

Sophisticated, flexible routines for statistical model building.

Regression Analysis



Applications of the Hewlett-Packard System 45

Over the last several years, HP has worked with a local university to develop a comprehensive library of statistics programs. These programs, together with HP's experience developing desktop computing systems, give you the most advanced solutions available on the market today to your statistical problems.

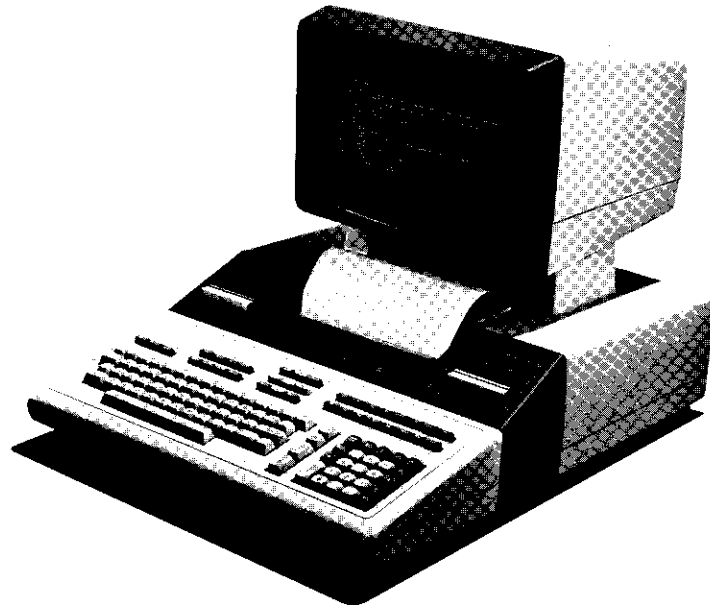
Now, Hewlett-Packard offers you a Regression Analysis software package with unprecedented flexibility for handling your data. It is designed to run on HP's System 45 — a powerful, friendly desktop computer which features a typewriter-like keyboard, a CRT screen, a built-in, high-speed magnetic tape drive and an enhanced BASIC language processor — all integrated into a single, easy-to-use package.

The Regression Analysis programs give you a choice of three regression techniques and a collection of routines for entering, editing and transforming your data for later analysis. They provide, as well, an extensive set of summary statistics routines. Besides all these, there is a program for analyzing residuals from any of the regression programs.

A variety of routines for extensive data manipulation.

An important part of this software package is the collection of routines for entering, editing, transforming and otherwise manipulating your data to prepare it for later analysis. Or, if preliminary analysis indicates the data should be modified in some way, these routines can be used, followed by a second analysis.

Uniform data structure. The data array in this program is compatible with all other statistics packages available from Hewlett-Packard. This means you need not reenter your data



when you want to analyze it with this program or any other HP program. In fact, your data can be collected independent of any program, stored on tape or disk and then analyzed using any of the regression programs described above.

Summary statistics. The following statistical routines for up to 1500 data points are included:

Basic Statistics:

- Mean
- Variance
- Standard Deviation
- Second Moment
- Skewness
- Kurtosis
- t
- Standard Error
- Confidence Interval for Mean
- Coefficient of Variation
- Correlation Coefficient for each pair of variables

Order Statistics:

- Range
- Midrange
- Median
- Percentiles
- Tukey's Middlemeans

Transformation routines. These useful routines allow you to choose more sophisticated models than simple linear regression. You create new variables by using any of these 16 transformations:

1. $aX^b + c$
2. $a \log(bX) + c$
3. $a \ln(bX) + c$
4. $a \exp(bX) + c$
5. $a(b^{cX})$
6. $a \cos(bX) + c$
7. $a \sin(bX) + c$
8. $a + \arcsin(bX) + c$
9. $aX + bY + c$
10. $aX^b Y^c$
11. $a \log(bX + cY)$
12. $a \ln(bX + cY)$
13. $a \cos(bX + cY)$
14. $a \sin(bX + cY)$
15. $\text{PROUND}(X, a)$
16. $\text{DROUND}(X, a)$

You may also enter a transformation routine of your own definition.

