LIBRARY	PROGRAM	PRESENT	CHANGE
STATISTICS			
A & B	09100-70801 (Prog. Step, pp 1)	25 - X Code 30	25 - X Code 36
A & B	09100-70802 (UI 1st entry point)	After last entry: SET FLAG before pressing CONTINUE	f_i Y, X_i + X
A & B	09100-70808 (Title)	$(E_i \neq E_i)$	$(E_i \neq E_j)$
A & B	09100-70811 (Prog. Steps, pp 2)	Step 4-7 Expression $\ln X_i - \frac{\ln X_i}{N}$	Step 4-7 Exprcssion $(\ln x)^2 - \frac{(\ln x)^2}{N}$
A & B	09100-70811 (Prog. Step pp 3)	Step 6-1 Expression $(\ln X_i - \frac{\ln X_i}{N}$ $(\ln Y_i - \frac{\ln Y_i}{N}$	Step 6-1 Expression (ln x ln y) $(\ln x)(\ln y)$ N
A & B	09100-70811 (Prog. Step, pp 2)	Step 5-3 Expression $\ln Y_i - \frac{\ln Y_i}{N}^2$	Step 5-3 Expression $(\ln y)^2 - \frac{(\ln y)^2}{N}$

HP Computer Museum
www.hpmuseum.net

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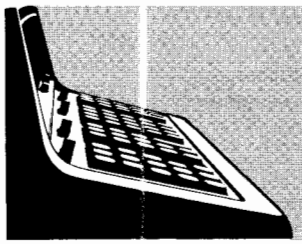


9100B STATISTICS PROGRAM LISTING

70801 - MEAN AND STANDARD DEVIATION
Calculates the mean and standard deviation of n data points.

70802 - STANDARD DEVIATION AND MEAN OF GROUPED DATA
Calculates the mean and standard deviation of data points of certain frequencies.

70803 - LINEAR REGRESSION
Calculates the best fit of a set of data points to the line $y = ax + b$, i.e., the program computes the estimates $\hat{a}$ and $\hat{b}$. It also gives the correlation coefficient r.

70804 - NORMAL PROBABILITY INTEGRAL
Evaluates the integral of the normal density function.

70805 - χ^2 - CHI SQUARE DISTRIBUTION
Calculates the integral of the Chi Square distribution from 0 up to a value of χ^2 for a given number of degrees of freedom.

70806 - χ^2 - CHI SQUARE EVALUATION EXPECTED VALUES EQUAL ($E_i = E$)
Chi square calculation where the expected value of each observation is equal.

70808 - χ^2 - CHI SQUARE EVALUATION EXPECTED VALUES UNEQUAL ($E_i \neq E_j$)
Chi square calculation where the expected values of the observations are not necessarily equal.

70811 - LEAST SQUARES FIT-POWER CURVE
Calculates coefficients fitting data points (x_i, y_i) to an equation of the form: $y = ax^b$

70812 - LEAST SQUARES FIT-EXPONENTIAL
Calculates coefficients fitting data points (x_i, y_i) to an equation of the form: $y = ae^{bx}$

70813 - POISSON DENSITY
Calculates the various summations associated with the Poisson density to give a probability based on an input parameter and summation endpoints.

70816 - RANDOM NUMBER GENERATOR
Random numbers (RN) in the range $0 \leq RN \leq 1$ are calculated; more than 10,000 random numbers may be generated before any previous value is repeated.

70904 - NORMAL (GAUSSIAN) CURVE PLOT
Given mean (M) and variance σ^2, this program generates a normal curve given by

$$y = \frac{1}{\sigma\sqrt{2\pi}} \; e^{\frac{-(X - M)^2}{2\sigma^2}}$$

This program can be used with Program 70905, Histogram Generation.

70907 - ONE WAY ANALYSIS OF VARIANCE
This program separates the total variance in a table of data into that due to chance and that due to differences between the population means underlying each column of sample data.

9100B ONLY

70901 - MULTIPLE LINEAR REGRESSIONS
Given a set of data points (X_i, Y_i, Z_i), this program determines the coefficients of the linear equation

$$Z = a_0 + a_1X + a_2Y$$

70902 - WEIBULL DISTRIBUTION PARAMETER CALCULATION FOR FAILURE DATA
Calculates the parameters for the Weibull distribution and thus estimates of times to failure percentages may be made.

70903 - NON-LINEAR REGRESSION - LEAST SQUARES PARABOLA
Calculates coefficients fitting data points (x_i, y_i) to an equation of the form:

$$y = a_0 + a_1x + a_2x^2$$

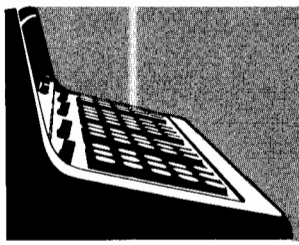
70905 - HISTOGRAM GENERATION WITH PLOT
This program generates and plots a histogram of ten windows given a set of positive numbers. The mean M and variance σ^2 of the data set are computed and stored for use by Program 70904, NORMAL CURVE PLOT.

70906 - HISTOGRAM GENERATION
This program generates a histogram table of ten windows given a data set of positive numbers. In addition it determines the mean M and the variance σ^2 of the data set.

70908 - F DISTRIBUTION
This program evaluates the F distribution density function for given values of F, V_1, and V_2.

70909 - TWO WAY ANALYSIS OF VARIANCE (m X 4)
This program analyses the total statistical variance in a table of data by separating the total variance into two parts, the variance among rows, and the variance between columns. These variances are then compared to the variance due to random influence.

70910 - TWO WAY ANALYSIS OF VARIANCE WITH REPLICATES
This program analyses the total statistical variance of a table of data by separating the total variance into three parts; the variance among rows, the variance between columns, and the variance due to interaction.



PART NO.
09100-70801

MEAN AND STANDARD DEVIATION

This program calculates the mean, $\overline{X}$, and standard deviation, S, of a set of data points by the equations

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

and

$$S = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}}$$

Reference: Introduction to the Theory of Statistics
by Mood and Graybill

McGraw - Hill 1963

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) or END

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
i ——— X

(i indicates point to be entered)

ENTER DATA: $X_i \longrightarrow X$

PRESS: CONTINUE

NO — Last Entry? — YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

S ——— Z
$\overline{X}$ ——— Y
n ——— X

S = Standard deviation

$\overline{X}$ = Mean

n = Number of data points

EXAMPLES

SAMPLE DATA

5.036
5.085
4.991
4.935
4.999
5.031
5.064
4.942
5.051
5.011

S = .0493
$\overline{X}$ = 5.0145
n = 10

SAMPLE DATA

4
5
6
4
7
2

S = 1.7512
$\overline{X}$ = 4.6667
n = 6

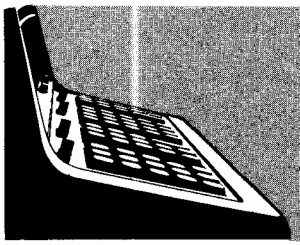
JUNE 1969
PART NO. 09100-70801

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
0	0	CLEAR	20										
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY									
	2	x→( )	23										
	3	d	17										
	4	STOP	41	X_i	0	0	ENTER X_i						
	5	IF FLAG	43										
	6	1	01	BRANCH AFTER LAST ENTRY									
	7	7	07										
	8	↑	27										
	9	×	36	CALCULATE ΣX_i AND ΣX_i^2									
	a	ACC +	60										
	b	d	17										
	c	x⇄y	30										
	d	1	01										
1	0	+	33	INCREMENT i									
	1	y→( )	40										
	2	d	17										
	3	↓	25										
	4	GO TO( )( )	44										
	5	0	00										
	6	4	04										
	7	d	17										
	8	↑	27										
	9	1	01										
	a	−	34	CHANGE n +1 TO n									
	b	y→( )	40										
	c	d	17										
	d	↑	27										
2	0	RECALL	61										
	1	ROLL ↑	22										
	2	÷	35	CALCULATE $\overline{X}$									
	3	y→( )	40										
	4	f	15										
	5	×	36										
	6	f	15										
	7	×	36										
	8	↓	25										
	9	−	34										
	a	d	17										
	b	↑	27	CALCULATE S									
	c	1	01										
	d	−	34										

