23649	66
1·16	·8769756	20239	236	·2035714	23578	70
1·17	·8789995	20004	235	·2012135	23504	74
1·18	·8809999	19769	235	·1988631	23426	78
1·19	·8829768	19535	234	·1965205	23344	82
1·20	·8849303	19302	233	·1941861	23259	85
1·21	·8868606	19070	232	·1918602	23170	89
1·22	·8887676	18839	231	·1895432	23077	93
1·23	·8906514	18609	230	·1872354	22981	96
1·24	·8925123	18379	229	·1849373	22882	99
1·25	·8943502	18151	228	·1826491	22779	103
1·26	·8961653	17924	227	·1803712	22673	106
1·27	·8979577	17697	226	·1781038	22564	109
1·28	·8997274	17472	225	·1758474	22452	112
1·29	·9014747	17248	224	·1736022	22337	115
1·30	·9031995	17026	223	·1713686	22218	118
1·31	·9049021	16804	222	·1691468	22097	121
1·32	·9065825	16584	220	·1669370	21973	124
1·33	·9082409	16365	219	·1647397	21847	127
1·34	·9098773	16147	218	·1625551	21717	129
1·35	·9114920	15930	217	·1603833	21585	132
1·36	·9130850	15715	215	·1582248	21451	134
1·37	·9146565	15501	214	·1560797	21314	137
1·38	·9162067	15289	212	·1539483	21175	139
1·39	·9177356	15078	211	·1518308	21033	142
1·40	·9192433	14868	210	·1497275	20890	144
1·41	·9207302	14660	208	·1476385	20744	146
1·42	·9221962	14453	207	·1455641	20596	148
1·43	·9236415	14248	205	·1435046	20446	150
1·44	·9250663	14044	204	·1414600	20294	152
1·45	·9264707	13842	202	·1394306	20140	154
1·46	·9278550	13642	201	·1374165	19985	155
1·47	·9292191	13443	199	·1354181	19828	157
1·48	·9305634	13245	197	·1334353	19669	159
1·49	·9318879	13049	196	·1314684	19508	160
1·50	·9331928	—	194	·1295176	—	162

Note sign of second difference, δ^2.

The Normal Probability Function (continued)

X	$P(X)$	δ +	δ^2 −	$Z(X)$	δ −	δ^2 +
1·50	·9331928	12855	194	·1295176	19346	162
1·51	·9344783	12662	193	·1275830	19183	163
1·52	·9357445	12471	191	·1256646	19018	165
1·53	·9369916	12282	189	·1237628	18853	166
1·54	·9382198	12094	188	·1218775	18685	167
1·55	·9394292	11908	186	·1200090	18517	168
1·56	·9406201	11724	184	·1181573	18348	169
1·57	·9417924	11541	183	·1163225	18177	170
1·58	·9429466	11360	181	·1145048	18006	171
1·59	·9440826	11181	179	·1127042	17834	172
1·60	·9452007	11004	177	·1109208	17661	173
1·61	·9463011	10828	176	·1091548	17487	174
1·62	·9473839	10654	174	·1074061	17312	174
1·63	·9484493	10482	172	·1056748	17137	175
1·64	·9494974	10311	170	·1039611	16962	176
1·65	·9505285	10142	169	·1022649	16786	176
1·66	·9515428	9975	167	·1005864	16609	177
1·67	·9525403	9810	165	·0989255	16432	177
1·68	·9535213	9647	163	·0972823	16255	177
1·69	·9544860	9485	162	·0956568	16077	178
1·70	·9554345	9326	160	·0940491	15899	178
1·71	·9563671	9167	158	·0924591	15722	178
1·72	·9572838	9011	156	·0908870	15544	178
1·73	·9581849	8856	155	·0893326	15366	178
1·74	·9590705	8704	153	·0877961	15188	178
1·75	·9599408	8553	151	·0862773	15010	178
1·76	·9607961	8403	149	·0847764	14832	178
1·77	·9616364	8256	147	·0832932	14654	178
1·78	·9624620	8110	146	·0818278	14477	177
1·79	·9632730	7966	144	·0803801	14300	177
1·80	·9640697	7824	142	·0789502	14123	177
1·81	·9648521	7684	140	·0775379	13946	176
1·82	·9656205	7545	139	·0761433	13770	176
1·83	·9663750	7409	137	·0747663	13594	176
1·84	·9671159	7273	135	·0734068	13419	175
1·85	·9678432	7140	133	·0720649	13245	175
1·86	·9685572	7009	132	·0707404	13071	174
1·87	·9692581	6879	130	·0694333	12897	173
1·88	·9699460	6751	128	·0681436	12725	173
1·89	·9706210	6624	126	·0668711	12553	172
1·90	·9712834	6500	125	·0656158	12382	171
1·91	·9719334	6377	123	·0643777	12211	170
1·92	·9725711	6255	121	·0631566	12041	170
1·93	·9731966	6136	120	·0619524	11873	169
1·94	·9738102	6018	118	·0607652	11705	168
1·95	·9744119	5902	116	·0595947	11538	167
1·96	·9750021	5787	115	·0584409	11372	166
1·97	·9755808	5674	113	·0573038	11206	165
1·98	·9761482	5563	111	·0561831	11042	164
1·99	·9767045	5453	110	·0550789	10879	163
2·00	·9772499		108	·0539910		162

X	$P(X)$	δ +	δ^2 −
2·00	·9772499	5345	108
2·01	·9777844	5239	106
2·02	·9783083	5134	105
2·03	·9788217	5031	103
2·04	·9793248	4929	102
2·05	·9798178	4829	100
2·06	·9803007	4731	98
2·07	·9807738	4634	97
2·08	·9812372	4539	95
2·09	·9816911	4445	94
2·10	·9821356	4352	92
2·11	·9825708	4262	91
2·12	·9829970	4172	89
2·13	·9834142	4084	88
2·14	·9838226	3998	86
2·15	·9842224	3913	85
2·16	·9846137	3829	84
2·17	·9849966	3747	82
2·18	·9853713	3666	81
2·19	·9857379	3587	79
2·20	·9860966	3509	78
2·21	·9864474	3432	77
2·22	·9867906	3357	75
2·23	·9871263	3283	74
2·24	·9874545	3210	73
2·25	·9877755	3138	71
2·26	·9880894	3068	70
2·27	·9883962	2999	69
2·28	·9886962	2932	68
2·29	·9889893	2865	66
2·30	·9892759	2800	65
2·31	·9895559	2736	64
2·32	·9898296	2674	63
2·33	·9900969	2612	62
2·34	·9903581	2552	60
2·35	·9906133	2492	59
2·36	·9908625	2434	58
2·37	·9911060	2377	57
2·38	·9913437	2321	56
2·39	·9915758	2267	55
2·40	·9918025	2213	54
2·41	·9920237	2160	53
2·42	·9922397	2108	52
2·43	·9924506	2058	51
2·44	·9926564	2008	50
2·45	·9928572	1960	49
2·46	·9930531	1912	48
2·47	·9932443	1865	47
2·48	·9934309	1820	46
2·49	·9936128	1775	45
2·50	·9937903		44

$$Z(X) = e^{-\frac{1}{2}X^2}/\sqrt{(2\pi)}, \quad P(X) = 1 - Q(X) = \int_{-\infty}^{X} Z(u)\,du.$$

The Normal Probability Function (continued)

$Z(X)$	δ −	δ^2 +
·0539910	10717	162
·0529192	10557	161
·0518636	10397	160
·0508239	10238	159
·0498001	10081	157
·0487920	9924	156
·0477996	9769	155
·0468226	9616	154
·0458611	9463	153
·0449148	9312	151
·0439836	9162	150
·0430674	9013	149
·0421661	8866	147
·0412795	8720	146
·0404076	8575	145
·0395500	8432	143
·0387069	8290	142
·0378779	8149	140
·0370629	8010	139
·0362619	7873	138
·0354746	7737	136
·0347009	7602	135
·0339408	7468	133
·0331939	7337	132
·0324603	7206	130
·0317397	7077	129
·0310319	6950	127
·0303370	6824	126
·0296546	6699	125
·0289847	6576	123
·0283270	6455	122
·0276816	6335	120
·0270481	6216	119
·0264265	6099	117
·0258166	5984	116
·0252182	5870	114
·0246313	5757	113
·0240556	5646	111
·0234910	5536	110
·0229374	5428	108
·0223945	5322	107
·0218624	5217	105
·0213407	5113	104
·0208294	5011	102
·0203284	4910	101
·0198374	4811	99
·0193563	4713	98
·0188850	4617	96
·0184233	4522	95
·0179711	4428	93
·0175283		92

X	$P(X)$	δ +	δ^2 −	$Z(X)$	δ −	δ^2 +
2·50	·9937903	1731	44	·0175283	4336	92
2·51	·9939634	1688	43	·0170947	4246	91
2·52	·9941323	1646	42	·0166701	4157	89
2·53	·9942969	1605	41	·0162545	4069	88
2·54	·9944574	1565	40	·0158476	3982	86
2·55	·9946139	1525	39	·0154493	3897	85
2·56	·9947664	1487	39	·0150596	3814	84
2·57	·9949151	1449	38	·0146782	3731	82
2·58	·9950600	1412	37	·0143051	3650	81
2·59	·9952012	1376	36	·0139401	3571	80
2·60	·9953388	1341	35	·0135830	3493	78
2·61	·9954729	1306	35	·0132337	3416	77
2·62	·9956035	1272	34	·0128921	3340	76
2·63	·9957308	1239	33	·0125581	3266	74
2·64	·9958547	1207	32	·0122315	3193	73
2·65	·9959754	1176	32	·0119122	3121	72
2·66	·9960930	1145	31	·0116001	3051	70
2·67	·9962074	1115	30	·0112951	2981	69
2·68	·9963189	1085	29	·0109969	2913	68
2·69	·9964274	1056	29	·0107056	2847	67
2·70	·9965330	1028	28	·0104209	2781	66
2·71	·9966358	1001	27	·0101428	2717	64
2·72	·9967359	974	27	·0098712	2654	63
2·73	·9968333	948	26	·0096058	2592	62
2·74	·9969280	922	26	·0093466	2531	61
2·75	·9970202	897	25	·0090936	2471	60
2·76	·9971099	873	24	·0088465	2413	59
2·77	·9971972	849	24	·0086052	2355	57
2·78	·9972821	825	23	·0083697	2299	56
2·79	·9973646	803	23	·0081398	2244	55
2·80	·9974449	781	22	·0079155	2189	54
2·81	·9975229	759	22	·0076965	2136	53
2·82	·9975988	738	21	·0074829	2084	52
2·83	·9976726	717	21	·0072744	2033	51
2·84	·9977443	697	20	·0070711	1983	50
2·85	·9978140	678	20	·0068728	1934	49
2·86	·9978818	658	19	·0066793	1886	48
2·87	·9979476	640	19	·0064907	1839	47
2·88	·9980116	622	18	·0063067	1793	46
2·89	·9980738	604	18	·0061274	1748	45
2·90	·9981342	587	17	·0059525	1704	44
2·91	·9981929	570	17	·0057821	1661	43
2·92	·9982498	553	16	·0056160	1619	42
2·93	·9983052	537	16	·0054541	1578	41
2·94	·9983589	522	16	·0052963	1537	40
2·95	·9984111	507	15	·0051426	1497	40
2·96	·9984618	492	15	·0049929	1459	39
2·97	·9985110	478	14	·0048470	1421	38
2·98	·9985588	464	14	·0047050	1384	37
2·99	·9986051	450	14	·0045666	1347	36
3·00	·9986501		13	·0044318		35

Note sign of second difference, δ^2.

