Optimize the allocation of your resources . . . automatically.

Linear Programming

hp museum

Applications of the Hewlett-Packard System 45

Desktop Computer Solution for Linear Programming

Linear programming is used extensively in such industries as manufacturing, transportation, agriculture and chemical for applications in production scheduling, profit optimization blending, assignment problems, and nutrition. It is a mathematical method of optimizing a linear function of a number of variables. The variables, representing resources, are subject to constraints, expressed as linear inequalities.

Now Hewlett-Packard provides a Linear Programming package for use with System 45. It gives the user a simple and convenient method of optimizing linear programming models.

A General Purpose Software Package

The System 45 Linear Programming package has been designed for use in many different industries. Since the format of the input and output has not been tailored or structured for any particular application, it can be used for many diverse applications requiring LP capability.

The complete System 45 Linear Programming Package consists of the following parts, all housed in a convenient binder:

- a prerecorded cartridge containing the programs for LP,
- an instruction manual describing the programs and giving detailed operating instructions as well as examples of software operation,
- a Special Function keys template which overlays the desktop computer to define keys for easy user access.

Size of Problems

The size of problems that can be handled differs depending upon the number of variables, the number of constraints, and the number of "greater than" constraints. The table in Figure 1 will give you an idea of the size of problems which can be handled by the program. The instruction manual contains the formula to be used in determining which problems can be solved.

Solving an LP Problem

The first step in solving an LP problem is to set up your input in the format used by the System 45 LP program. A worksheet, included in the manual, assists you in formatting the input.

From the worksheet you enter:

- Problem name
- Maximize or minimize
- Number of constraints
- Number of variables
- Names of the constraints
- Names of the variables
- Objective function coefficients for the variable
- Upper bound on the variables
- Lower bound on the variable
- Variable coefficients for each constraint
- Constraint values

Figure 2 shows a sample worksheet for a typical problem of trying to determine the lowest cost for a food mix.

NUMBER OF CONSTRAINTS		NUMBER OF VARIABLES		
Total	Greater than or Equal to	29k Memory	46k Memory	62k Memory
5	2	127	270	465
10	5	83	190	303
15	7	55	139	229
20	10	34	103	177
25	12	16	76	139
30	15		53	109
35	17		35	84
40	20		17	62
45	22		2	43
50	25			25

Figure 1: This table indicates the size of problems which can be handled with the different memory configurations.

PROBLEM NAME: FEED

No. Constraints 17 Max. or Min. Min
No. Variables 13
No. <= 7 No. = 1 No. >= 9

Variable Name	Corn	Fishm	Alf	Rbran	M&B	Soya	Gcnut	Tapioc	Calpho	Lime	Mol	Oil	WhP	Constraint Type (<=, =, >=)	Constraint Values
Objective Function	23	60	21.5	19.5	51	45	37	15	22.42	1.5	7.14	70	21		
Upper Bounds	40	6	9	-1	1	20	10	15	5	5	5	1	-1		
Lower Bounds	0	0	7	0	0	0	0	0	0	0	0	0	0		
Constraint Name															
PrtnUP	8.5	60	15	13	45	43	45	2	0	0	3	0	16	<=	2050
FatUP	3.8	7.5	1.5	5	9	3.5	6	0	0	0	0	0	4	<=	500
FibrUP	12.5	1	3.3	12	2.5	6	11	0	0	0	0	0	8	<=	650
AshUP	1.5	20	9	16	35	5.5	5.5	0	0	0	11	0	5.5	<=	650
CalUP	.01	5	1.2	.6	11	.2	.17	0	13	38	.1	0	.1	<=	110
PhosUP	.25	3	.2	1.8	5.5	.6	.56	0	9	0	.05	0	.9	<=	70
EnrgUP	1560	1360	400	860	810	1100	1200	1300	0	0	900	8000	810	<=	130000
Weight	1	1	1	1	1	1	1	1	1	1	1	1	1	=	100
PrtnLO	8.5	60	15	13	45	43	45	2	0	0	3	0	16	>=	190
FatLO	3.8	7.5	1.5	5	9	3.5	6	0	0	0	0	0	4	>=	250
FibrLO	12.5	1	3.3	12	2.5	6	11	0	0	0	0	0	8	>=	400
AshLO	1.5	20	9	16	35	5.5	5.5	0	0	0	11	0	5.5	>=	300
CalLO	.01	5	1.2	.6	11	.2	.17	0	13	38	.1	0	.1	>=	80
PhosLO	.25	3	.2	1.8	5.5	.6	.55	0	9	0	.05	0	.9	>=	55
Meth	.2	1.8	0	.24	.5	.74	.46	0	0	0	0	0	.25	>=	30
Gly	.2	5.3	0	.5	2.2	2.7	1.3	0	0	0	0	0	7	>=	80
EnrgLO	1560	1350	400	860	810	1100	1200	1300	0	0	900	8000	810	>=	120000