▲ Denotes Revision

PART NO. 09100-70801

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3	0	f	15									
	1	ROLL ↓	31									
	2	÷	35									
	3	d	17									
	4	ROLL ↓	31									
	5	$\sqrt{x}$	76									
	6	$x \rightleftarrows y$	30									
	7	ROLL ↑	22									
	8	END	46	n	$\overline{X}$	S	DISPLAY					
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



PART NO.
09100-70802

MEAN AND STANDARD DEVIATION OF GROUPED DATA

This program calculates the mean, $\bar{X}$, and standard deviation, S, of a set of data points X_1, X_2, $\cdots$, X_K with frequencies f_1, f_2, $\cdots$, f_K respectively. The equations used are

$$\bar{X} = \frac{\sum_{i=1}^{K} f_i X_i}{\sum_{i=1}^{K} f_i}$$

$$S = \sqrt{\frac{\sum_{i=1}^{K} f_i (X_i - \bar{X})^2}{\sum_{i=1}^{K} f_i - 1}}$$

Reference: Introduction to the Theory of Statistics
by Mood and Graybill

McGraw - Hill 1963

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
i ——— X

(i indicates pair of points to be entered)

ENTER DATA: f_i → Y, X_i → X

PRESS: CONTINUE

NO ← Last Entry? → YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

S ——— Z
$\overline{X}$ ——— Y
n ——— X

S = Standard Deviation

$\overline{X}$ = Mean

n = Number of Data Points

EXAMPLES

SAMPLE DATA

X_i	f_i
1	3
2	3
3	1
4	2
5	1

S = 1.4337
$\overline{X}$ = 2.5
n = 10

SAMPLE DATA

X_i	f_i
41	3
38	5
37	2
39	18
40	22

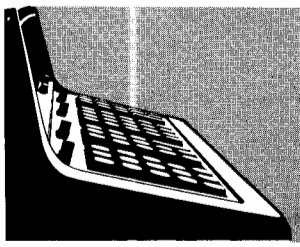
S = .9010
$\overline{X}$ = 39.38
n = 50

PART NO. 09100-70802

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a	
0 0	CLEAR	20										
1	$x\rightarrow(\ )$	23										
2	$\subset$	16										
3	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY									
4	$x\rightarrow(\ )$	23										
5	d	17										
6	STOP	41	X_i	f_i	0	ENTER X_i AND f_i						
7	IF FLAG	43										
8	2	02	BRANCH AFTER LAST ENTRY									
9	5	05										
a	ROLL ↑	22										
b	$\subset$	16										
c	ROLL ↑	22	CALCULATE Σf_i									
d	+	33										
1 0	$y\rightarrow(\ )$	40										
1	$\subset$	16										
2	ROLL ↑	22										
3	×	36										
4	$x\rightleftarrows y$	30	CALCULATE $\Sigma f_i X_i$ AND $\Sigma f_i X_i^2$									
5	×	36										
6	ACC +	60										
7	0	00										
8	↑	27	CLEAR AND RECALL i									
9	d	17										
a	↑	27										
b	1	01	INCREMENT i									
c	+	33										
d	$y\rightarrow(\ )$	40										
2 0	d	17										
1	↓	25										
2	GO TO( )( )	44										
3	0	00										
4	6	06										
5	$y\rightleftarrows(\ )$	24										
6	F	15										
7	↑	27										
8	↓	25	CALCULATE $\overline{X}$									
9	$\subset$	16										
a	÷	35										
b	ROLL ↓	31										
c	×	36										
d	$x\rightarrow(\ )$	23										

PART NO. 09100-70802

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	f	15									
	1	1	01									
	2	ROLL ↑	22									
	3	−	34									
	4	↓	25									
	5	÷	35									
	6	y≷( )	24	CALCULATE S								
	7	E	12									
	8	CHG SIGN	32									
	9	÷	35									
	a	E	12									
	b	+	33									
	c	↓	25									
	d	√x	76									
4	0	↑	27									
	1	f	15									
	2	↑	27									
	3	c	16									
	4	END	46	n	$\bar{X}$	S	DISPLAY					
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



PART NO.
09100-70803

LINEAR REGRESSION AND CORRELATION COEFFICIENT

This program calculates the equation of the straight line of best fit of a set of data points. The best fit is determined by minimizing the sum of the squares of the deviations of the data points from the line.

The program calculates m and b for the equation

$$Y = mX + b.$$

The program also calculates a correlation coefficient r, an indication of goodness of fit. Note $-1 \leq r \leq 1$ where the sign corresponds to the slope m. If $r = 0$ there is no correlation, and if $r = \pm 1$ there is perfect correlation or a perfect fit.

The defining equations are

$$m = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$

$$b = \overline{Y} - m\overline{X}$$

where $\overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$ and $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$

$$r = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2 \sum (Y_i - \overline{Y})^2}}$$

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

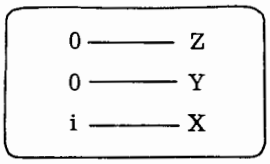
USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

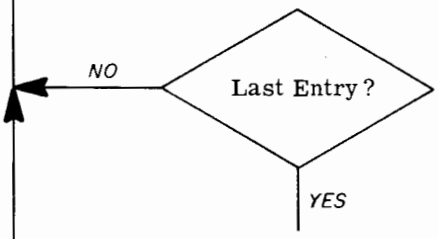
DISPLAY



(i indicates pair of points to be entered)

ENTER DATA: $Y_i \rightarrow Y$, $X_i \rightarrow X$

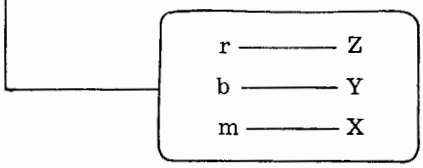
PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY



EXAMPLES

X	Y
26	92
30	85
44	78
50	81
62	54
68	51
74	40

r = -.96

b = 121.04

m = -1.03

Y = -1.03X + 121.04

X	Y
0	1
1	3
2	2
3	4
4	5

r = .9

b = 1.2

m = .9

Y = .9X + 1.2

PART NO. 09100-70803

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0	0	CLEAR	20									
	1	x→( )	23									
	2	d	17									
	3	x→( )	23									
	4	c	16									
	5	x→( )	23									
	6	b	14									
	7	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	8	x→( )	23									
	9	a	13									
	a	STOP	41	X_i	Y_i	0	ENTER X_i AND Y_i					
	b	IF FLAG	43									
	c	3	03									
	d	7	07									
1	0	ACC +	60	CALCULATE ΣX_i AND ΣY_i								
	1	↑	27									
	2	×	36									
	3	x⇄y	30									
	4	y⇄( )	24									
	5	d	17									
	6	+	33									
	7	y⇄( )	24									
	8	d	17									
	9	↓	25									
	a	×	36									
	b	b	14									
	c	+	33									
	d	y→( )	40									
2	0	b	14									
	1	ROLL ↑	22									
	2	↑	27									
	3	×	36									
	4	c	16									
	5	+	33									
	6	y→( )	40									
	7	c	16									
	8	a	13									
	9	↑	27									
	a	1	01									
	b	+	33									
	c	y→( )	40									
	d	a	13									

CLEAR STORAGE REGISTERS

BRANCH AFTER LAST ENTRY

CALCULATE ΣX_i^2

CALCULATE $\Sigma X_i Y_i$

CALCULATE ΣY_i^2

INCREMENT COUNTER

FROM 3-6

PART NO. 09100-70803

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
3	0	0	00										CLEAR DISPLAY REGISTERS AND RECALL COUNTER
	1	ROLL ↓	31										
	2	$x \rightleftarrows y$	30										
	3	↓	25										
	4	GO TO ( )( )	44										
	5	0	00										
	6	a	13										
	7	a	13										DECREMENT COUNT
	8	↑	27										
	9	1	01										
	a	−	34										
	b	↓	25										
	c	$x \rightarrow ( )$	23										
	d	a	13										
4	0	e	12										CALCULATE $\overline{Y}$
	1	↑	27										
	2	a	13										
	3	÷	35										
	4	$y \rightleftarrows ( )$	24										CALCULATE $\overline{X}$
	5	f	15										
	6	÷	35										
	7	$y \rightarrow ( )$	40										CALCULATE $\Sigma(X_i - \overline{X})^2$
	8	e	12										
	9	×	36										
	a	e	12										
	b	×	36										
	c	d	17										
	d	$x \rightleftarrows y$	30										
5	0	−	34										CALCULATE $\Sigma(Y_i - \overline{Y})^2$
	1	$y \rightarrow ( )$	40										
	2	d	17										
	3	c	16										
	4	↑	27										
	5	f	15										
	6	↑	27										
	7	×	36										
	8	a	13										
	9	×	36										
	a	↓	25										
	b	−	34										
	c	$y \rightarrow ( )$	40										
	d	c	16										