The Normal Probability Function (continued)

X	$P(X)$	δ +	δ^2 −	$Z(X)$	δ −	δ^2 +
3·00	·9986501	437	13	·0044318	1312	35
3·01	·9986938	424	13	·0043007	1277	35
3·02	·9987361	411	13	·0041729	1243	34
3·03	·9987772	399	12	·0040486	1210	33
3·04	·9988171	387	12	·0039276	1178	32
3·05	·9988558	375	12	·0038098	1146	32
3·06	·9988933	364	11	·0036951	1115	31
3·07	·9989297	353	11	·0035836	1085	30
3·08	·9989650	342	11	·0034751	1056	29
3·09	·9989992	332	10	·0033695	1027	29
3·10	·9990324	322	10	·0032668	999	28
3·11	·9990646	312	10	·0031669	971	27
3·12	·9990957	302	10	·0030698	944	27
3·13	·9991260	293	9	·0029754	918	26
3·14	·9991553	284	9	·0028835	893	26
3·15	·9991836	275	9	·0027943	868	25
3·16	·9992112	267	9	·0027075	843	24
3·17	·9992378	258	8	·0026231	820	24
3·18	·9992636	250	8	·0025412	797	23
3·19	·9992886	242	8	·0024615	774	23
3·20	·9993129	235	8	·0023841	752	22
3·21	·9993363	227	7	·0023089	731	21
3·22	·9993590	220	7	·0022358	710	21
3·23	·9993810	213	7	·0021649	689	20
3·24	·9994024	206	7	·0020960	669	20
3·25	·9994230	200	7	·0020290	650	19
3·26	·9994429	193	6	·0019641	631	19
3·27	·9994623	187	6	·0019010	612	18
3·28	·9994810	181	6	·0018397	595	18
3·29	·9994991	175	6	·0017803	577	17
3·30	·9995166	169	6	·0017226	560	17
3·31	·9995335	164	6	·0016666	543	17
3·32	·9995499	159	5	·0016122	527	16
3·33	·9995658	153	5	·0015595	512	16
3·34	·9995811	148	5	·0015084	496	15
3·35	·9995959	143	5	·0014587	481	15
3·36	·9996103	139	5	·0014106	467	15
3·37	·9996242	134	5	·0013639	453	14
3·38	·9996376	130	4	·0013187	439	14
3·39	·9996505	125	4	·0012748	426	13
3·40	·9996631	121	4	·0012322	413	13
3·41	·9996752	117	4	·0011910	400	13
3·42	·9996869	113	4	·0011510	388	12
3·43	·9996982	109	4	·0011122	376	12
3·44	·9997091	106	4	·0010747	364	12
3·45	·9997197	102	4	·0010383	353	11
3·46	·9997299	99	3	·0010030	342	11
3·47	·9997398	95	3	·0009689	331	11
3·48	·9997493	92	3	·0009358	320	10
3·49	·9997585	89	3	·0009037	310	10
3·50	·9997674		3	·0008727		10

X	$P(X)$	δ +	δ^2 −
3·50	·9997674	86	3
3·51	·9997759	83	3
3·52	·9997842	80	3
3·53	·9997922	77	3
3·54	·9997999	74	3
3·55	·9998074	72	3
3·56	·9998146	69	3
3·57	·9998215	67	2
3·58	·9998282	65	2
3·59	·9998347	62	2
3·60	9998409	60	2
3·61	9998469	58	2
3·62	·9998527	56	2
3·63	·9998583	54	2
3·64	·9998637	52	2
3·65	·9998689	50	2
3·66	·9998739	48	2
3·67	·9998787	47	2
3·68	·9998834	45	2
3·69	·9998879	43	2
3·70	·9998922	42	2
3·71	·9998964	40	2
3·72	·9999004	39	1
3·73	·9999043	37	1
3·74	·9999080	36	1
3·75	·9999116	35	1
3·76	·9999150	33	1
3·77	·9999184	32	1
3·78	·9999216	31	1
3·79	·9999247	30	1
3·80	·9999277	29	1
3·81	·9999305	28	1
3·82	·9999333	27	1
3·83	·9999359	26	1
3·84	·9999385	25	1
3·85	·9999409	24	1
3·86	·9999433	23	1
3·87	·9999456	22	1
3·88	·9999478	21	1
3·89	·9999499	20	1
3·90	·9999519	19	1
3·91	·9999539	19	1
3·92	·9999557	18	1
3·93	·9999575	17	1
3·94	·9999593	17	1
3·95	·9999609	16	1
3·96	·9999625	15	1
3·97	·9999641	15	1
3·98	·9999655	14	1
3·99	·9999670	14	1
4·00	·9999683		1

$$Z(X) = e^{-\frac{1}{2}X^2}/\sqrt{(2\pi)}, \quad P(X) = 1 - Q(X) = \int_{-\infty}^{X} Z(u)\,du.$$

The Normal Probability Function (continued)

$Z(X)$	δ −	δ^2 +
·0008727	301	10
·0008426	291	10
·0008135	282	9
·0007853	273	9
·0007581	264	9
·0007317	256	8
·0007061	247	8
·0006814	239	8
·0006575	232	8
·0006343	224	8
·0006119	217	7
·0005902	210	7
·0005693	203	7
·0005490	196	7
·0005294	189	6
·0005105	183	6
·0004921	177	6
·0004744	171	6
·0004573	165	6
·0004408	160	6
·0004248	155	5
·0004093	149	5
·0003944	144	5
·0003800	139	5
·0003661	135	5
·0003526	130	5
·0003396	125	4
·0003271	121	4
·0003149	117	4
·0003032	113	4
·0002919	109	4
·0002810	105	4
·0002705	102	4
·0002604	98	4
·0002506	95	3
·0002411	91	3
·0002320	88	3
·0002232	85	3
·0002147	82	3
·0002065	79	3
·0001987	76	3
·0001910	73	3
·0001837	71	3
·0001766	68	3
·0001698	66	2
·0001633	63	2
·0001569	61	2
·0001508	59	2
·0001449	57	2
·0001393	55	2
·0001338		2

X	$P(X)$	δ +	δ^2 −	$Z(X)$	δ −	δ^2 +
4·00	·9999683	13	1	·0001338	53	2
4·01	·9999696	13	1	·0001286	51	2
4·02	·9999709	12	0	·0001235	49	2
4·03	·9999721	12		·0001186	47	2
4·04	·9999733	11		·0001140	45	2
4·05	·9999744	11		·0001094	43	2
4·06	·9999755	10		·0001051	42	2
4·07	·9999765	10		·0001009	40	2
4·08	·9999775	9		·0000969	39	2
4·09	·9999784	9		·0000930	37	1
4·10	·9999793	9		·0000893	36	1
4·11	·9999802	8		·0000857	35	1
4·12	·9999811	8		·0000822	33	1
4·13	·9999819	8		·0000789	32	1
4·14	·9999826	7		·0000757	31	1
4·15	·9999834	7		·0000726	30	1
4·16	·9999841	7		·0000697	28	1
4·17	·9999848	7		·0000668	27	1
4·18	·9999854	6		·0000641	26	1
4·19	·9999861	6		·0000615	25	1
4·20	·9999867	6		·0000589	24	1
4·21	·9999872	6		·0000565	23	1
4·22	·9999878	5		·0000542	22	1
4·23	·9999883	5		·0000519	22	1
4·24	·9999888	5		·0000498	21	1
4·25	·9999893	5		·0000477	20	1
4·26	·9999898	4		·0000457	19	1
4·27	·9999902	4		·0000438	18	1
4·28	·9999907	4		·0000420	18	1
4·29	·9999911	4		·0000402	17	1
4·30	·9999915	4		·0000385	16	1
4·31	·9999918	4		·0000369	16	1
4·32	·9999922	3		·0000354	15	1
4·33	·9999925	3		·0000339	14	1
4·34	·9999929	3		·0000324	14	1
4·35	·9999932	3		·0000310	13	1
4·36	·9999935	3		·0000297	13	1
4·37	·9999938	3		·0000284	12	1
4·38	·9999941	3		·0000272	12	0
4·39	·9999943	3		·0000261	11	
4·40	·9999946	2		·0000249	11	
4·41	·9999948	2		·0000239	10	
4·42	·9999951	2		·0000228	10	
4·43	·9999953	2		·0000218	9	
4·44	·9999955	2		·0000209	9	
4·45	·9999957	2		·0000200	9	
4·46	·9999959	2		·0000191	8	
4·47	·9999961	2		·0000183	8	
4·48	·9999963	2		·0000175	8	
4·49	·9999964	2		·0000167	7	
4·50	·9999966			·0000160		

Note sign of second difference, δ^2.