A choice of regression methods.

The program package gives you a choice of three regression techniques to analyze your data: polynomial regression (up to degree nine); multiple linear regression (up to 20 variables); and multiple regression with four variable selection procedures.

Polynomial regression. This program fits a polynomial of the form $Y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ to your data, using the least squares method to determine the coefficients. Program output includes:

- mean, variance, standard deviation and coefficient of variation of both variables,
- the correlation coefficient and standard error of estimate,
- a preliminary analysis of variance table to assist in selecting the degree of polynomial desired,
- a final analysis of variance table which includes the F-ratio associated with each degree,
- the computed regression coefficients together with their associated standard errors and t-values,
- the confidence interval for each coefficient.

You can also obtain a scatter plot of your data, together with plots of any of the polynomials you have chosen to examine. Figure 1 draws a sample data set with the results from polynomial regression.

Multiple linear regression. This program fits a model of the form $Y = a_0 + a_1x_1 + a_2x_2 + \dots + a_mx_m$ to your data, using the least squares method. Output includes a matrix which gives the correlation between each pair of variables; the coefficient of determination and standard error of estimate; an analysis of variance table including the F-ratio associated with each variable; and the regression coefficients with their associated standard errors, t-values and confidence intervals.

```

Bus Passenger Service Time

Variable # 1      Variable # 2
RES#
1      1.00000      1.40000
2      1.00000      2.00000
3      1.00000      3.00000
4      1.00000      1.80000
5      1.00000      2.00000
6      2.00000      4.70000
7      2.00000      8.00000
8      2.00000      3.00000
27     13.00000     25.20000
28     17.00000     33.50000
29     19.00000     33.70000

*****
POLYNOMIAL REGRESSION ON DATA SET:
Bus Passenger Service Time
*****
Dependent variable = TIME
Independent variable = NUMBER

VARIABLE      N      MEAN      VARIANCE      STANDARD      COEFFICIENT
NUMBER        29     6.00000     23.07143     4.80327      80.05454
TIME          29    12.10690     82.79495     9.09917      75.15694

CORRELATION = .980337144461

Selected degree of regression = 1
R-SQUARED = .961050916826
STANDARD ERROR OF ESTIMATE = 1.82848718352

SOURCE      DF      SUM OF SQUARES      MEAN SQUARE      F-VALUE
TOTAL      28      2318.25962
REGRESSION  1      2227.98776      2227.98776      666.39
X=1        1      2227.98776      2227.98776      666.39
RESIDUAL    27      90.27087      3.34337

VARIABLE      REGRESSION COEFFICIENTS      STANDARD ERROR      T-VALUE
E-FORMAT      E-FORMAT      REG. COEFFICIENT
'CONSTANT'    .96417      .9641723353400E+00      .54919      1.75
X=1           1.85712      .185712074305E+01      .07194      25.91

          COEFFICIENT      95 % CONFIDENCE INTERVAL
          COEFFICIENT      LOWER LIMIT      UPPER LIMIT
'CONSTANT'    .96417      -.16293      2.09128
X=1           1.85712      1.70948      2.00477
    
```