FROM 0-d

PART NO. 09100-70803

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
6	b	14										CALCULATE $\Sigma(X_i - \bar{X})(Y_i - \bar{Y})$
	↑	27										
	f	15										
	↑	27										
	e	12										
	×	36										
	a	13										
	×	36										
	↓	25										
	−	34										
	d	17										
	↑	27										
	↓	25										
	$\sqrt{x}$	76										CALCULATE r
7	÷	35										
	c	16										
	$\sqrt{x}$	76										
	÷	35										
	d	17										
	ROLL ↑	22										CALCULATE m
	$x \rightleftarrows y$	30										
	÷	35										
	$y \rightleftarrows ()$	24										
	e	12										
	e	12										
	×	36										CALCULATE b
	f	15										
	$x \rightleftarrows y$	30										
8	−	34										
	e	12										
	END	46	m	b	r							DISPLAY

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
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a											
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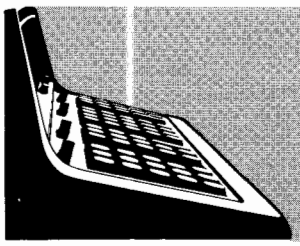
c											
d											



NORMAL PROBABILITY INTEGRAL

This program computes the integral of the standardized normal distribution

$$P(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}z^2} dz$$

The following equations are used

$$P(X) = \frac{\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)}{2} + \frac{1}{2}$$

where

$$\mathrm{erf}(X) = \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \cdots (2n+1)} X^{2n+1}$$

Reference: Handbook of Mathematical Functions
by Abramowitz and Stegan

National Bureau of Standards 1964

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) or END

PRESS: CONTINUE

DISPLAY

0 ——— Z

0 ——— Y

1 ——— X

ENTER DATA: X ——— X

PRESS: CONTINUE

DISPLAY

0 ——— Z

P(X) ——— Y

X ——— X

EXAMPLES

X = .3	P(X) = .618
X = -.3	P(X) = .382
X = 0	P(X) = .5
X = 3	P(X) = .999

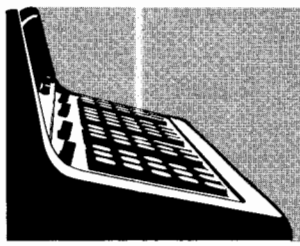
PART NO. 09100-70804

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0	0	CLEAR	20									
	1	1	01	DISPLAY 1 TO INDICATE FIRST DATA ENTRY								
	2	STOP	41	X	0	0	ENTER X					
	3	IF $x=y$	50									
	4	0	00	TEST IF X=0								
	5	9	11									
	6	GO TO( )( )	44									
	7	1	01									
	8	1	01									
	9	·	21									
	a	5	05									
	b	$x \rightleftarrows y$	30	ENTER .5 AND BRANCH TO DISPLAY								
	c	GO TO( )( )	44									
	d	5	05									
1	0	d	17									
	1	$x \rightarrow ( )$	23	STORE X								
	2	c	16									
	3	IF $x<y$	52									
	4	SET FLAG	54	IF X IS NEGATIVE, SET FLAG AND CHANGE SIGN								
	5	CHG SIGN	32									
	6	↑	27									
	7	2	02									
	8	$\sqrt{x}$	76	CALCULATE $\frac{X}{\sqrt{2}} = X'$								
	9	÷	35									
	a	↓	25									
	b	↑	27									
	c	×	36									
	d	↑	27									
2	0	↓	25									
	1	ROLL ↑	22									
	2	e^x	74	CALCULATE $\frac{2X' e^{-X'^2}}{\sqrt{\pi}}$								
	3	÷	35									
	4	2	02									
	5	×	36									
	6	π	56									
	7	$\sqrt{x}$	76									
	8	÷	35									
	9	1	01									
	a	$y \rightarrow ( )$	40									
	b	a	13									
	c	ROLL ↑	22	CALCULATE $\frac{1}{X'^2}$								
	d	÷	35									

PART NO. 09100-70804

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
3 0	$y\rightarrow(\,)$	40										
1	b	14										
2	2	02										CALCULATE $\frac{2}{X'^2}$
3	÷	35										
4	ENTER EXP	26										
5	1	01										ENTER TOLERANCE 10^{-10}
6	0	00										
7	CHG SIGN	32										
8	ROLL ↑	22										
9	$y\rightleftarrows(\,)$	24										RECALL $\frac{1}{X'^2}$
a	b	14										
b	ROLL ↓	31										
c	+	33										CALCULATE NEXT TERM OF SERIES
d	ROLL ↓	31										erf (X′)
4 0	÷	35										
1	ROLL ↓	31										
2	$y\rightleftarrows(\,)$	24										
3	a	13										
4	+	33										ADD CALCULATED TERM TO SERIES
5	$y\rightleftarrows(\,)$	24										
6	a	13										
7	$y\rightleftarrows(\,)$	24										RECALL TOLERANCE
8	b	14										
9	IF $x>y$	53										BRANCH WHEN TERM IS LESS THAN TOLERANCE 10^{-10}
a	3	03										
b	9	11										
c	a	13										RECALL erf (X′)
d	↑	27										
5 0	2	02										
1	÷	35										
2	·	21										CALCULATE P(X) USING erf (X′)
3	5	05										
4	+	33										
5	1	01										
6	IF FLAG	43										MODIFY P(X) WHEN X IS NEGATIVE
7	$x\rightleftarrows y$	30										
8	−	34										
9	CLEAR x	37										
a	ROLL ↓	31										
b	$x\rightleftarrows y$	30										
c	C	16										
d	END	46	X	P(X)	0	DISPLAY						

FROM 1-0



PART NO.
09100-70805

χ^2 - CHI SQUARE DISTRIBUTION

This program evaluates the Chi Square Distribution Integral for a given value of χ^2 and ν degrees of freedom; i.e., the program evaluates

$$P(\chi^2,\nu)= \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})} \int_0^{\chi^2} T^{\nu/2-1} e^{-T/2}\, dT \qquad 0 \le \chi^2$$

The series approximation used to evaluate the integral is

$$P(\chi^2,\nu)=(\tfrac{1}{2}\chi^2)^{\nu/2} \frac{e^{-\chi^2/2}}{\Gamma(\frac{\nu+2}{2})} \left\{1+\sum_{r=1}^{\infty} \frac{\chi^{2r}}{(\nu+2)(\nu+4)\cdots(\nu+2r)}\right\}$$

Reference: Handbook of Mathematical Functions
by Abramowitz and Stegan

National Bureau of Standards 1964

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

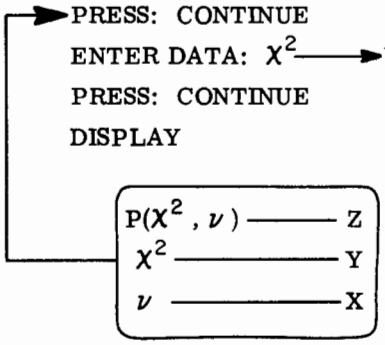
PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