The Normal Probability Function (continued)

X	$P(X)$*	$Z(X)$*
4·50	66023	159837
4·51	67586	152797
4·52	69080	146051
4·53	70508	139590
4·54	71873	133401
4·55	73177	127473
4·56	74423	121797
4·57	75614	116362
4·58	76751	111159
4·59	77838	106177
4·60	78875	101409
4·61	79867	96845
4·62	80813	92477
4·63	81717	88297
4·64	82580	84298
4·65	83403	80472
4·66	84190	76812
4·67	84940	73311
4·68	85656	69962
4·69	86340	66760
4·70	86992	63698
4·71	87614	60771
4·72	88208	57972
4·73	88774	55296
4·74	89314	52739
4·75	89829	50295
4·76	90320	47960
4·77	90789	45728
4·78	91235	43596
4·79	91661	41559
4·80	92067	39613
4·81	92453	37755
4·82	92822	35980
4·83	93173	34285
4·84	93508	32667
4·85	93827	31122
4·86	94131	29647
4·87	94420	28239
4·88	94696	26895
4·89	94958	25613
4·90	95208	24390
4·91	95446	23222
4·92	95673	22108
4·93	95889	21046
4·94	96094	20033
4·95	96289	19066
4·96	96475	18144
4·97	96652	17265
4·98	96821	16428
4·99	96981	15629

X	$P(X)$*	$Z(X)$*
5·00	97133	14867
5·01	97278	14141
5·02	97416	13450
5·03	97548	12791
5·04	97672	12162
5·05	97791	11564
5·06	97904	10994
5·07	98011	10451
5·08	98113	9934
5·09	98210	9441
5·10	98302	8972
5·11	98389	8526
5·12	98472	8101
5·13	98551	7696
5·14	98626	7311
5·15	98698	6944
5·16	98765	6595
5·17	98830	6263
5·18	98891	5947
5·19	98949	5647
5·20	99004	5361
5·21	99056	5089
5·22	99105	4831
5·23	99152	4585
5·24	99197	4351
5·25	99240	4128
5·26	99280	3917
5·27	99318	3716
5·28	99354	3525
5·29	99388	3344
5·30	99421	3171
5·31	99452	3007
5·32	99481	2852
5·33	99509	2704
5·34	99535	2563
5·35	99560	2430
5·36	99584	2303
5·37	99606	2183
5·38	99628	2069
5·39	99648	1960
5·40	99667	1857
5·41	99685	1760
5·42	99702	1667
5·43	99718	1579
5·44	99734	1495
5·45	99748	1416
5·46	99762	1341
5·47	99775	1270
5·48	99787	1202
5·49	99799	1138

X	$P(X)$*	$Z(X)$*
5·50	99810	1077
5·51	99821	1019
5·52	99831	965
5·53	99840	913
5·54	99849	864
5·55	99857	817
5·56	99865	773
5·57	99873	731
5·58	99880	691
5·59	99886	654
5·60	99893	618
5·61	99899	585
5·62	99905	553
5·63	99910	522
5·64	99915	494
5·65	99920	467
5·66	99924	441
5·67	99929	417
5·68	99933	394
5·69	99936	372
5·70	99940	351
5·71	99944	332
5·72	99947	313
5·73	99950	296
5·74	99953	280
5·75	99955	264
5·76	99958	249
5·77	99960	235
5·78	99963	222
5·79	99965	210
5·80	99967	198
5·81	99969	187
5·82	99971	176
5·83	99972	166
5·84	99974	157
5·85	99975	148
5·86	99977	139
5·87	99978	131
5·88	99979	124
5·89	99981	117
5·90	99982	110
5·91	99983	104
5·92	99984	98
5·93	99985	92
5·94	99986	87
5·95	99987	82
5·96	99987	77
5·97	99988	73
5·98	99989	68
5·99	99990	65
6·00	99990	61

$$Z(X) = e^{-\frac{1}{2}X^2}/\sqrt{(2\pi)}, \quad P(X) = 1 - Q(X) = \int_{-\infty}^{X} Z(u)\,du.$$

* The entries for $P(X)$ and $Z(X)$ on this page are given to 10 decimal places; thus 0·99999 should be prefixed to each entry for $P(X)$ and a decimal point, followed by four, five, ..., eight zeros, as appropriate, to $Z(X)$.

Percentage Points of the F-distribution (Variance Ratio)

Upper 25 % points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	5·83	7·50	8·20	8·58	8·82	8·98	9·10	9·19	9·26	9·32	9·41	9·49	9·58	9·63	9·67	9·71	9·76	9·80	9·85
2	2·57	3·00	3·15	3·23	3·28	3·31	3·34	3·35	3·37	3·38	3·39	3·41	3·43	3·43	3·44	3·45	3·46	3·47	3·48
3	2·02	2·28	2·36	2·39	2·41	2·42	2·43	2·44	2·44	2·44	2·45	2·46	2·46	2·46	2·47	2·47	2·47	2·47	2·47
4	1·81	2·00	2·05	2·06	2·07	2·08	2·08	2·08	2·08	2·08	2·08	2·08	2·08	2·08	2·08	2·08	2·08	2·08	2·08
5	1·69	1·85	1·88	1·89	1·89	1·89	1·89	1·89	1·89	1·89	1·89	1·89	1·88	1·88	1·88	1·88	1·87	1·87	1·87
6	1·62	1·76	1·78	1·79	1·79	1·78	1·78	1·78	1·77	1·77	1·77	1·76	1·76	1·75	1·75	1·75	1·74	1·74	1·74
7	1·57	1·70	1·72	1·72	1·71	1·71	1·70	1·70	1·69	1·69	1·68	1·68	1·67	1·67	1·66	1·66	1·65	1·65	1·65
8	1·54	1·66	1·67	1·66	1·66	1·65	1·64	1·64	1·63	1·63	1·62	1·62	1·61	1·60	1·60	1·59	1·59	1·58	1·58
9	1·51	1·62	1·63	1·63	1·62	1·61	1·60	1·60	1·59	1·59	1·58	1·57	1·56	1·56	1·55	1·54	1·54	1·53	1·53
10	1·49	1·60	1·60	1·59	1·59	1·58	1·57	1·56	1·56	1·55	1·54	1·53	1·52	1·52	1·51	1·51	1·50	1·49	1·48
11	1·47	1·58	1·58	1·57	1·56	1·55	1·54	1·53	1·53	1·52	1·51	1·50	1·49	1·49	1·48	1·47	1·47	1·46	1·45
12	1·46	1·56	1·56	1·55	1·54	1·53	1·52	1·51	1·51	1·50	1·49	1·48	1·47	1·46	1·45	1·45	1·44	1·43	1·42
13	1·45	1·55	1·55	1·53	1·52	1·51	1·50	1·49	1·49	1·48	1·47	1·46	1·45	1·44	1·43	1·42	1·42	1·41	1·40
14	1·44	1·53	1·53	1·52	1·51	1·50	1·49	1·48	1·47	1·46	1·45	1·44	1·43	1·42	1·41	1·41	1·40	1·39	1·38
15	1·43	1·52	1·52	1·51	1·49	1·48	1·47	1·46	1·46	1·45	1·44	1·43	1·41	1·41	1·40	1·39	1·38	1·37	1·36
16	1·42	1·51	1·51	1·50	1·48	1·47	1·46	1·45	1·44	1·44	1·43	1·41	1·40	1·39	1·38	1·37	1·36	1·35	1·34
17	1·42	1·51	1·50	1·49	1·47	1·46	1·45	1·44	1·43	1·43	1·41	1·40	1·39	1·38	1·37	1·36	1·35	1·34	1·33
18	1·41	1·50	1·49	1·48	1·46	1·45	1·44	1·43	1·42	1·42	1·40	1·39	1·38	1·37	1·36	1·35	1·34	1·33	1·32
19	1·41	1·49	1·49	1·47	1·46	1·44	1·43	1·42	1·41	1·41	1·40	1·38	1·37	1·36	1·35	1·34	1·33	1·32	1·30
20	1·40	1·49	1·48	1·47	1·45	1·44	1·43	1·42	1·41	1·40	1·39	1·37	1·36	1·35	1·34	1·33	1·32	1·31	1·29
21	1·40	1·48	1·48	1·46	1·44	1·43	1·42	1·41	1·40	1·39	1·38	1·37	1·35	1·34	1·33	1·32	1·31	1·30	1·28
22	1·40	1·48	1·47	1·45	1·44	1·42	1·41	1·40	1·39	1·39	1·37	1·36	1·34	1·33	1·32	1·31	1·30	1·29	1·28
23	1·39	1·47	1·47	1·45	1·43	1·42	1·41	1·40	1·39	1·38	1·37	1·35	1·34	1·33	1·32	1·31	1·30	1·28	1·27
24	1·39	1·47	1·46	1·44	1·43	1·41	1·40	1·39	1·38	1·38	1·36	1·35	1·33	1·32	1·31	1·30	1·29	1·28	1·26
25	1·39	1·47	1·46	1·44	1·42	1·41	1·40	1·39	1·38	1·37	1·36	1·34	1·33	1·32	1·31	1·29	1·28	1·27	1·25
26	1·38	1·46	1·45	1·44	1·42	1·41	1·39	1·38	1·37	1·37	1·35	1·34	1·32	1·31	1·30	1·29	1·28	1·26	1·25
27	1·38	1·46	1·45	1·43	1·42	1·40	1·39	1·38	1·37	1·36	1·35	1·33	1·32	1·31	1·30	1·28	1·27	1·26	1·24
28	1·38	1·46	1·45	1·43	1·41	1·40	1·39	1·38	1·37	1·36	1·34	1·33	1·31	1·30	1·29	1·28	1·27	1·25	1·24
29	1·38	1·45	1·45	1·43	1·41	1·40	1·38	1·37	1·36	1·35	1·34	1·32	1·31	1·30	1·29	1·27	1·26	1·25	1·23
30	1·38	1·45	1·44	1·42	1·41	1·39	1·38	1·37	1·36	1·35	1·34	1·32	1·30	1·29	1·28	1·27	1·26	1·24	1·23
40	1·36	1·44	1·42	1·40	1·39	1·37	1·36	1·35	1·34	1·33	1·31	1·30	1·28	1·26	1·25	1·24	1·22	1·21	1·19
60	1·35	1·42	1·41	1·38	1·37	1·35	1·33	1·32	1·31	1·30	1·29	1·27	1·25	1·24	1·22	1·21	1·19	1·17	1·15
120	1·34	1·40	1·39	1·37	1·35	1·33	1·31	1·30	1·29	1·28	1·26	1·24	1·22	1·21	1·19	1·18	1·16	1·13	1·10
∞	1·32	1·39	1·37	1·35	1·33	1·31	1·29	1·28	1·27	1·25	1·24	1·22	1·19	1·18	1·16	1·14	1·12	1·08	1·00

$F = \frac{s_1^2}{s_2^2} = \frac{S_1}{\nu_1} \Big/ \frac{S_2}{\nu_2}$, where $s_1^2 = S_1/\nu_1$ and $s_2^2 = S_2/\nu_2$ are independent mean squares estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively.