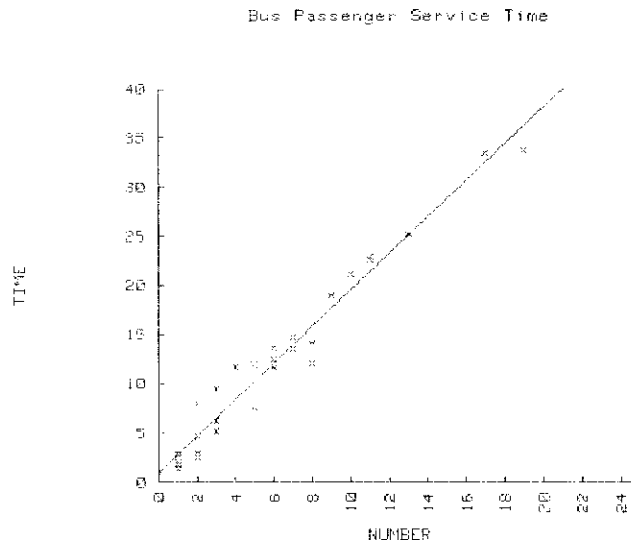


Figure 1. Sample polynomial regression showing the relationship between the number of passengers and service time.

```

*****
STEPWISE REGRESSION ON
SAMPLE
*****

Dependent variable:Water Use
Independent variable(s) = Temp(C)
                        Production
                        Days
                        Payroll

Tolerance = .01
F-value for inclusion = 4
F-value for deletion = 4

CORRELATION MATRIX

Temp(C)      Production      Days      Payroll      Water Use
Temp(C)      1.0000000      -.0923502      .2685939      -.1073726      .2503744
Production    1.0000000      1.0000000      .1857436      .9184717      .6308869
Days          1.0000000      1.0000000      1.0000000      .6318312      -.0887635
Payroll      1.0000000      1.0000000      1.0000000      1.0000000      .4134486
Water Use    1.0000000      1.0000000      1.0000000      1.0000000      1.0000000

*****
#--VARIABLE   F TO PART   F TO   REGRESSION COEFFICIENTS   STD
ENTER CORR  TOL DELETE  STD.FORMAT  E-FORMAT  ERROR
1.Temp(C)    1.00      .258  1.000
2.Production  9.92      .631  1.000
3.Days       .12      .089  1.000
4.Payroll    3.09      .410  1.000

*****
STEP NUMBER 1
VARIABLE 'Production' ADDED
R-SQUARED = .39801925079

ANOVA
SOURCE      DF      SUM OF SQUARES      MEAN SQUARE      F-VALUE
TOTAL      16      3861967.52900
REGRESSION  1      1537133.56850      1537133.56850      9.92
RESIDUAL    15      2324833.96050      154988.93123

STANDARD ERROR = 393.686336102

#--VARIABLE   F TO PART   F TO   REGRESSION COEFFICIENTS   STD
ENTER CORR  TOL DELETE  STD.FORMAT  E-FORMAT  ERROR
1.Temp(C)    2.66      .399  .991
2.Production  9.92      .631  1.000
3.Days       .59      .202  .989
4.Payroll    3.79      .541  .156

Constant = 2495.97697139

*****
STEP NUMBER 2
VARIABLE 'Payroll' ADDED
R-SQUARED = .57420302157

ANOVA
SOURCE      DF      SUM OF SQUARES      MEAN SQUARE      F-VALUE
TOTAL      16      3861967.52900
REGRESSION  2      2217553.42436      1108776.71218      9.44
RESIDUAL    14      1644414.10465      117458.15033

STANDARD ERROR = 342.721600568

#--VARIABLE   F TO PART   F TO   REGRESSION COEFFICIENTS   STD
ENTER CORR  TOL DELETE  STD.FORMAT  E-FORMAT  ERROR
1.Temp(C)    3.10      .439  .988
2.Production  9.92      .631  1.000
3.Days       1.83      .351  .962
4.Payroll    5.79      .541  .156

Constant = 5177.19960983

Tolerance value too small and/or F-values insufficient to proceed.

```

Multiple linear regression with four variable selection procedures. A limitation of multiple linear regression is that one or more of the independent variables may fail to explain a sufficient amount of the variation of the dependent variable. This program overcomes this limitation by allowing you to examine an analysis of variance table at each step of the routine and select the next variable to be deleted from those previously added. This is called a manual selection procedure. The program also includes three automatic procedures. Forward selection begins with no variables and adds variables until no more are significant. Backward selection begins with all variables and eliminates insignificant ones. Stepwise selection combines these by starting with no variables and then adding or deleting until all variables in the model are significant and the variables omitted are insignificant. Program output includes the correlation matrix, an analysis of variance table, and the regression coefficients with their associated standard errors. Figure 2 shows a sample data set using stepwise selection.