ENTER DATA: $\chi^2 \longrightarrow Y$, $\nu \longrightarrow X$

PRESS: CONTINUE

DISPLAY



EXAMPLES

P(7.88, 1) = .995

P(10.6, 2) = .995

P(12.8, 17) = .251

GENERAL FORM

$P(\chi^2, \nu)$

PART NO. 09100-70805

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	ν	χ^2	0	ENTER ν AND χ^2					
	2	$x\rightarrow( )$	23									
	3	d	17									
	4	$y\rightarrow( )$	40									
	5	c	16									
	6	2	02									
	7	÷	35									
	8	$y\rightarrow( )$	40									
	9	f	15									
	a	↓	25									
	b	ln x	65									
	c	↑	27									
	d	d	17									
1	0	↑	27									
	1	1	01									
	2	+	33									
	3	2	02									
	4	÷	35									
	5	$y\rightarrow( )$	40									
	6	e	12									
	7	↓	25									
	8	×	36									
	9	↓	25									
	a	e^x	74									
	b	↑	27									
	c	f	15									
	d	CHG SIGN	32									
2	0	e^x	74									
	1	×	36									
	2	e	12									
	3	↑	27									
	4	int x	64									
	5	IF $x=y$	50									
	6	4	04									
	7	3	03									
	8	2	02									
	9	ln x	65									
	a	×	36									
	b	↓	25									
	c	e^x	74									
	d	×	36									

STORE DATA

CALCULATE $\left(\frac{\chi^2}{2}\right)^{\frac{\nu+1}{2}}$

CALCULATE $e^{\frac{-\chi^2}{2}}\left(\frac{\chi^2}{2}\right)^{\frac{\nu+1}{2}}$

TRUE IF ν IS ODD

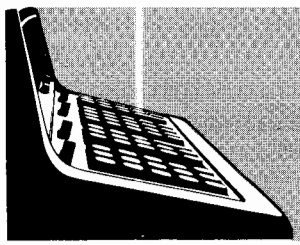
BRANCHES HERE IF ν IS EVEN

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
3	0	c	16										
	1	$\sqrt{x}$	76										
	2	÷	35										CALCULATE COEFFICIENT $(\frac{1}{2}\chi^2)^{\frac{\nu}{2}} \frac{e^{-\frac{\chi^2}{2}}}{\Gamma(\frac{\nu+2}{2})}$
	3	d	17										
	4	÷	35										
	5	↑	27										DECREMENT ν_i BY 2
	6	2	02										
	7	−	34										
	8	0	00										
	9	IF x=y	50										TEST IF ν_i IS DECREMENTED TO ZERO
	a	6	06										
	b	9	11										
	c	↓	25										
	d	÷	35										
4	0	GO TO( )( )	44										
	1	3	03										CONTINUE DECREMENTING ν_i AND DIVIDING
	2	5	05										
	3	4	04										BRANCHES HERE IF ν IS ODD
	4	ln x	65										
	5	×	36										
	6	↓	25										
	7	e^x	74										
	8	×	36										
	9	d	17										
	a	↑	27										
	b	2	02										
	c	×	36										
	d	↓	25										
5	0	÷	35										
	1	↑	27										
	2	4	04										DIVIDE BY $(2\nu)\cdots 10\cdot 6\cdot 2\cdot 1$
	3	−	34										
	4	IF x<y	52										
	5	4	04										
	6	d	17										
	7	2	02										
	8	IF x>y	53										CALCULATE COEFFICIENT $(\frac{1}{2}\chi^2)^{\frac{\nu}{2}} \frac{e^{-\frac{\chi^2}{2}}}{\Gamma(\frac{\nu+2}{2})}$
	9	6	06										
	a	0	00										
	b	↓	25										
	c	÷	35										
	d	↑	27										

FROM 2-7

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a	
6	0	C	16										
	1	|y|	55										
	2	÷	35										
	3	π	56										
	4	÷	35										
	5	↓	25										
	6	√x	76										
	7	×	36										
	8	↑	27										
	9	d	17										FROM 3-b
	A	x→( )	23										
	B	f	15										
	C	0	00										
	D	x→( )	23										
7	0	e	12										
	1	1	01										
	2	x⇄y	30										
	3	2	02										
	4	ACC +	60	CALCULATE $\left\{1+\sum \frac{\chi^{2r}}{(\nu+2)(\nu+4)\cdots(\nu+2r)}\right\}$ AND $\nu+2r$									
	5	C	16										
	6	×	36										
	7	f	15	CALCULATE $\frac{\chi^2}{\nu_i+2}$									
	8	÷	35										
	9	ENTER EXP	26	COMPARE LAST TERM IN SERIES TO 10^{-9} AND STOP SUMMATION IF SMALLER									
	A	9	11										
	B	CHG SIGN	32										
	C	IF x<y	52										
	D	7	07										
8	0	3	03										
	1	e	12										
	2	+	33	CALCULATE $P(\chi^2,\nu)$									
	3	↓	25										
	4	×	36										
	5	C	16										
	6	↑	27	RECALL χ^2 AND ν									
	7	d	17										
	8	END	46	ν	χ^2	$P(\chi^2,\nu)$	DISPLAY						
	9												
	A												
	B												
	C												
	D												

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70806

χ^2 CHI SQUARE EVALUATION
EXPECTED VALUES EQUAL ($E_i = E$)

This program calculates the value of χ^2 by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E)^2}{E}$$

where

O_i — observed frequency

E — expected frequency of O_i is $E = \frac{\sum_{i=1}^{n} O_i}{n}$

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
i ——— X

(i indicates point to be entered)

ENTER DATA: O_i ⟶ X

PRESS: CONTINUE

NO — Last Entry? — YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

E ——— Z
χ^2 ——— Y
n ——— X

EXAMPLES

The table shows the observed and expected frequencies in tossing a die 120 times. Calculate χ^2 for testing if the die is fair.

FACE	i	1	2	3	4	5	6
O_i Observed Frequency		25	17	15	23	24	16
E_i Expected Frequency		20	20	20	20	20	20

E = 20

χ^2 = 5.0

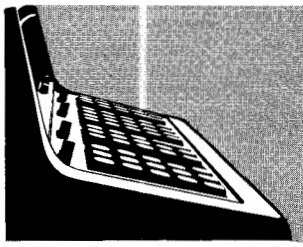
n = 6

PART NO. 09100-70806

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
0	0	CLEAR	20										
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY									
	2	x→( )	23										
	3	d	17										
	4	STOP	41	0_i	0	0							
	5	IF FLAG	43										BRANCH AFTER LAST ENTRY
	6	1	01										
	7	9	11										
	8	↑	27										CALCULATE $\sum O_i$ AND $\sum O_i^2$
	9	×	36										
	a	ACC +	60										
	b	0	00										
	c	↑	27										
	d	d	17										
1	0	↑	27										CLEAR AND INCREMENT n_i
	1	1	01										
	2	+	33										
	3	y→( )	40										
	4	d	17										
	5	↓	25										
	6	GO TO( )( )	44										
	7	0	00										
	8	4	04										
	9	y⇄( )	24										DECREMENT n
	a	d	17										
	b	1	01										
	c	−	34										
	d	y→( )	40										
2	0	d	17										
	1	RCL	61										CALCULATE E
	2	↑	27										
	3	d	17										
	4	÷	35										
	5	y→( )	40										
	6	f	15										
	7	×	36										CALCULATE X^2
	8	f	15										
	9	×	36										
	a	ROLL ↓	31										
	b	−	34										
	c	f	15										
	d	÷	35										

PART NO. 09100-70806

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	d	17									
	1	END	46	n	χ^2	E	DISPLAY					
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



X^2 CHI SQUARE EVALUATION
▲ EXPECTED VALUES UNEQUAL ($E_i \neq E_j$)

June 1969
PART NO.
09100-70808

This program calculates the value of X^2 by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i — observed frequency

E_i — expected frequency of O_i

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

▲ Denotes Revision

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
i ——— X

(i indicates points to be entered)

ENTER DATA: $O_i \rightarrow Y$, $E_i \rightarrow X$

PRESS: CONTINUE

NO — Last Entry? — YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 ——— Z
χ^2 ——— Y
n ——— X

EXAMPLES

The table shows the observed and expected frequencies of some numbers. Calculate χ^2.

i	1	2	3	4	5	6
O_i - Observed Frequency	8	50	47	56	5	14
E_i - Expected Frequency	9.6	46.75	51.85	54.4	8.25	9.15