Figure 2: A sample worksheet for a typical, small LP problem of mixing feed and trying to determine the lowest cost. These worksheets are provided in the instruction manual.

```
CONSTRAINT PrtnUP
       8.500 Corn          60.000 Fishm        15.000 Alf
      13.000 Rbran         45.000 M&B          43.000 Soya
      45.000 Grnut          2.000 Tapioc        3.000 Mol
      16.000 WhP                                             <=      2050.000

CONSTRAINT FatUP
       3.800 Corn           7.500 Fishm         1.500 Alf
       5.000 Rbran          9.000 M&B           3.500 Soya
       6.000 Grnut          4.000 WhP                        <=       500.000

CONSTRAINT FibrUP
      12.500 Corn           1.000 Fishm         3.300 Alf
      12.000 Rbran          2.500 M&B           6.000 Soya
      11.000 Grnut          8.000 WhP                        <=       650.000

CONSTRAINT AshUP
       1.500 Corn          20.000 Fishm         9.000 Alf
      16.000 Rbran         35.000 M&B           5.500 Soya
       5.500 Grnut         11.000 Mol           5.500 WhP    <=       650.000

CONSTRAINT CalUP
        .010 Corn           5.000 Fishm         1.200 Alf
        .600 Rbran         11.000 M&B            .200 Soya
        .170 Grnut         13.000 Calpho       38.000 Lime


     860.000 Rbran        810.000 M&B        1100.000 Soya
    1200.000 Grnut       1300.000 Tapioc      900.000 Mol
    8000.000 Oil          810.000 WhP                        >=     12000.000

OBJECTIVE FUNCTION
   MINIMIZE
     23.000 Corn          60.000 Fishm         21.500 Alf
     19.500 Rbran         51.000 M&B           45.000 Soya
     37.000 Grnut         15.000 Tapioc        22.420 Calpho
      1.500 Lime           7.140 Mol           70.000 Oil
     21.000 WhP

            VARIABLE LIMITS
            ---------------
     0.00     <= Corn     <=      40.00
     0.00     <= Fishm    <=       6.00
     7.00     <= Alf      <=       7.00
     0.00     <= M&B      <=       1.00
     0.00     <= Soya     <=      20.00
     0.00     <= Grnut    <=      10.00
     0.00     <= Tapioc   <=      15.00
     0.00     <= Calpho   <=       5.00
     0.00     <= Lime     <=       5.00
     0.00     <= Mol      <=       5.00
     0.00     <= Oil      <=       1.00
```

Figure 3: Printout of the data for the sample feed-mix problem.

```
                 FEED

*********************************
YOUR VARIABLES 1 THROUGH  13
SURPLUS VARIABLES  14 THROUGH   22
SLACK VARIABLES  23 THROUGH   29
ARTIFICIAL VARIABLES  30 THROUGH   39


Print Initial Tableau?
Yes


TABLEAU AFTER   0 ITERATIONS
      8.50     60.00     15.00     13.00     45.00     43.00     45.00      2.00
      0.00      0.00      3.00      0.00     16.00      0.00      0.00      6.00
      0.00      0.00      0.00      0.00      0.00      0.00      1.00      9.00
      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
      0.00      0.00      0.00      0.00      0.00      0.00      0.00   1945.00


Print Final Tableau?
Yes
```

Figure 4: Initial tableau for the sample problem before solution. Note that surplus, slack and artificial variables are automatically calculated and entered into the tableau.

```
TABLEAU AFTER  21 ITERATIONS
      0.00     33.50      0.00      8.30     19.12      0.00     24.75     26.76
      0.00      0.00     23.42     65.65      0.00      0.00      0.00      0.00
      0.00      0.00     -1.58     41.34      0.00       .01      1.00      0.00
      2.36      0.00       .57      0.00      0.00    -21.60      0.00      0.00
      0.00      0.00      0.00      1.58    -41.34      0.00      -.01    460.53


      0.00     -4.32      0.00     -1.38      8.28      0.00       .69     -4.33
      0.00      0.00     -2.74    -31.89      0.00      0.00      0.00      0.00
      0.00      0.00      -.07      4.13      0.00      -.00      0.00      1.00
      -.68      0.00       .03      0.00      0.00      -.96      0.00      0.00
      0.00      0.00      0.00       .07     -4.13      0.00       .00    203.64
```

Figure 5: Final tableau for the sample problem showing the number of iterations before a solution is reached.

Check Input on the CRT Display

The print section of the program allows the user to view the data on the CRT or print it on one of the optional printers. The printout (see Figure 3) can be used as a check on the data entered or just to provide a hard-copy record of the problem. If any errors have been made during entry of the problem, the modify routine provides a simple way to make corrections. The routine also allows the user to create essentially new problems by just adding or deleting constraints from a previously created problem.