Percentage Points of the F-distribution (Variance Ratio) (continued)

Upper 10% *points*

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	39·86	49·50	53·59	55·83	57·24	58·20	58·91	59·44	59·86	60·19	60·71	61·22	61·74	62·00	62·26	62·53	62·79	63·06	63·33
2	8·53	9·00	9·16	9·24	9·29	9·33	9·35	9·37	9·38	9·39	9·41	9·42	9·44	9·45	9·46	9·47	9·47	9·48	9·49
3	5·54	5·46	5·39	5·34	5·31	5·28	5·27	5·25	5·24	5·23	5·22	5·20	5·18	5·18	5·17	5·16	5·15	5·14	5·13
4	4·54	4·32	4·19	4·11	4·05	4·01	3·98	3·95	3·94	3·92	3·90	3·87	3·84	3·83	3·82	3·80	3·79	3·78	3·76
5	4·06	3·78	3·62	3·52	3·45	3·40	3·37	3·34	3·32	3·30	3·27	3·24	3·21	3·19	3·17	3·16	3·14	3·12	3·10
6	3·78	3·46	3·29	3·18	3·11	3·05	3·01	2·98	2·96	2·94	2·90	2·87	2·84	2·82	2·80	2·78	2·76	2·74	2·72
7	3·59	3·26	3·07	2·96	2·88	2·83	2·78	2·75	2·72	2·70	2·67	2·63	2·59	2·58	2·56	2·54	2·51	2·49	2·47
8	3·46	3·11	2·92	2·81	2·73	2·67	2·62	2·59	2·56	2·54	2·50	2·46	2·42	2·40	2·38	2·36	2·34	2·32	2·29
9	3·36	3·01	2·81	2·69	2·61	2·55	2·51	2·47	2·44	2·42	2·38	2·34	2·30	2·28	2·25	2·23	2·21	2·18	2·16
10	3·29	2·92	2·73	2·61	2·52	2·46	2·41	2·38	2·35	2·32	2·28	2·24	2·20	2·18	2·16	2·13	2·11	2·08	2·06
11	3·23	2·86	2·66	2·54	2·45	2·39	2·34	2·30	2·27	2·25	2·21	2·17	2·12	2·10	2·08	2·05	2·03	2·00	1·97
12	3·18	2·81	2·61	2·48	2·39	2·33	2·28	2·24	2·21	2·19	2·15	2·10	2·06	2·04	2·01	1·99	1·96	1·93	1·90
13	3·14	2·76	2·56	2·43	2·35	2·28	2·23	2·20	2·16	2·14	2·10	2·05	2·01	1·98	1·96	1·93	1·90	1·88	1·85
14	3·10	2·73	2·52	2·39	2·31	2·24	2·19	2·15	2·12	2·10	2·05	2·01	1·96	1·94	1·91	1·89	1·86	1·83	1·80
15	3·07	2·70	2·49	2·36	2·27	2·21	2·16	2·12	2·09	2·06	2·02	1·97	1·92	1·90	1·87	1·85	1·82	1·79	1·76
16	3·05	2·67	2·46	2·33	2·24	2·18	2·13	2·09	2·06	2·03	1·99	1·94	1·89	1·87	1·84	1·81	1·78	1·75	1·72
17	3·03	2·64	2·44	2·31	2·22	2·15	2·10	2·06	2·03	2·00	1·96	1·91	1·86	1·84	1·81	1·78	1·75	1·72	1·69
18	3·01	2·62	2·42	2·29	2·20	2·13	2·08	2·04	2·00	1·98	1·93	1·89	1·84	1·81	1·78	1·75	1·72	1·69	1·66
19	2·99	2·61	2·40	2·27	2·18	2·11	2·06	2·02	1·98	1·96	1·91	1·86	1·81	1·79	1·76	1·73	1·70	1·67	1·63
20	2·97	2·59	2·38	2·25	2·16	2·09	2·04	2·00	1·96	1·94	1·89	1·84	1·79	1·77	1·74	1·71	1·68	1·64	1·61
21	2·96	2·57	2·36	2·23	2·14	2·08	2·02	1·98	1·95	1·92	1·87	1·83	1·78	1·75	1·72	1·69	1·66	1·62	1·59
22	2·95	2·56	2·35	2·22	2·13	2·06	2·01	1·97	1·93	1·90	1·86	1·81	1·76	1·73	1·70	1·67	1·64	1·60	1·57
23	2·94	2·55	2·34	2·21	2·11	2·05	1·99	1·95	1·92	1·89	1·84	1·80	1·74	1·72	1·69	1·66	1·62	1·59	1·55
24	2·93	2·54	2·33	2·19	2·10	2·04	1·98	1·94	1·91	1·88	1·83	1·78	1·73	1·70	1·67	1·64	1·61	1·57	1·53
25	2·92	2·53	2·32	2·18	2·09	2·02	1·97	1·93	1·89	1·87	1·82	1·77	1·72	1·69	1·66	1·63	1·59	1·56	1·52
26	2·91	2·52	2·31	2·17	2·08	2·01	1·96	1·92	1·88	1·86	1·81	1·76	1·71	1·68	1·65	1·61	1·58	1·54	1·50
27	2·90	2·51	2·30	2·17	2·07	2·00	1·95	1·91	1·87	1·85	1·80	1·75	1·70	1·67	1·64	1·60	1·57	1·53	1·49
28	2·89	2·50	2·29	2·16	2·06	2·00	1·94	1·90	1·87	1·84	1·79	1·74	1·69	1·66	1·63	1·59	1·56	1·52	1·48
29	2·89	2·50	2·28	2·15	2·06	1·99	1·93	1·89	1·86	1·83	1·78	1·73	1·68	1·65	1·62	1·58	1·55	1·51	1·47
30	2·88	2·49	2·28	2·14	2·05	1·98	1·93	1·88	1·85	1·82	1·77	1·72	1·67	1·64	1·61	1·57	1·54	1·50	1·46
40	2·84	2·44	2·23	2·09	2·00	1·93	1·87	1·83	1·79	1·76	1·71	1·66	1·61	1·57	1·54	1·51	1·47	1·42	1·38
60	2·79	2·39	2·18	2·04	1·95	1·87	1·82	1·77	1·74	1·71	1·66	1·60	1·54	1·51	1·48	1·44	1·40	1·35	1·29
120	2·75	2·35	2·13	1·99	1·90	1·82	1·77	1·72	1·68	1·65	1·60	1·55	1·48	1·45	1·41	1·37	1·32	1·26	1·19
∞	2·71	2·30	2·08	1·94	1·85	1·77	1·72	1·67	1·63	1·60	1·55	1·49	1·42	1·38	1·34	1·30	1·24	1·17	1·00

$F = \frac{s_1^2}{s_2^2} = \frac{S_1}{\nu_1} \Big/ \frac{S_2}{\nu_2}$, where $s_1^2 = S_1/\nu_1$ and $s_2^2 = S_2/\nu_2$ are independent mean squares estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively.

Percentage Points of the F-distribution (Variance Ratio) (continued)

Upper 5 % points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161·4	199·5	215·7	224·6	230·2	234·0	236·8	238·9	240·5	241·9	243·9	245·9	248·0	249·1	250·1	251·1	252·2	253·3	254·3
2	18·51	19·00	19·16	19·25	19·30	19·33	19·35	19·37	19·38	19·40	19·41	19·43	19·45	19·45	19·46	19·47	19·48	19·49	19·50
3	10·13	9·55	9·28	9·12	9·01	8·94	8·89	8·85	8·81	8·79	8·74	8·70	8·66	8·64	8·62	8·59	8·57	8·55	8·53
4	7·71	6·94	6·59	6·39	6·26	6·16	6·09	6·04	6·00	5·96	5·91	5·86	5·80	5·77	5·75	5·72	5·69	5·66	5·63
5	6·61	5·79	5·41	5·19	5·05	4·95	4·88	4·82	4·77	4·74	4·68	4·62	4·56	4·53	4·50	4·46	4·43	4·40	4·36
6	5·99	5·14	4·76	4·53	4·39	4·28	4·21	4·15	4·10	4·06	4·00	3·94	3·87	3·84	3·81	3·77	3·74	3·70	3·67
7	5·59	4·74	4·35	4·12	3·97	3·87	3·79	3·73	3·68	3·64	3·57	3·51	3·44	3·41	3·38	3·34	3·30	3·27	3·23
8	5·32	4·46	4·07	3·84	3·69	3·58	3·50	3·44	3·39	3·35	3·28	3·22	3·15	3·12	3·08	3·04	3·01	2·97	2·93
9	5·12	4·26	3·86	3·63	3·48	3·37	3·29	3·23	3·18	3·14	3·07	3·01	2·94	2·90	2·86	2·83	2·79	2·75	2·71
10	4·96	4·10	3·71	3·48	3·33	3·22	3·14	3·07	3·02	2·98	2·91	2·85	2·77	2·74	2·70	2·66	2·62	2·58	2·54
11	4·84	3·98	3·59	3·36	3·20	3·09	3·01	2·95	2·90	2·85	2·79	2·72	2·65	2·61	2·57	2·53	2·49	2·45	2·40
12	4·75	3·89	3·49	3·26	3·11	3·00	2·91	2·85	2·80	2·75	2·69	2·62	2·54	2·51	2·47	2·43	2·38	2·34	2·30
13	4·67	3·81	3·41	3·18	3·03	2·92	2·83	2·77	2·71	2·67	2·60	2·53	2·46	2·42	2·38	2·34	2·30	2·25	2·21
14	4·60	3·74	3·34	3·11	2·96	2·85	2·76	2·70	2·65	2·60	2·53	2·46	2·39	2·35	2·31	2·27	2·22	2·18	2·13
15	4·54	3·68	3·29	3·06	2·90	2·79	2·71	2·64	2·59	2·54	2·48	2·40	2·33	2·29	2·25	2·20	2·16	2·11	2·07
16	4·49	3·63	3·24	3·01	2·85	2·74	2·66	2·59	2·54	2·49	2·42	2·35	2·28	2·24	2·19	2·15	2·11	2·06	2·01
17	4·45	3·59	3·20	2·96	2·81	2·70	2·61	2·55	2·49	2·45	2·38	2·31	2·23	2·19	2·15	2·10	2·06	2·01	1·96
18	4·41	3·55	3·16	2·93	2·77	2·66	2·58	2·51	2·46	2·41	2·34	2·27	2·19	2·15	2·11	2·06	2·02	1·97	1·92
19	4·38	3·52	3·13	2·90	2·74	2·63	2·54	2·48	2·42	2·38	2·31	2·23	2·16	2·11	2·07	2·03	1·98	1·93	1·88
20	4·35	3·49	3·10	2·87	2·71	2·60	2·51	2·45	2·39	2·35	2·28	2·20	2·12	2·08	2·04	1·99	1·95	1·90	1·84
21	4·32	3·47	3·07	2·84	2·68	2·57	2·49	2·42	2·37	2·32	2·25	2·18	2·10	2·05	2·01	1·96	1·92	1·87	1·81
22	4·30	3·44	3·05	2·82	2·66	2·55	2·46	2·40	2·34	2·30	2·23	2·15	2·07	2·03	1·98	1·94	1·89	1·84	1·78
23	4·28	3·42	3·03	2·80	2·64	2·53	2·44	2·37	2·32	2·27	2·20	2·13	2·05	2·01	1·96	1·91	1·86	1·81	1·76
24	4·26	3·40	3·01	2·78	2·62	2·51	2·42	2·36	2·30	2·25	2·18	2·11	2·03	1·98	1·94	1·89	1·84	1·79	1·73
25	4·24	3·39	2·99	2·76	2·60	2·49	2·40	2·34	2·28	2·24	2·16	2·09	2·01	1·96	1·92	1·87	1·82	1·77	1·71
26	4·23	3·37	2·98	2·74	2·59	2·47	2·39	2·32	2·27	2·22	2·15	2·07	1·99	1·95	1·90	1·85	1·80	1·75	1·69
27	4·21	3·35	2·96	2·73	2·57	2·46	2·37	2·31	2·25	2·20	2·13	2·06	1·97	1·93	1·88	1·84	1·79	1·73	1·67
28	4·20	3·34	2·95	2·71	2·56	2·45	2·36	2·29	2·24	2·19	2·12	2·04	1·96	1·91	1·87	1·82	1·77	1·71	1·65
29	4·18	3·33	2·93	2·70	2·55	2·43	2·35	2·28	2·22	2·18	2·10	2·03	1·94	1·90	1·85	1·81	1·75	1·70	1·64
30	4·17	3·32	2·92	2·69	2·53	2·42	2·33	2·27	2·21	2·16	2·09	2·01	1·93	1·89	1·84	1·79	1·74	1·68	1·62
40	4·08	3·23	2·84	2·61	2·45	2·34	2·25	2·18	2·12	2·08	2·00	1·92	1·84	1·79	1·74	1·69	1·64	1·58	1·51
60	4·00	3·15	2·76	2·53	2·37	2·25	2·17	2·10	2·04	1·99	1·92	1·84	1·75	1·70	1·65	1·59	1·53	1·47	1·39
120	3·92	3·07	2·68	2·45	2·29	2·17	2·09	2·02	1·96	1·91	1·83	1·75	1·66	1·61	1·55	1·50	1·43	1·35	1·25
∞	3·84	3·00	2·60	2·37	2·21	2·10	2·01	1·94	1·88	1·83	1·75	1·67	1·57	1·52	1·46	1·39	1·32	1·22	1·00