$\chi^2 = 4.844$

$n = 6$

PART NO. 09100-70808

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
0 0	CLEAR	20										
1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY									
2	ACC +	60										
3	f	15	RECALL i TO INDICATE NEXT ENTRY									
4	STOP	41	Ei	Oi	O							ENTER EXPECTED VALUE AND OBSERVED VALUE
5	IF FLAG	43										
6	1	01	BRANCH AFTER LAST ENTRY									
7	9	11										
8	−	34										
9	x⇄y	30										
a	↑	27										
b	×	36										
c	↓	25	CALCULATE X^2 AND INCREMENT i									
d	x⇄y	30										
1 0	÷	35										
1	1	01										
2	ACC +	60										
3	0	00										
4	↑	27										
5	↑	27	CLEAR AND BRANCH FOR NEXT ENTRY									
6	GO TO ( )( )	44										
7	0	00										
8	3	03										
9	RECALL	61										
a	↑	27										
b	1	01	RECALL X^2, CLEAR AND CALCULATE n									
c	−	34										
d	0	00										
2 0	ROLL ↓	31										
1	END	46	n	X^2	O							DISPLAY
2												
3												
4												
5												
6												
7												
8												
9												
a												
b												
c												
d												

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
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6											
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9											
a											
b											
c											
d											
0											
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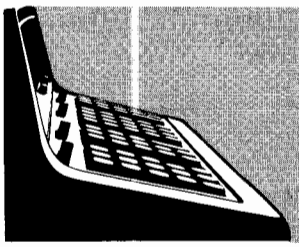
9											
a											
b											
c											
d											



LEAST SQUARES FIT - POWER CURVE

This program computes the least squares fit and correlation coefficient of N pairs of data points for a power curve of the form:

$$Y = aX^b$$

The equation is linearized into $\ln Y = b \ln X + \ln a$

where

$$b = \frac{N\sum(\ln X \ln Y) - \sum \ln X \sum \ln Y}{N\sum(\ln X)^2 - (\sum \ln X)^2}$$

and

$$r = \frac{N\sum \ln X \ln Y - (\sum \ln X)(\sum \ln Y)}{\sqrt{\left[N\sum(\ln X)^2 - (\sum \ln X)^2\right]\left[N\sum(\ln Y)^2 - (\sum \ln Y)^2\right]}}$$

$$\ln a = \frac{\sum \ln Y}{N} - \frac{\sum \ln X}{N} b$$

Note: $X_i > 0$ and $Y_i > 0$, $i = 1, \cdots, N$

Reference: Statistical Theory and Methodology in Science and Engineering
by K.A. Brownlee

John Wiley and Sons 1965

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
i —— X

(i indicates pair of points to be entered)

ENTER DATA: $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE

NO ← Last Entry? → YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r —— Z
b —— Y
a —— X

EXAMPLES

$Y = aX^b$

X	Y
1.0001	25.58
3.16	14.55
10	9.26
31.6	5.63
100	3.48
316	2.12
1000	1.7

r = -0.9964

b = -0.4022

a = 23.5871

$Y = 23.5871X^{-.4022}$

X	Y
1	3
2	4.2574
3	5.2248
4	6.0417
5	6.7624

r = 1.000

b = .505

a = 3.000

$Y = 3X^{.505}$

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a	
0	CLEAR	20										
	x→( )	23										
	d	17										
	x→( )	23										
	c	16										
	x→( )	23										CLEAR STORAGE AND DISPLAY 1 TO INDICATE FIRST ENTRY
	b	14										
	1	01										
	x→( )	23										
	a	13										
	STOP	41	X_i	Y_i	0							ENTER X_i AND Y_i
	IF FLAG	43										
	3	03										BRANCH AFTER ALL DATA HAS BEEN ENTERED
	b	14										
1	ln x	65										FROM 3-a
	x⇄y	30										
	ln x	65										
	x⇄y	30										
	ACC +	60										CALCULATE $\sum \ln X_i$, $\sum \ln Y_i$
	↑	27										
	×	36										
	x⇄y	30										CALCULATE $\sum (\ln X_i)^2$
	y⇄( )	24										
	d	17										
	+	33										
	y⇄( )	24										
	d	17										
	↓	25										
2	×	36										CALCULATE $\sum (\ln X_i)(\ln Y_i)$
	b	14										
	+	33										
	y→( )	40										
	b	14										
	ROLL ↑	22										
	↑	27										
	×	36										
	c	16										CALCULATE $\sum (\ln Y_i)^2$
	+	33										
	y→( )	40										
	c	16										
	a	13										
	↑	27										

JUNE 1969
PART NO. 09100-70811

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	1	01									
	1	+	33									
	2	y→( )	40									
	3	a	13									
	4	CLEAR x	37									
	5	ROLL ↓	31									
	6	x⇄y	30									
	7	↓	25									
	8	GO TO( )( )	44									
	9	0	00									
	a	a	13									
	b	a	13									
	c	↑	27									
	d	1	01									
4	0	−	34									
	1	y→( )	40									
	2	a	13									
	3	e	12									
	4	↑	27									
	5	a	13									
	6	÷	35									
	7	y⇄( )	24									
	8	f	15									
	9	÷	35									
	a	y→( )	40									
	b	e	12									
	c	×	36									
	d	e	12									
5	0	×	36									
	1	d	17									
	2	x⇄y	30									
	3	−	34									
	4	y→( )	40									
	5	d	17									
	6	c	16									
	7	↑	27									
	8	f	15									
	9	↑	27									
	a	×	36									
	b	a	13									
	c	×	36									
	d	↓	25									

FROM 0-d

INCREMENT COUNTER

CLEAR DISPLAY AND RECALL COUNTER

DECREMENT COUNTER

CALCULATE $\sum \frac{\ln Y_i}{N}$

CALCULATE $\sum \frac{\ln X_i}{N}$

CALCULATE $\sum (\ln x)^2 - \frac{(\sum \ln x)^2}{N}$

CALCULATE $\sum (\ln y)^2 - \frac{(\sum \ln y)^2}{N}$

▲ Denotes Revision

JUNE 1969
PART NO. 09100-70811

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6 0	−	34									
1	y→()	40									
2	C	16									
3	b	14									
4	↑	27									
5	F	15									
6	↑	27									
7	E	12	CALCULATE $\Sigma(\ln x \ \ln y) - \frac{(\Sigma \ln x)(\Sigma \ln y)}{N}$								
8	×	36									
9	a	13									
a	×	36									
b	↓	25									
c	−	34									
d	d	17									
7 0	↑	27									
1	↓	25									
2	√x	76	CALCULATE r								
3	÷	35									
4	C	16									
5	√x	76									
6	÷	35									
7	d	17									
8	ROLL ↑	22	CALCULATE b								
9	x⇄y	30									
a	÷	35									
b	y⇄()	24									
c	E	12									
d	E	12									
8 0	×	36	CALCULATE ln a								
1	F	15									
2	x⇄y	30									
3	−	34									
4	E	12									
5	x⇄y	30									
6	e^x	74									
7	END	46	a	b	r	DISPLAY					
8											
9											
a											
b											
c											
d											

▲ Denotes Revision

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

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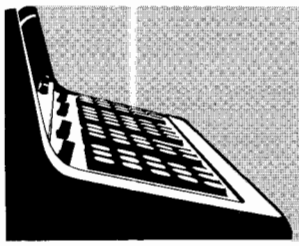

PART NO.
09100-70812

LEAST SQUARES FIT - EXPONENTIAL

This program computes the least squares fit and a correlation coefficient of n pairs of data points for an exponential function of the form:

$$y = ae^{bx}$$

The equation is linearized into

$$\ln y = \ln a + bx$$

or

$$Y = A + bx$$

Using a linear regression method,

$$b = \frac{n\sum xY - \sum x \sum Y}{n\sum x^2 - (\sum x)^2}$$

$$A = \frac{\sum Y - b\sum x}{n}$$

$$a = e^{A}$$

the correlation coefficient is given by

$$r = \frac{n\sum xY - \sum x \sum Y}{\sqrt{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum Y^2 - (\sum Y)^2\right]}}$$