Figure 2. Sample stepwise regression showing the effect of average external temperature, production yield, number of working days in the month, and number of employees on water use.

Residual analysis. The residual analysis program may be used after any regression program. It will automatically determine the differences between actual Y values and predicted Y values. This program prints a table of residuals and standardized residuals and plots the standardized residuals either against the observation number, or against any variable. Figure 3 shows the residual analysis of the data in Figure 2.

OBS#	OBSERVED Y	PREDICTED Y	RESIDUAL	STAND. RES.	SIGNIF.
1	2973.00000	3598.86969	-217.86969	-.43335	
2	3118.00000	3141.55475	-31.55475	-.09207	
3	3188.00000	2951.51323	228.48677	.66668	
4	3293.00000	3182.87562	110.12438	.32132	
5	3398.00000	3791.88564	-401.88564	-1.17263	
6	4227.00000	3935.76492	451.23508	1.31662	
7	3852.00000	3586.10897	265.89103	.77582	
8	3566.00000	3674.51316	-98.51316	-.29919	
9	3532.00000	3385.88984	146.11016	.42866	
10	3614.00000	3783.51944	-169.51944	-.49463	
11	3886.00000	4169.20368	-273.20368	-.79716	
12	3427.00000	3434.24056	-2.75944	-.08805	
13	3224.00000	3418.90788	-86.90788	-.25358	
14	3214.00000	3767.12980	-553.12980	-1.61393	
15	4345.00000	3786.36312	558.63688	1.63000	
16	4956.00000	4471.67328	484.32672	1.35482	
17	3624.00000	3818.59316	-188.59316	-.54448	

Here's how to get started with the HP Regression Analysis pack.

To order this software package, specify Part No. 09845-15010. The package includes a user instruction manual, Special Function key overlays, and a program cartridge. An additional Source Documentation manual, containing annotated program listings, tape file descriptions and definitions of variables may be ordered to accompany the software pack by specifying Part No. 09845-15012.

To run this software, you'll need the HP 9845A Desktop Computer with 29k bytes of user memory (Opt. 201). The following hardware may be ordered to enhance your system:

- The internal Thermal Line Printer (Opt. 500)
- The Graphics package (Opt. 370 and 700) or the HP 9872A Plotter for plotting
- The I/O ROM (Opt. 320) for entering data from external devices
- The HP 9885M/S Flexible Disk Drive or the HP 7900 Series Disc Drives for storing larger amounts of data.

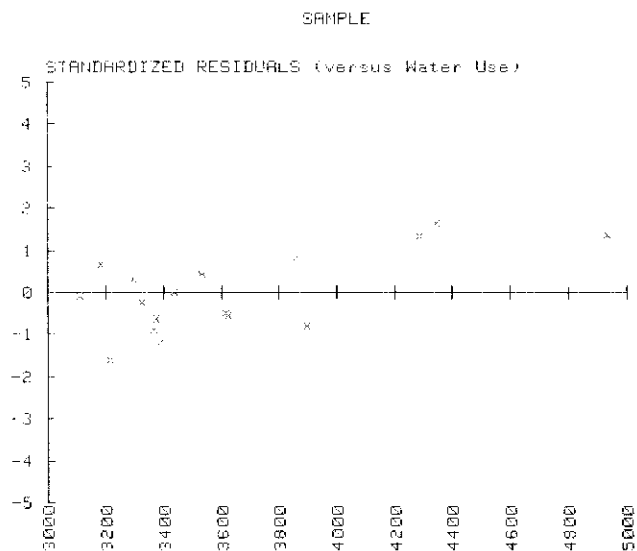


Figure 3. Residual table and plot using the data from Figure 2.



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