$F=\frac{s_1^2}{s_2^2}=\frac{S_1}{\nu_1}\Big/\frac{S_2}{\nu_2}$, where $s_1^2=S_1/\nu_1$ and $s_2^2=S_2/\nu_2$ are independent mean squares estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively.

Percentage Points of the F-distribution (Variance Ratio) (continued)

Upper 2·5 % points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	647·8	799·5	864·2	899·6	921·8	937·1	948·2	956·7	963·3	968·6	976·7	984·9	993·1	997·2	1001	1006	1010	1014	1018
2	38·51	39·00	39·17	39·25	39·30	39·33	39·36	39·37	39·39	39·40	39·41	39·43	39·45	39·46	39·46	39·47	39·48	39·49	39·50
3	17·44	16·04	15·44	15·10	14·88	14·73	14·62	14·54	14·47	14·42	14·34	14·25	14·17	14·12	14·08	14·04	13·99	13·95	13·90
4	12·22	10·65	9·98	9·60	9·36	9·20	9·07	8·98	8·90	8·84	8·75	8·66	8·56	8·51	8·46	8·41	8·36	8·31	8·26
5	10·01	8·43	7·76	7·39	7·15	6·98	6·85	6·76	6·68	6·62	6·52	6·43	6·33	6·28	6·23	6·18	6·12	6·07	6·02
6	8·81	7·26	6·60	6·23	5·99	5·82	5·70	5·60	5·52	5·46	5·37	5·27	5·17	5·12	5·07	5·01	4·96	4·90	4·85
7	8·07	6·54	5·89	5·52	5·29	5·12	4·99	4·90	4·82	4·76	4·67	4·57	4·47	4·42	4·36	4·31	4·25	4·20	4·14
8	7·57	6·06	5·42	5·05	4·82	4·65	4·53	4·43	4·36	4·30	4·20	4·10	4·00	3·95	3·89	3·84	3·78	3·73	3·67
9	7·21	5·71	5·08	4·72	4·48	4·32	4·20	4·10	4·03	3·96	3·87	3·77	3·67	3·61	3·56	3·51	3·45	3·39	3·33
10	6·94	5·46	4·83	4·47	4·24	4·07	3·95	3·85	3·78	3·72	3·62	3·52	3·42	3·37	3·31	3·26	3·20	3·14	3·08
11	6·72	5·26	4·63	4·28	4·04	3·88	3·76	3·66	3·59	3·53	3·43	3·33	3·23	3·17	3·12	3·06	3·00	2·94	2·88
12	6·55	5·10	4·47	4·12	3·89	3·73	3·61	3·51	3·44	3·37	3·28	3·18	3·07	3·02	2·96	2·91	2·85	2·79	2·72
13	6·41	4·97	4·35	4·00	3·77	3·60	3·48	3·39	3·31	3·25	3·15	3·05	2·95	2·89	2·84	2·78	2·72	2·66	2·60
14	6·30	4·86	4·24	3·89	3·66	3·50	3·38	3·29	3·21	3·15	3·05	2·95	2·84	2·79	2·73	2·67	2·61	2·55	2·49
15	6·20	4·77	4·15	3·80	3·58	3·41	3·29	3·20	3·12	3·06	2·96	2·86	2·76	2·70	2·64	2·59	2·52	2·46	2·40
16	6·12	4·69	4·08	3·73	3·50	3·34	3·22	3·12	3·05	2·99	2·89	2·79	2·68	2·63	2·57	2·51	2·45	2·38	2·32
17	6·04	4·62	4·01	3·66	3·44	3·28	3·16	3·06	2·98	2·92	2·82	2·72	2·62	2·56	2·50	2·44	2·38	2·32	2·25
18	5·98	4·56	3·95	3·61	3·38	3·22	3·10	3·01	2·93	2·87	2·77	2·67	2·56	2·50	2·44	2·38	2·32	2·26	2·19
19	5·92	4·51	3·90	3·56	3·33	3·17	3·05	2·96	2·88	2·82	2·72	2·62	2·51	2·45	2·39	2·33	2·27	2·20	2·13
20	5·87	4·46	3·86	3·51	3·29	3·13	3·01	2·91	2·84	2·77	2·68	2·57	2·46	2·41	2·35	2·29	2·22	2·16	2·09
21	5·83	4·42	3·82	3·48	3·25	3·09	2·97	2·87	2·80	2·73	2·64	2·53	2·42	2·37	2·31	2·25	2·18	2·11	2·04
22	5·79	4·38	3·78	3·44	3·22	3·05	2·93	2·84	2·76	2·70	2·60	2·50	2·39	2·33	2·27	2·21	2·14	2·08	2·00
23	5·75	4·35	3·75	3·41	3·18	3·02	2·90	2·81	2·73	2·67	2·57	2·47	2·36	2·30	2·24	2·18	2·11	2·04	1·97
24	5·72	4·32	3·72	3·38	3·15	2·99	2·87	2·78	2·70	2·64	2·54	2·44	2·33	2·27	2·21	2·15	2·08	2·01	1·94
25	5·69	4·29	3·69	3·35	3·13	2·97	2·85	2·75	2·68	2·61	2·51	2·41	2·30	2·24	2·18	2·12	2·05	1·98	1·91
26	5·66	4·27	3·67	3·33	3·10	2·94	2·82	2·73	2·65	2·59	2·49	2·39	2·28	2·22	2·16	2·09	2·03	1·95	1·88
27	5·63	4·24	3·65	3·31	3·08	2·92	2·80	2·71	2·63	2·57	2·47	2·36	2·25	2·19	2·13	2·07	2·00	1·93	1·85
28	5·61	4·22	3·63	3·29	3·06	2·90	2·78	2·69	2·61	2·55	2·45	2·34	2·23	2·17	2·11	2·05	1·98	1·91	1·83
29	5·59	4·20	3·61	3·27	3·04	2·88	2·76	2·67	2·59	2·53	2·43	2·32	2·21	2·15	2·09	2·03	1·96	1·89	1·81
30	5·57	4·18	3·59	3·25	3·03	2·87	2·75	2·65	2·57	2·51	2·41	2·31	2·20	2·14	2·07	2·01	1·94	1·87	1·79
40	5·42	4·05	3·46	3·13	2·90	2·74	2·62	2·53	2·45	2·39	2·29	2·18	2·07	2·01	1·94	1·88	1·80	1·72	1·64
60	5·29	3·93	3·34	3·01	2·79	2·63	2·51	2·41	2·33	2·27	2·17	2·06	1·94	1·88	1·82	1·74	1·67	1·58	1·48
120	5·15	3·80	3·23	2·89	2·67	2·52	2·39	2·30	2·22	2·16	2·05	1·94	1·82	1·76	1·69	1·61	1·53	1·43	1·31
∞	5·02	3·69	3·12	2·79	2·57	2·41	2·29	2·19	2·11	2·05	1·94	1·83	1·71	1·64	1·57	1·48	1·39	1·27	1·00

$F=\dfrac{s_1^2}{s_2^2}=\dfrac{S_1}{\nu_1}\Big/\dfrac{S_2}{\nu_2}$, where $s_1^2=S_1/\nu_1$ and $s_2^2=S_2/\nu_2$ are independent mean squares estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively.