Note: $Y_i > 0 \quad i = 1, \cdots, n$

Reference: Statistical Theory and Methodology in Science and Engineering
by K.A. Brownlee

John Wiley and Sons 1965

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
i	——	X

(i indicates pair of points to be entered)

ENTER DATA: $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE

NO ← Last Entry? → YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r	——	Z
b	——	Y
a	——	X

EXAMPLES

GENERAL FORM: $Y = ae^{bX}$

X	Y
.5	7.12
1.2	11.67
3.1	44.75
7.4	935.64

r = 1.000
b = .707
a = 4.998
$Y = 4.998e^{.707X}$

X	Y
.72	2.16
1.31	1.61
1.95	1.16
2.58	.85

r = -1.000
b = -.503
a = 3.103
$Y = 3.103e^{-.503X}$

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
0	0	CLEAR	20										CLEAR STORAGE AND DISPLAY 1 TO INDICATE FIRST ENTRY
	1	x→( )	23										
	2	a	13										
	3	x→( )	23										
	4	b	14										
	5	x→( )	23										
	6	c	16										
	7	1	01										
	8	x→( )	23										
	9	d	17										
	a	STOP	41	x_i	y_i	0							ENTER x_i AND y_i
	b	IF FLAG	43										
	c	3	03										TEST FOR LAST ENTRY
	d	6	06										
1	0	x⇄y	30										
	1	ln x	65										CALCULATE $\ln y_i$
	2	ACC +	60										
	3	↑	27										
	4	×	36										
	5	x⇄y	30										
	6	y⇄( )	24										
	7	b	14										
	8	+	33										
	9	y⇄( )	24										
	a	b	14										CALCULATE, ACCUMULATE AND STORE $\sum x$, $\sum \ln y$, $\sum \ln^2 y$, $\sum x \ln y$
	b	↓	25										
	c	×	36										
	d	c	16										
2	0	+	33										
	1	y→( )	40										
	2	c	16										
	3	ROLL ↑	22										
	4	↑	27										
	5	×	36										
	6	a	13										CALCULATE, ACCUMULATE AND STORE $\sum x^2$
	7	+	33										
	8	y→( )	40										
	9	a	13										
	a	CLEAR x	37										
	b	↑	27										
	c	d	17										INCREMENT COUNTER
	d	↑	27										

FROM 3-5

FROM d

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
3	0	1	01										
	1	+	33										
	2	↓	25										
	3	GO TO( )( )	44										
	4	0	00										
	5	8	10										
	6	d	17										
	7	↑	27										
	8	1	01										DECREMENT COUNTER
	9	−	34										
	a	y→( )	40										
	b	d	17										
	c	c	16										CALCULATE $n\sum x \ln y$
	d	×	36										
4	0	↑	27										
	1	RCL	61										
	2	×	36										CALCULATE $\sum x \sum \ln y$
	3	↓	25										
	4	−	34										
	5	y→( )	40										CALCULATE AND STORE $n\sum x \ln y - \sum x \sum \ln y$
	6	c	16										
	7	d	17										
	8	↑	27										
	9	a	13										
	a	×	36										
	b	e	12										
	c	↑	27										CALCULATE AND STORE $n\sum x^2 - (\sum x)^2$
	d	×	36										
5	0	↓	25										
	1	−	34										
	2	y→( )	40										
	3	a	13										
	4	d	17										
	5	↑	27										
	6	b	14										
	7	×	36										
	8	f	15										CALCULATE AND STORE $n\sum \ln y - (\sum \ln y)^2$
	9	↑	27										
	a	×	36										
	b	↓	25										
	c	−	34										
	d	a	13										

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6 0	×	36									
1	↓	25	CALCULATE $\sqrt{\left[n\sum x^2-(\sum x)^2\right]\left[n\sum \ln^2 y-(\sum \ln y)^2\right]}$								
2	$\sqrt{x}$	76									
3	↑	27									
4	c	16									
5	$x\rightleftarrows y$	30									
6	÷	35	CALCULATE AND STORE r								
7	$y\to( )$	40									
8	b	14									
9	c	16									
a	↑	27	CALCULATE b								
b	a	13									
c	÷	35									
d	e	12									
7 0	↑	27									
1	↓	25									
2	×	36									
3	f	15	CALCULATE A								
4	$x\rightleftarrows y$	30									
5	−	34									
6	d	17									
7	÷	35									
8	b	14									
9	ROLL ↓	31	RECALL, POSITION, AND CALCULATE a								
a	e^x	74									
b	END	46	a	b	r	DISPLAY					
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
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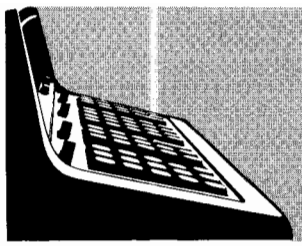
d											



PART NO.
09100-70813

POISSON DENSITY

The Poisson density function is defined by $f(n;\lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$ where λ may be estimated by

$\hat{\lambda}$ = the expected value of X_i $(n = 0, 1, 2, \ldots)$

$$\sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} = e^{-\lambda} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{-\lambda} e^{\lambda} = 1$$

The Poisson density function is a descrete density which is used to evaluate such things as component failure probabilities. It is also used to approximate the Binomial Distribution when the number of events (N) is large and the probability that an event will happen (p) is small i.e., a general rule of thumb is for $p \leq .1$ and $\hat{\lambda} = Np \leq 5$.

Reference: Mathematical Statistics
by John E. Freund

1962 Prentice-Hall

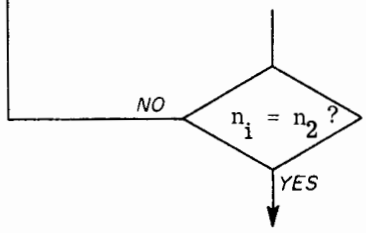
USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)
PRESS: GO TO (0) (0) [or END]
PRESS: CONTINUE
ENTER DATA: $n_2 \rightarrow Z$, $n_1 \rightarrow Y$, $\hat{\lambda} \rightarrow X$
PRESS: CONTINUE
DISPLAY

$Z \longleftarrow n_i$

$Y \longleftarrow \sum_{n=n_1}^{n_i} f(n;\hat{\lambda})$

$X \longleftarrow f(n_i;\hat{\lambda})$



PRESS: CONTINUE

EXAMPLES

A Poisson distribution is given by

$$f(n;\ .72) = \frac{(.72)^n e^{-.72}}{n!}$$

Find: a) $f(0;\ .72)$

$\hat{\lambda} = .72,\ n_1 = 0,\ n_2 = 0$

$f(0;\ .72) = .48675226$

b) $f(3;\ .72)$

$\hat{\lambda} = .72,\ n_1 = 3,\ n_2 = 3$

$f(3;\ .72) = .03027988$

If 3% of the electric bulbs manufactured by a company are defective find the probability that in a sample of 100 bulbs

a) Between 1 and 3 bulbs will be defective.