Problem Automatically Stored

After a problem is entered, it is automatically stored on the tape cartridge under the problem name given at the start of the data entry.

The user then has the opportunity to enter another problem, to print, modify, or solve the problem just entered, or to solve any previously entered problem.

Solutions

The solve routine performs the optimization, using a modified simplex method, prints the solution and, if desired, a sensitivity analysis. The tableau, which is used to provide a beginning solution in optimizing the problem, will show not only the variables which have been entered, but also the slack, surplus and artificial variables. The system automatically computes these variables in order to convert the inequalities to equalities. Printing the tableau is an option which you may specify either before or after optimization. (See Figures 4 and 5).

After the optimization, the system will automatically print out the basis and solution variables, including the dual variables, and indicate both the value of the objective function and the number of iterations required as shown in Figure 6. If you wish to see a final tableau, the system will print one out at this point.

Finally, this LP system will provide a sensitivity analysis to determine the range for variables to stay in or out of the basis and the critical level of the constraint values. This part of the program is an important tool for analyzing the results of LP and looking at various alternatives. The sensitivity analysis for the sample problem is shown in Figure 7.

Equipment Required

To use the System 45 Linear Programming package, the following hardware is required:

- 9845A Desktop Computer
- 29,882 bytes of read/write memory (Opt. 201)
- Internal thermal line printer (Opt. 500)
- Second internal tape drive (Opt. 600)

The size of linear programming problem which can be handled can be increased without any changes to the program by having a larger internal memory option.

Ordering Information

To order the Linear Programming Package, specify Part No. 09845-10500. For further information on this program package or on the equipment, call or write your nearest Hewlett-Packard Sales Office. Ask to talk with your desktop computer representative.

ANSWERS:

BASIS AFTER ITERATION 23

VARIABLE	VALUE
SLK 1	460.554
SLK 2	203.644
Soya	10.470
SLK 4	174.426
SUR 13	30.000
SLK 6	15.000
SLK 7	10000.000
SUR 10	46.356
Alf	8.910
SUR 12	175.574
SUR 9	1399.446
WhP	21.464
Lime	1.627
Calpho	.199
SUR 16	136.405
Corn	30.432
SUR 11	250.000
Fishm U	6.000
Tapioc U	15.000
Mol U	5.000
Oil U	1.000

DUAL VARIABLES

COLUMN	CONSTRAINT #	VALUE
14	1	0.000
15	2	0.000
16	3	0.000
17	4	0.000
18	5	0.000
19	6	.712
20	7	19.929
21	8	0.000
22	9	.027
23	10	0.000
24	11	0.000
25	12	3.663
26	13	0.000
27	14	.591
28	15	0.000
29	16	0.000
30	17	-23.560

OBJECTIVE FUNCTION VALUE = 2508.90

Figure 6: Printout of the basis and solution variables, including dual variables, the value of the objective function, and the number of iterations required for optimization.

SENSITIVITY ANALYSIS

RIGHT-HAND-SIDE RANGING

CONSTRAINT NO.	LL	B(I)	UL
1	545.554	1945.000	UNBOUNDED
2	443.144	489.500	UNBOUNDED
3	376.900	626.900	UNBOUNDED
4	411.426	587.000	UNBOUNDED
5	71.600	101.600	UNBOUNDED
6	53.600	68.600	70.344
7	127199.003	127200.000	127204.484
8	-43.405	93.000	UNBOUNDED
9	-5312.847	85.000	866.448
10	-221.054	239.500	UNBOUNDED
11	173.256	376.900	UNBOUNDED
12	204.074	237.000	243.946
13	-102.826	71.600	UNBOUNDED
14	23.600	53.600	593.960
15	15.000	30.000	UNBOUNDED
16	-9920.000	80.000	UNBOUNDED
17	117185.780	117200.000	117205.379

BASIS VARIABLE COEFFICIENT RANGING

VARIABLE	LL	C(J)	UL
Soya	35.053	45.000	54.129
Alf	8.640	21.500	28.796
WhP	12.585	21.000	25.587
Lime	UNBOUNDED	1.500	21.447
Calpho	16.146	22.420	35.233
Corn	10.984	23.000	36.768

NON-BASIS VARIABLE COEFFICIENT RANGING

VARIABLE	LL	C(J)	UL
Fishm	31.087	60.000	UNBOUNDED
Rbran	7.833	19.500	UNBOUNDED
M&B	42.824	51.000	UNBOUNDED
Grnut	24.014	37.000	UNBOUNDED
Tapioc	UNBOUNDED	15.000	UNBOUNDED
Mol	UNBOUNDED	7.140	UNBOUNDED
Oil	UNBOUNDED	70.000	UNBOUNDED

Figure 7: This sensitivity analysis allows you to evaluate the effect of various constraints on the basis and indicates the range of variables to stay in or out of that basis.

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