Percentage Points of the F-distribution (Variance Ratio) (continued)

Upper 1% points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999·5	5403	5625	5764	5859	5928	5981	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98·50	99·00	99·17	99·25	99·30	99·33	99·36	99·37	99·39	99·40	99·42	99·43	99·45	99·46	99·47	99·47	99·48	99·49	99·50
3	34·12	30·82	29·46	28·71	28·24	27·91	27·67	27·49	27·35	27·23	27·05	26·87	26·69	26·60	26·50	26·41	26·32	26·22	26·13
4	21·20	18·00	16·69	15·98	15·52	15·21	14·98	14·80	14·66	14·55	14·37	14·20	14·02	13·93	13·84	13·75	13·65	13·56	13·46
5	16·26	13·27	12·06	11·39	10·97	10·67	10·46	10·29	10·16	10·05	9·89	9·72	9·55	9·47	9·38	9·29	9·20	9·11	9·02
6	13·75	10·92	9·78	9·15	8·75	8·47	8·26	8·10	7·98	7·87	7·72	7·56	7·40	7·31	7·23	7·14	7·06	6·97	6·88
7	12·25	9·55	8·45	7·85	7·46	7·19	6·99	6·84	6·72	6·62	6·47	6·31	6·16	6·07	5·99	5·91	5·82	5·74	5·65
8	11·26	8·65	7·59	7·01	6·63	6·37	6·18	6·03	5·91	5·81	5·67	5·52	5·36	5·28	5·20	5·12	5·03	4·95	4·86
9	10·56	8·02	6·99	6·42	6·06	5·80	5·61	5·47	5·35	5·26	5·11	4·96	4·81	4·73	4·65	4·57	4·48	4·40	4·31
10	10·04	7·56	6·55	5·99	5·64	5·39	5·20	5·06	4·94	4·85	4·71	4·56	4·41	4·33	4·25	4·17	4·08	4·00	3·91
11	9·65	7·21	6·22	5·67	5·32	5·07	4·89	4·74	4·63	4·54	4·40	4·25	4·10	4·02	3·94	3·86	3·78	3·69	3·60
12	9·33	6·93	5·95	5·41	5·06	4·82	4·64	4·50	4·39	4·30	4·16	4·01	3·86	3·78	3·70	3·62	3·54	3·45	3·36
13	9·07	6·70	5·74	5·21	4·86	4·62	4·44	4·30	4·19	4·10	3·96	3·82	3·66	3·59	3·51	3·43	3·34	3·25	3·17
14	8·86	6·51	5·56	5·04	4·69	4·46	4·28	4·14	4·03	3·94	3·80	3·66	3·51	3·43	3·35	3·27	3·18	3·09	3·00
15	8·68	6·36	5·42	4·89	4·56	4·32	4·14	4·00	3·89	3·80	3·67	3·52	3·37	3·29	3·21	3·13	3·05	2·96	2·87
16	8·53	6·23	5·29	4·77	4·44	4·20	4·03	3·89	3·78	3·69	3·55	3·41	3·26	3·18	3·10	3·02	2·93	2·84	2·75
17	8·40	6·11	5·18	4·67	4·34	4·10	3·93	3·79	3·68	3·59	3·46	3·31	3·16	3·08	3·00	2·92	2·83	2·75	2·65
18	8·29	6·01	5·09	4·58	4·25	4·01	3·84	3·71	3·60	3·51	3·37	3·23	3·08	3·00	2·92	2·84	2·75	2·66	2·57
19	8·18	5·93	5·01	4·50	4·17	3·94	3·77	3·63	3·52	3·43	3·30	3·15	3·00	2·92	2·84	2·76	2·67	2·58	2·49
20	8·10	5·85	4·94	4·43	4·10	3·87	3·70	3·56	3·46	3·37	3·23	3·09	2·94	2·86	2·78	2·69	2·61	2·52	2·42
21	8·02	5·78	4·87	4·37	4·04	3·81	3·64	3·51	3·40	3·31	3·17	3·03	2·88	2·80	2·72	2·64	2·55	2·46	2·36
22	7·95	5·72	4·82	4·31	3·99	3·76	3·59	3·45	3·35	3·26	3·12	2·98	2·83	2·75	2·67	2·58	2·50	2·40	2·31
23	7·88	5·66	4·76	4·26	3·94	3·71	3·54	3·41	3·30	3·21	3·07	2·93	2·78	2·70	2·62	2·54	2·45	2·35	2·26
24	7·82	5·61	4·72	4·22	3·90	3·67	3·50	3·36	3·26	3·17	3·03	2·89	2·74	2·66	2·58	2·49	2·40	2·31	2·21
25	7·77	5·57	4·68	4·18	3·85	3·63	3·46	3·32	3·22	3·13	2·99	2·85	2·70	2·62	2·54	2·45	2·36	2·27	2·17
26	7·72	5·53	4·64	4·14	3·82	3·59	3·42	3·29	3·18	3·09	2·96	2·81	2·66	2·58	2·50	2·42	2·33	2·23	2·13
27	7·68	5·49	4·60	4·11	3·78	3·56	3·39	3·26	3·15	3·06	2·93	2·78	2·63	2·55	2·47	2·38	2·29	2·20	2·10
28	7·64	5·45	4·57	4·07	3·75	3·53	3·36	3·23	3·12	3·03	2·90	2·75	2·60	2·52	2·44	2·35	2·26	2·17	2·06
29	7·60	5·42	4·54	4·04	3·73	3·50	3·33	3·20	3·09	3·00	2·87	2·73	2·57	2·49	2·41	2·33	2·23	2·14	2·03
30	7·56	5·39	4·51	4·02	3·70	3·47	3·30	3·17	3·07	2·98	2·84	2·70	2·55	2·47	2·39	2·30	2·21	2·11	2·01
40	7·31	5·18	4·31	3·83	3·51	3·29	3·12	2·99	2·89	2·80	2·66	2·52	2·37	2·29	2·20	2·11	2·02	1·92	1·80
60	7·08	4·98	4·13	3·65	3·34	3·12	2·95	2·82	2·72	2·63	2·50	2·35	2·20	2·12	2·03	1·94	1·84	1·73	1·60
120	6·85	4·79	3·95	3·48	3·17	2·96	2·79	2·66	2·56	2·47	2·34	2·19	2·03	1·95	1·86	1·76	1·66	1·53	1·38
∞	6·63	4·61	3·78	3·32	3·02	2·80	2·64	2·51	2·41	2·32	2·18	2·04	1·88	1·79	1·70	1·59	1·47	1·32	1·00

$F = \frac{s_1^2}{s_2^2} = \frac{S_1}{\nu_1} \Big/ \frac{S_2}{\nu_2}$, where $s_1^2 = S_1/\nu_1$ and $s_2^2 = S_2/\nu_2$ are independent mean squares estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively.

Percentage Points of the F-distribution (Variance Ratio) (continued)

Upper 0·5 % *points*

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	16211	20000	21615	22500	23056	23437	23715	23925	24091	24224	24426	24630	24836	24940	25044	25148	25253	25359	25465
2	198·5	199·0	199·2	199·2	199·3	199·3	199·4	199·4	199·4	199·4	199·4	199·4	199·4	199·5	199·5	199·5	199·5	199·5	199·5
3	55·55	49·80	47·47	46·19	45·39	44·84	44·43	44·13	43·88	43·69	43·39	43·08	42·78	42·62	42·47	42·31	42·15	41·99	41·83
4	31·33	26·28	24·26	23·15	22·46	21·97	21·62	21·35	21·14	20·97	20·70	20·44	20·17	20·03	19·89	19·75	19·61	19·47	19·32
5	22·78	18·31	16·53	15·56	14·94	14·51	14·20	13·96	13·77	13·62	13·38	13·15	12·90	12·78	12·66	12·53	12·40	12·27	12·14
6	18·63	14·54	12·92	12·03	11·46	11·07	10·79	10·57	10·39	10·25	10·03	9·81	9·59	9·47	9·36	9·24	9·12	9·00	8·88
7	16·24	12·40	10·88	10·05	9·52	9·16	8·89	8·68	8·51	8·38	8·18	7·97	7·75	7·65	7·53	7·42	7·31	7·19	7·08
8	14·69	11·04	9·60	8·81	8·30	7·95	7·69	7·50	7·34	7·21	7·01	6·81	6·61	6·50	6·40	6·29	6·18	6·06	5·95
9	13·61	10·11	8·72	7·96	7·47	7·13	6·88	6·69	6·54	6·42	6·23	6·03	5·83	5·73	5·62	5·52	5·41	5·30	5·19
10	12·83	9·43	8·08	7·34	6·87	6·54	6·30	6·12	5·97	5·85	5·66	5·47	5·27	5·17	5·07	4·97	4·86	4·75	4·64
11	12·23	8·91	7·60	6·88	6·42	6·10	5·86	5·68	5·54	5·42	5·24	5·05	4·86	4·76	4·65	4·55	4·44	4·34	4·23
12	11·75	8·51	7·23	6·52	6·07	5·76	5·52	5·35	5·20	5·09	4·91	4·72	4·53	4·43	4·33	4·23	4·12	4·01	3·90
13	11·37	8·19	6·93	6·23	5·79	5·48	5·25	5·08	4·94	4·82	4·64	4·46	4·27	4·17	4·07	3·97	3·87	3·76	3·65
14	11·06	7·92	6·68	6·00	5·56	5·26	5·03	4·86	4·72	4·60	4·43	4·25	4·06	3·96	3·86	3·76	3·66	3·55	3·44
15	10·80	7·70	6·48	5·80	5·37	5·07	4·85	4·67	4·54	4·42	4·25	4·07	3·88	3·79	3·69	3·58	3·48	3·37	3·26
16	10·58	7·51	6·30	5·64	5·21	4·91	4·69	4·52	4·38	4·27	4·10	3·92	3·73	3·64	3·54	3·44	3·33	3·22	3·11
17	10·38	7·35	6·16	5·50	5·07	4·78	4·56	4·39	4·25	4·14	3·97	3·79	3·61	3·51	3·41	3·31	3·21	3·10	2·98
18	10·22	7·21	6·03	5·37	4·96	4·66	4·44	4·28	4·14	4·03	3·86	3·68	3·50	3·40	3·30	3·20	3·10	2·99	2·87
19	10·07	7·09	5·92	5·27	4·85	4·56	4·34	4·18	4·04	3·93	3·76	3·59	3·40	3·31	3·21	3·11	3·00	2·89	2·78
20	9·94	6·99	5·82	5·17	4·76	4·47	4·26	4·09	3·96	3·85	3·68	3·50	3·32	3·22	3·12	3·02	2·92	2·81	2·69
21	9·83	6·89	5·73	5·09	4·68	4·39	4·18	4·01	3·88	3·77	3·60	3·43	3·24	3·15	3·05	2·95	2·84	2·73	2·61
22	9·73	6·81	5·65	5·02	4·61	4·32	4·11	3·94	3·81	3·70	3·54	3·36	3·18	3·08	2·98	2·88	2·77	2·66	2·55
23	9·63	6·73	5·58	4·95	4·54	4·26	4·05	3·88	3·75	3·64	3·47	3·30	3·12	3·02	2·92	2·82	2·71	2·60	2·48
24	9·55	6·66	5·52	4·89	4·49	4·20	3·99	3·83	3·69	3·59	3·42	3·25	3·06	2·97	2·87	2·77	2·66	2·55	2·43
25	9·48	6·60	5·46	4·84	4·43	4·15	3·94	3·78	3·64	3·54	3·37	3·20	3·01	2·92	2·82	2·72	2·61	2·50	2·38
26	9·41	6·54	5·41	4·79	4·38	4·10	3·89	3·73	3·60	3·49	3·33	3·15	2·97	2·87	2·77	2·67	2·56	2·45	2·33
27	9·34	6·49	5·36	4·74	4·34	4·06	3·85	3·69	3·56	3·45	3·28	3·11	2·93	2·83	2·73	2·63	2·52	2·41	2·29
28	9·28	6·44	5·32	4·70	4·30	4·02	3·81	3·65	3·52	3·41	3·25	3·07	2·89	2·79	2·69	2·59	2·48	2·37	2·25
29	9·23	6·40	5·28	4·66	4·26	3·98	3·77	3·61	3·48	3·38	3·21	3·04	2·86	2·76	2·66	2·56	2·45	2·33	2·21
30	9·18	6·35	5·24	4·62	4·23	3·95	3·74	3·58	3·45	3·34	3·18	3·01	2·82	2·73	2·63	2·52	2·42	2·30	2·18
40	8·83	6·07	4·98	4·37	3·99	3·71	3·51	3·35	3·22	3·12	2·95	2·78	2·60	2·50	2·40	2·30	2·18	2·06	1·93
60	8·49	5·79	4·73	4·14	3·76	3·49	3·29	3·13	3·01	2·90	2·74	2·57	2·39	2·29	2·19	2·08	1·96	1·83	1·69
120	8·18	5·54	4·50	3·92	3·55	3·28	3·09	2·93	2·81	2·71	2·54	2·37	2·19	2·09	1·98	1·87	1·75	1·61	1·43
∞	7·88	5·30	4·28	3·72	3·35	3·09	2·90	2·74	2·62	2·52	2·36	2·19	2·00	1·90	1·79	1·67	1·53	1·36	1·00