Expected no. of bulbs that are defective $= \hat{\lambda} = (.03)(100) = 3$

$$\sum_{n=1}^{3} f(n;3) = .59744482$$

where $\hat{\lambda} = 3,\ n_1 = 1,\ n_2 = 3$

b) Less than or equal to 2 bulbs are defective

$$\sum_{n=0}^{2} f(n;3) = .42319008$$

where $\hat{\lambda} = 3,\ n_1 = 0,\ n_2 = 2$

PART NO. 09100-70813

HEWLETT·PACKARD [00] FROM 4-7

HEWLETT·PACKARD [02] FROM 5-2

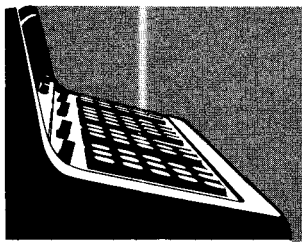
HEWLETT·PACKARD [04]

HEWLETT·PACKARD [06]

HEWLETT·PACKARD [07]

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
0 C	CLEAR	20										
1	STOP	41	$\hat{\lambda}$	n_1	n_2							ENTER DATA; $n_2, n_1, \hat{\lambda}$
2	$x\rightarrow( )$	23										STORE DATA
3	c	16										
4	$y\rightarrow( )$	40										
5	b	14										
6	ROLL ↑	22										
7	$x\rightarrow( )$	23										
8	d	17										
9	$x\rightleftarrows y$	30										
A	CHG SIGN	32										
B	e^x	74										
C	↑	27										
D	c	16										
1 0	ln x	65										CALCULATE $e^{-\lambda}\hat{\lambda}^{n_i}$
1	↑	27										
1 2	b	14										
1 3	×	36										
1 4	$x\rightleftarrows y$	30										
1 5	e^x	74										
1 6	ROLL ↑	22										
1 7	×	36										
1 8	$y\rightarrow( )$	40										
1 9	a	13										DECREMENT n_i
1 A	b	14										
1 B	↑	27										
1 C	↑	27										
1 D	1	01										
2 0	−	34										
2 1	IF $x>y$	53										
2 2	2	02										
2 3	a	13										CONTINUE CALCULATING $n_i\cdot(n_i-1)\cdots 1$
2 4	ROLL ↓	31										
2 5	×	36										
2 6	ROLL ↑	22										
2 7	GO TO( )( )	44										
2 8	2	02										
2 9	0	00										
2 A	a	13										
2 B	ROLL ↑	22										
2 C	↑	27										
2 D	0	00										

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	$x \rightleftarrows y$	30									
	1	IF $x=y$	50									
	2	arc ▾	72	IF $n_i! = 0$, ENTER 1								
	3	1	01									
	4	$x \rightleftarrows y$	30									
	5	↓	25	CALCULATE $\frac{e^{-\hat{x}} \hat{x}^{n_i}}{n_i!}$ AND $\sum_{n=n_1}^{n_i} f(n, \hat{x})$								
	6	÷	35									
	7	$x \rightleftarrows y$	30									
	8	↑	27									
	9	ACC +	60									
	a	b	14									
	b	ROLL ↓	31	POSITION DISPLAY								
	c	RCL	61									
	d	STOP	41	$\frac{e^{-\hat{x}} \hat{x}^{n_i}}{n_i!}$	$\sum_{n=n_1}^{n_i} f(n, \hat{x})$	0						
4	0	CLEAR x	37									
	1	$x \rightarrow ( )$	23	ZERO STORAGE REGISTER f								
	2	f	15									
	3	d	17									
	4	ROLL ↑	22									
	5	IF $x=y$	50									
	6	0	00	BRANCH TO INITIAL ENTRY WHEN THROUGH								
	7	0	00									
	8	↑	27									
	9	1	01									
	a	+	33	INCREMENT n_i								
	b	$y \rightarrow ( )$	40									
	c	b	14									
	d	c	16	RECALL $\hat{x}$ AND CONTINUE SUMMATION								
5	0	GO TO ( ) ( )	44									
	1	0	00									
	2	a	13									
	3	END	46									
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



RANDOM NUMBER GENERATOR

This program calculates random numbers (RN) in the range $0 \leq RN_i \leq 1$ using the formula given below:

$$RN_i = \left[\pi + RN_{(i-1)}\right]^8 - \text{Int.} \left\{ \left[\pi + RN_{(i-1)}\right]^8 \right\}$$

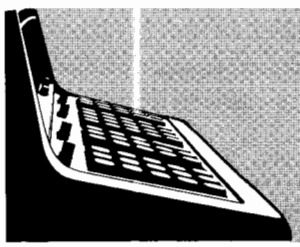
RN_i is the current random number and $RN_{(i-1)}$ is the last calculated random number. More than 10,000 random numbers may be generated before values are repeated.

HEWLETT·PACKARD HEWLETT·PACKARD HEWLETT·PACKARD HEWLETT·PA

0
1
2
3
4
5
6
7
8
9
a
b
c
d

0
1
2
3
4
5
6
7
8
9
a
b
c
d

0
1
2
3
4
5
6
7
8
9
a
b
c
d



PART NO.
09100-70904

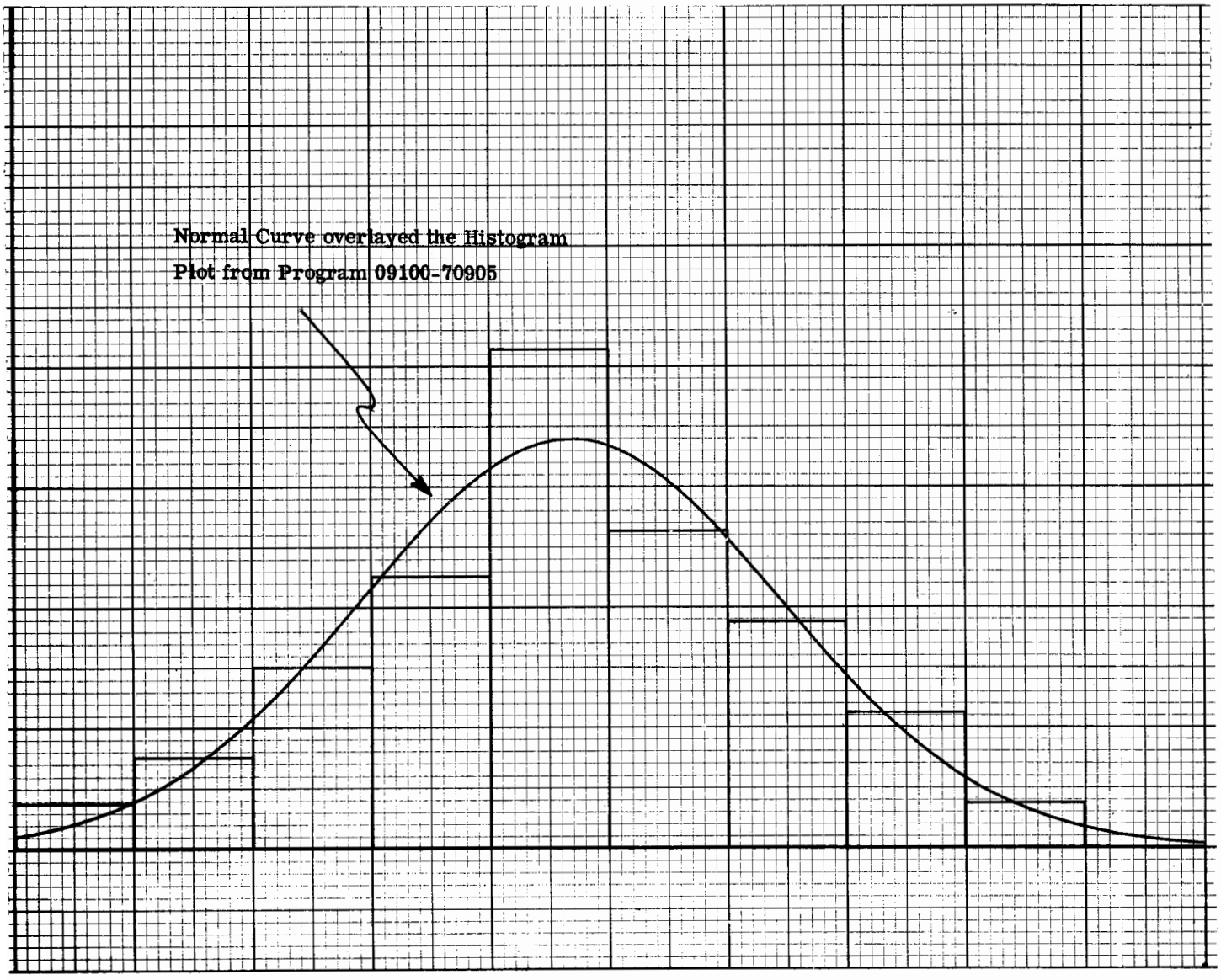
NORMAL CURVE PLOT

This program generates a normal curve given mean M_h and variance σ^2_h. The program determines Y from:

$$Y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(h - M_h)^2}{2\sigma^2_h}}$$

by varying h from 0 to 10 in increments of 0.1. The program requires that M_h and σ^2_h be stored in the f and e registers respectively prior to execution. This program was intended to be used in conjunction with program 09100-70905 Histogram Generation (with Plot). To plot in units of centimeters, place a 2 in locations (2) (5) and (2) (c).