$F=\frac{s_1^2}{s_2^2}=\frac{S_1}{\nu_1}\Big/\frac{S_2}{\nu_2}$, where $s_1^2=S_1/\nu_1$ and $s_2^2=S_2/\nu_2$ are independent mean squares estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively

Percentage Points of the F-distribution (Variance Ratio) (continued)

Upper 0·1 % points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4053*	5000*	5404*	5625*	5764*	5859*	5929*	5981*	6023*	6056*	6107*	6158*	6209*	6235*	6261*	6287*	6313*	6340*	6366*
2	998·5	999·0	999·2	999·2	999·3	999·3	999·4	999·4	999·4	999·4	999·4	999·4	999·4	999·5	999·5	999·5	999·5	999·5	999·5
3	167·0	148·5	141·1	137·1	134·6	132·8	131·6	130·6	129·9	129·2	128·3	127·4	126·4	125·9	125·4	125·0	124·5	124·0	123·5
4	74·14	61·25	56·18	53·44	51·71	50·53	49·66	49·00	48·47	48·05	47·41	46·76	46·10	45·77	45·43	45·09	44·75	44·40	44·05
5	47·18	37·12	33·20	31·09	29·75	28·84	28·16	27·64	27·24	26·92	26·42	25·91	25·39	25·14	24·87	24·60	24·33	24·06	23·79
6	35·51	27·00	23·70	21·92	20·81	20·03	19·46	19·03	18·69	18·41	17·99	17·56	17·12	16·89	16·67	16·44	16·21	15·99	15·75
7	29·25	21·69	18·77	17·19	16·21	15·52	15·02	14·63	14·33	14·08	13·71	13·32	12·93	12·73	12·53	12·33	12·12	11·91	11·70
8	25·42	18·49	15·83	14·39	13·49	12·86	12·40	12·04	11·77	11·54	11·19	10·84	10·48	10·30	10·11	9·92	9·73	9·53	9·33
9	22·86	16·39	13·90	12·56	11·71	11·13	10·70	10·37	10·11	9·89	9·57	9·24	8·90	8·72	8·55	8·37	8·19	8·00	7·81
10	21·04	14·91	12·55	11·28	10·48	9·92	9·52	9·20	8·96	8·75	8·45	8·13	7·80	7·64	7·47	7·30	7·12	6·94	6·76
11	19·69	13·81	11·56	10·35	9·58	9·05	8·66	8·35	8·12	7·92	7·63	7·32	7·01	6·85	6·68	6·52	6·35	6·17	6·00
12	18·64	12·97	10·80	9·63	8·89	8·38	8·00	7·71	7·48	7·29	7·00	6·71	6·40	6·25	6·09	5·93	5·76	5·59	5·42
13	17·81	12·31	10·21	9·07	8·35	7·86	7·49	7·21	6·98	6·80	6·52	6·23	5·93	5·78	5·63	5·47	5·30	5·14	4·97
14	17·14	11·78	9·73	8·62	7·92	7·43	7·08	6·80	6·58	6·40	6·13	5·85	5·56	5·41	5·25	5·10	4·94	4·77	4·60
15	16·59	11·34	9·34	8·25	7·57	7·09	6·74	6·47	6·26	6·08	5·81	5·54	5·25	5·10	4·95	4·80	4·64	4·47	4·31
16	16·12	10·97	9·00	7·94	7·27	6·81	6·46	6·19	5·98	5·81	5·55	5·27	4·99	4·85	4·70	4·54	4·39	4·23	4·06
17	15·72	10·66	8·73	7·68	7·02	6·56	6·22	5·96	5·75	5·58	5·32	5·05	4·78	4·63	4·48	4·33	4·18	4·02	3·85
18	15·38	10·39	8·49	7·46	6·81	6·35	6·02	5·76	5·56	5·39	5·13	4·87	4·59	4·45	4·30	4·15	4·00	3·84	3·67
19	15·08	10·16	8·28	7·26	6·62	6·18	5·85	5·59	5·39	5·22	4·97	4·70	4·43	4·29	4·14	3·99	3·84	3·68	3·51
20	14·82	9·95	8·10	7·10	6·46	6·02	5·69	5·44	5·24	5·08	4·82	4·56	4·29	4·15	4·00	3·86	3·70	3·54	3·38
21	14·59	9·77	7·94	6·95	6·32	5·88	5·56	5·31	5·11	4·95	4·70	4·44	4·17	4·03	3·88	3·74	3·58	3·42	3·26
22	14·38	9·61	7·80	6·81	6·19	5·76	5·44	5·19	4·99	4·83	4·58	4·33	4·06	3·92	3·78	3·63	3·48	3·32	3·15
23	14·19	9·47	7·67	6·69	6·08	5·65	5·33	5·09	4·89	4·73	4·48	4·23	3·96	3·82	3·68	3·53	3·38	3·22	3·05
24	14·03	9·34	7·55	6·59	5·98	5·55	5·23	4·99	4·80	4·64	4·39	4·14	3·87	3·74	3·59	3·45	3·29	3·14	2·97
25	13·88	9·22	7·45	6·49	5·88	5·46	5·15	4·91	4·71	4·56	4·31	4·06	3·79	3·66	3·52	3·37	3·22	3·06	2·89
26	13·74	9·12	7·36	6·41	5·80	5·38	5·07	4·83	4·64	4·48	4·24	3·99	3·72	3·59	3·44	3·30	3·15	2·99	2·82
27	13·61	9·02	7·27	6·33	5·73	5·31	5·00	4·76	4·57	4·41	4·17	3·92	3·66	3·52	3·38	3·23	3·08	2·92	2·75
28	13·50	8·93	7·19	6·25	5·66	5·24	4·93	4·69	4·50	4·35	4·11	3·86	3·60	3·46	3·32	3·18	3·02	2·86	2·69
29	13·39	8·85	7·12	6·19	5·59	5·18	4·87	4·64	4·45	4·29	4·05	3·80	3·54	3·41	3·27	3·12	2·97	2·81	2·64
30	13·29	8·77	7·05	6·12	5·53	5·12	4·82	4·58	4·39	4·24	4·00	3·75	3·49	3·36	3·22	3·07	2·92	2·76	2·59
40	12·61	8·25	6·60	5·70	5·13	4·73	4·44	4·21	4·02	3·87	3·64	3·40	3·15	3·01	2·87	2·73	2·57	2·41	2·23
60	11·97	7·76	6·17	5·31	4·76	4·37	4·09	3·87	3·69	3·54	3·31	3·08	2·83	2·69	2·55	2·41	2·25	2·08	1·89
120	11·38	7·32	5·79	4·95	4·42	4·04	3·77	3·55	3·38	3·24	3·02	2·78	2·53	2·40	2·26	2·11	1·95	1·76	1·54
∞	10·83	6·91	5·42	4·62	4·10	3·74	3·47	3·27	3·10	2·96	2·74	2·51	2·27	2·13	1·99	1·84	1·66	1·45	1·00

* Multiply these entries by 100.

This 0·1 % table is based on the following sources: Colcord & Deming (1935); Fisher & Yates (1953, Table V) used with the permission of the authors and of Messrs Oliver and Boyd; Norton (1952).

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Percentage Points of the t-distribution

ν	$Q=0{\cdot}4$ $2Q=0{\cdot}8$	0·25 0·5	0·05 0·1	0·025 0·05	0·005 0·01	0·0025 0·005	0·0005 0·001
1	0·325	1·000	6·314	12·706	63·657	127·32	636·62
2	·289	0·816	2·920	4·303	9·925	14·089	31·598
3	·277	·765	2·353	3·182	5·841	7·453	12·924
4	·271	·741	2·132	2·776	4·604	5·598	8·610
5	0·267	0·727	2·015	2·571	4·032	4·773	6·869
6	·265	·718	1·943	2·447	3·707	4·317	5·959
7	·263	·711	1·895	2·365	3·499	4·029	5·408
8	·262	·706	1·860	2·306	3·355	3·833	5·041
9	·261	·703	1·833	2·262	3·250	3·690	4·781
10	0·260	0·700	1·812	2·228	3·169	3·581	4·587
11	·260	·697	1·796	2·201	3·106	3·497	4·437
12	·259	·695	1·782	2·179	3·055	3·428	4·318
13	·259	·694	1·771	2·160	3·012	3·372	4·221
14	·258	·692	1·761	2·145	2·977	3·326	4·140
15	0·258	0·691	1·753	2·131	2·947	3·286	4·073
16	·258	·690	1·746	2·120	2·921	3·252	4·015
17	·257	·689	1·740	2·110	2·898	3·222	3·965
18	·257	·688	1·734	2·101	2·878	3·197	3·922
19	·257	·688	1·729	2·093	2·861	3·174	3·883
20	0·257	0·687	1·725	2·086	2·845	3·153	3·850
21	·257	·686	1·721	2·080	2·831	3·135	3·819
22	·256	·686	1·717	2·074	2·819	3·119	3·792
23	·256	·685	1·714	2·069	2·807	3·104	3·767
24	·256	·685	1·711	2·064	2·797	3·091	3·745
25	0·256	0·684	1·708	2·060	2·787	3·078	3·725
26	·256	·684	1·706	2·056	2·779	3·067	3·707
27	·256	·684	1·703	2·052	2·771	3·057	3·690
28	·256	·683	1·701	2·048	2·763	3·047	3·674
29	·256	·683	1·699	2·045	2·756	3·038	3·659
30	0·256	0·683	1·697	2·042	2·750	3·030	3·646
40	·255	·681	1·684	2·021	2·704	2·971	3·551
60	·254	·679	1·671	2·000	2·660	2·915	3·460
120	·254	·677	1·658	1·980	2·617	2·860	3·373
∞	·253	·674	1·645	1·960	2·576	2·807	3·291

$Q=1-P(t|\nu)$ is the upper-tail area of the distribution for ν degrees of freedom, appropriate for use in a single-tail test. For a two-tail test, $2Q$ must be used.

Percentage Points of the χ^2-Distribution

ν \ Q	0·995	0·990	0·975	0·950	0·900	0·750	0·500
1	392704.10^{-10}	157088.10^{-9}	982069.10^{-9}	393214.10^{-8}	0·0157908	0·1015308	0·454936
2	0·0100251	0·0201007	0·0506356	0·102587	0·210721	0·575364	1·38629
3	0·0717218	0·114832	0·215795	0·351846	0·584374	1·212534	2·36597
4	0·206989	0·297109	0·484419	0·710723	1·063623	1·92256	3·35669
5	0·411742	0·554298	0·831212	1·145476	1·61031	2·67460	4·35146
6	0·675727	0·872090	1·23734	1·63538	2·20413	3·45460	5·34812
7	0·989256	1·239043	1·68987	2·16735	2·83311	4·25485	6·34581
8	1·34441	1·64650	2·17973	2·73264	3·48954	5·07064	7·34412
9	1·73493	2·08790	2·70039	3·32511	4·16816	5·89883	8·34283
10	2·15586	2·55821	3·24697	3·94030	4·86518	6·73720	9·34182
11	2·60322	3·05348	3·81575	4·57481	5·57778	7·58414	10·3410
12	3·07382	3·57057	4·40379	5·22603	6·30380	8·43842	11·3403
13	3·56503	4·10692	5·00875	5·89186	7·04150	9·29907	12·3398
14	4·07467	4·66043	5·62873	6·57063	7·78953	10·1653	13·3393
15	4·60092	5·22935	6·26214	7·26094	8·54676	11·0365	14·3389
16	5·14221	5·81221	6·90766	7·96165	9·31224	11·9122	15·3385
17	5·69722	6·40776	7·56419	8·67176	10·0852	12·7919	16·3382
18	6·26480	7·01491	8·23075	9·39046	10·8649	13·6753	17·3379
19	6·84397	7·63273	8·90652	10·1170	11·6509	14·5620	18·3377
20	7·43384	8·26040	9·59078	10·8508	12·4426	15·4518	19·3374
21	8·03365	8·89720	10·28293	11·5913	13·2396	16·3444	20·3372
22	8·64272	9·54249	10·9823	12·3380	14·0415	17·2396	21·3370
23	9·26043	10·19567	11·6886	13·0905	14·8480	18·1373	22·3369
24	9·88623	10·8564	12·4012	13·8484	15·6587	19·0373	23·3367
25	10·5197	11·5240	13·1197	14·6114	16·4734	19·9393	24·3366
26	11·1602	12·1981	13·8439	15·3792	17·2919	20·8434	25·3365
27	11·8076	12·8785	14·5734	16·1514	18·1139	21·7494	26·3363
28	12·4613	13·5647	15·3079	16·9279	18·9392	22·6572	27·3362
29	13·1211	14·2565	16·0471	17·7084	19·7677	23·5666	28·3361
30	13·7867	14·9535	16·7908	18·4927	20·5992	24·4776	29·3360
40	20·7065	22·1643	24·4330	26·5093	29·0505	33·6603	39·3353
50	27·9907	29·7067	32·3574	34·7643	37·6886	42·9421	49·3349
60	35·5345	37·4849	40·4817	43·1880	46·4589	52·2938	59·3347
70	43·2752	45·4417	48·7576	51·7393	55·3289	61·6983	69·3345
80	51·1719	53·5401	57·1532	60·3915	64·2778	71·1445	79·3343
90	59·1963	61·7541	65·6466	69·1260	73·2911	80·6247	89·3342
100	67·3276	70·0649	74·2219	77·9295	82·3581	90·1332	99·3341
X	−2·5758	−2·3263	−1·9600	−1·6449	−1·2816	−0·6745	0·0000

$$Q = Q(\chi^2 \mid \nu) = 1 - P(\chi^2 \mid \nu) = 2^{-\frac{1}{2}\nu}\,\{\Gamma(\tfrac{1}{2}\nu)\}^{-1} \int_{\chi^2}^{\infty} e^{-\frac{1}{2}x}\, x^{\frac{1}{2}\nu - 1}\, dx.$$

Percentage Points of the X^2-Distribution (continued)

ν \ Q	0·250	0·100	0·050	0·025	0·010	0·005	0·001
1	1·32330	2·70554	3·84146	5·02389	6·63490	7·87944	10·828
2	2·77259	4·60517	5·99146	7·37776	9·21034	10·5966	13·816
3	4·10834	6·25139	7·81473	9·34840	11·3449	12·8382	16·266
4	5·38527	7·77944	9·48773	11·1433	13·2767	14·8603	18·467
5	6·62568	9·23636	11·0705	12·8325	15·0863	16·7496	20·515
6	7·84080	10·6446	12·5916	14·4494	16·8119	18·5476	22·458
7	9·03715	12·0170	14·0671	16·0128	18·4753	20·2777	24·322
8	10·2189	13·3616	15·5073	17·5345	20·0902	21·9550	26·125
9	11·3888	14·6837	16·9190	19·0228	21·6660	23·5894	27·877
10	12·5489	15·9872	18·3070	20·4832	23·2093	25·1882	29·588
11	13·7007	17·2750	19·6751	21·9200	24·7250	26·7568	31·264
12	14·8454	18·5493	21·0261	23·3367	26·2170	28·2995	32·909
13	15·9839	19·8119	22·3620	24·7356	27·6882	29·8195	34·528
14	17·1169	21·0641	23·6848	26·1189	29·1412	31·3194	36·123
15	18·2451	22·3071	24·9958	27·4884	30·5779	32·8013	37·697
16	19·3689	23·5418	26·2962	28·8454	31·9999	34·2672	39·252
17	20·4887	24·7690	27·5871	30·1910	33·4087	35·7185	40·790
18	21·6049	25·9894	28·8693	31·5264	34·8053	37·1565	42·312
19	22·7178	27·2036	30·1435	32·8523	36·1909	38·5823	43·820
20	23·8277	28·4120	31·4104	34·1696	37·5662	39·9968	45·315
21	24·9348	29·6151	32·6706	35·4789	38·9322	41·4011	46·797
22	26·0393	30·8133	33·9244	36·7807	40·2894	42·7957	48·268
23	27·1413	32·0069	35·1725	38·0756	41·6384	44·1813	49·728
24	28·2412	33·1962	36·4150	39·3641	42·9798	45·5585	51·179
25	29·3389	34·3816	37·6525	40·6465	44·3141	46·9279	52·618
26	30·4346	35·5632	38·8851	41·9232	45·6417	48·2899	54·052
27	31·5284	36·7412	40·1133	43·1945	46·9629	49·6449	55·476
28	32·6205	37·9159	41·3371	44·4608	48·2782	50·9934	56·892
29	33·7109	39·0875	42·5570	45·7223	49·5879	52·3356	58·301
30	34·7997	40·2560	43·7730	46·9792	50·8922	53·6720	59·703
40	45·6160	51·8051	55·7585	59·3417	63·6907	66·7660	73·402
50	56·3336	63·1671	67·5048	71·4202	76·1539	79·4900	86·661
60	66·9815	74·3970	79·0819	83·2977	88·3794	91·9517	99·607
70	77·5767	85·5270	90·5312	95·0232	100·425	104·215	112·317
80	88·1303	96·5782	101·879	106·629	112·329	116·321	124·839
90	98·6499	107·565	113·145	118·136	124·116	128·299	137·208
100	109·141	118·498	124·342	129·561	135·807	140·169	149·449
X	+0·6745	+1·2816	+1·6449	+1·9600	+2·3263	+2·5758	+3·0902

For $\nu > 100$ take

$$\chi^2=\nu\left\{1-\frac{2}{9\nu}+X\sqrt{\frac{2}{9\nu}}\right\}^3 \quad \text{or} \quad \chi^2=\tfrac{1}{2}\{X+\sqrt{(2\nu-1)}\}^2,$$

according to the degree of accuracy required. X is the standardized normal deviate corresponding to $P=1-Q$, and is shown in the bottom line of the table.

Appendix

HP-IB

"HP-IB" stands for Hewlett-Packard Interface Bus and is our version of IEEE Standard 488,1975 which is a universal standard interface. In this program package, it is used to refer to the interface between the computer and the 9872A Plotter. The select code may be found on the interface card. The bus address is found on the back of the plotter itself in binary form. For further information consult the Programming or Operating and Service Manual for the plotter.

Changing the Mass Storage Device

To change the mass storage to a different tape drive, or to a disk, follow these instructions:

1. Type: MASS STORAGE IS":unit specifier"
2. Press: EXECUTE

Some examples are ":T15" for the righthand side tape drive and ":F8" for the flexible disk master.

HEWLETT PACKARD

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