USER INSTRUCTIONS	EXAMPLE
SET: Decimal Wheel at 6 or less Using the orgin controls, locate the pen at X = 1 in., Y = 1 in. ENTER PROGRAM: (Starting Address is 0-0) PRESS: END PRESS: CONTINUE	$M_h = 4.679$ $\sigma_h^2 = 3.132$ See plot below



HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
00	e	12			
(+) 1	$\sqrt{x}$	76			
2	↑	27			
3	π	56			
4	↑	27			
5	2	02			
6	×	36			
7	↓	25			
8	$\sqrt{x}$	76			
9	×	36			
a	y→( )	40			
b	C	16			
c	0	00			
d	↑	27			
10	y→( )	40			
(+) 1	d	17			
2	F	15			
3	−	34			
4	↓	25			
5	↑	27			
6	×	36			
7	e	12			
8	↑	27			
9	2	02			
a	×	36			
b	↓	25			
c	÷	35			
d	↓	25			
20	CHG SIGN	32			
(+) 1	e^x	74			
2	↑	27			
3	C	16			
4	÷	35			
5	7	07			
6	5	05			
7	0	00			
8	0	00			
9	×	36			
a	d	17			
b	↑	27			
c	5	05			
d	0	00			

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
30	0	00			
(+) 1	×	36			
2	↓	25			
3	FMT	42			
4	↓	25			
5	d	17			
6	↑	27			
7	•	21			
8	1	01			
9	+	33			
a	1	01			
b	0	00			
c	•	21			
d	1	01			
40	IF x>y	53			
(+) 1	1	01			
2	0	00			
3	CLEAR	20			
4	FMT	42			
5	↑	27			
6	END	46			
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

f	M_h
e	σ_h^2
d	h
c	$\sigma_h\sqrt{2\pi}$
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
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5	
4	
3	
2	
1	
0	

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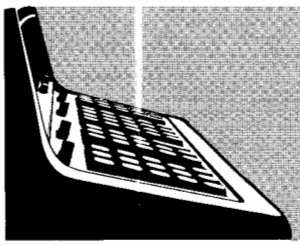

PART NO.
09100-70907

ONE WAY ANALYSIS OF VARIANCE m x n

This program separates the total variance in a table of data into a portion due to chance and a portion due to differences between and population means underlying each column of sample data. It then calculates the variance ratio.

$$F = \frac{nm(m-1) \sum_{j=1}^{n} (\bar{X}_j - \bar{\bar{X}})}{(n-1) \sum_{i=1}^{m} \sum_{j=1}^{n} (X_{ij} - \bar{X}_j)}$$

with $\nu_1 = n - 1$ degrees of freedom

$\nu_2 = n(m-1)$ degrees of freedom

where

$$\bar{\bar{X}} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij}$$

$$\bar{X}_j = \frac{1}{n} \sum_{j=1}^{n} X_{ij}$$

The equation used by the program is:

$$F = \frac{nm(m-1) \left\{ \sum_{j=1}^{n} \left[\sum_{i=1}^{m} X_{ij} \right]^2 - \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 \right\}}{(n-1) \left\{ \sum_{j=1}^{n} \sum_{i=1}^{m} X^2_{ij} - \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 - \frac{1}{m} \sum_{j=1}^{n} \left[\sum_{i=1}^{m} X_{ij} \right]^2 + \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 \right\}}$$

Reference: Freund, John E., Mathematical Statistics, Prentice Hall (1962)

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A at address 0-0

PRESS: CONTINUE

→ DISPLAY

0	Z
0	Y
0	X

ENTER DATA: n Columns → Y, m Rows → X

PRESS: CONTINUE

→ DISPLAY

0	Z	
j	Y	jth Column
i	X	ith Row

Enter data Column by Column

ENTER DATA: X_{ij} → X

PRESS: CONTINUE

NO ← Last data entered? → YES

DISPLAY

F	Z	F=variance ratio
ν_1	Y	ν_1=degrees of freedom in numerator
ν_2	X	ν_2=degrees of freedom in denominator

← PRESS: CONTINUE for new case

EXAMPLE

General form

i Rows \ j Columns	1	2	.	.	.	.	n
1	X_{11}						X_{1n}
2							
°							
i				X_{ij}			
•							
.							
m	X_{m1}						X_{mn}

Rows \ Columns			
	172	203	161
	185	172	149
	165	187	183
	194	183	156
	212	179	144

$F = 5.01$

$\nu_1 = 2$

$\nu_2 = 12$

PART NO. 09100-70907

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Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
(+) 01	x→( )	23			
02	9	11			
03	x→( )	23			
04	b	14	ENTER		
05	STOP	41	M	N	0
06	y→( )	40			
07	c	16			
08	↑	27			
09	1	01			
0a	+	33			
0b	y→( )	40			
0c	d	17			
0d	x⇄y	30			

Step	Key	Code	Display x	Display y	Display z
10	CLEAR x	37			
(+) 11	ROLL ↓	31			
12	x→( )	23			
13	a	13	ENTER		
14	STOP	41	X_{ij}	0	0
15	y⇄( )	24			
16	b	14			
17	+	33			
18	y⇄( )	24			
19	b	14			
1a	↑	27			
1b	×	36			
1c	ACC +	60			
1d	↓	25			

Step	Key	Code	Display x	Display y	Display z
20	a	13			
(+) 21	↑	27			
22	1	01			
23	+	33			
24	d	17			
25	IF x>y	53			
26	1	01			
27	0	00			
28	↓	25			
29	b	14			
2a	↑	27			
2b	×	36			
2c	↓	25			
2d	y⇄( )	24			

Step	Key	Code	Display x	Display y	Display z
30	9	11			
(+) 31	+	33			
32	y⇄( )	24			
33	9	11			
34	CLEAR x	37			
35	ROLL ↓	31			
36	1	01			
37	−	34			
38	x→( )	23			
39	a	13			
3a	CLEAR x	37			
3b	x→( )	23			
3c	b	14			
3d	IF x<y	52			

Step	Key	Code	Display x	Display y	Display z
40	1	01			
(+) 41	3	03			
42	d	17			
43	↑	27			
44	1	01			
45	−	34			
46	y→( )	40			
47	d	17			
48	RCL	61			
49	↑	27			
4a	×	36			
4b	c	16			
4c	÷	35			
4d	d	17			

Step	Key	Code	Display x	Display y	Display z
50	÷	35			
(+) 51	↓	25			
52	−	34			
53	y→( )	40			
54	e	12			
55	ROLL ↓	31			
56	y⇄( )	24			
57	9	11			
58	d	17			
59	÷	35			
5a	↓	25			
5b	x⇄y	30			
5c	−	34			
5d	e	12			

Step	Key	Code	Display x	Display y	Display z
60	x⇄y	30			
(+) 61	−	34			
62	x⇄y	30			
63	÷	35			
64	c	16			
65	×	36			
66	↑	27			
67	1	01			
68	−	34			
69	y→( )	40			
6a	f	15			
6b	↓	25			
6c	÷	35			
6d	d	17			

Step	Key	Code	Display x	Display y	Display z
70	↑	27			
(+) 71	1	01			
72	−	34			
73	↓	25			
74	×	36			
75	↑	27			
76	c	16			
77	×	36			
78	f	15			
79	x⇄y	30			
7a	CONT	47	DISPLAY		
7b	END	46	V_2	V_1	F
7c					
7d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

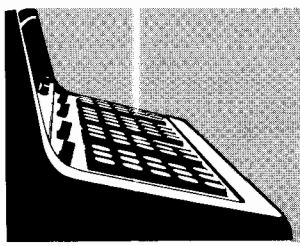
Step	Key	Code	x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



MULTIPLE LINEAR REGRESSION

This program fits any number of data points (X_i, Y_i, Z_i) to a linear, two variable equation of the form:

$$Z = a_o + a_1 X + a_2 Y$$

where X and Y are the independent variables.

Development:

The constants a_o, a_1, and a_2 of the equation may be found by solving simultaneously the following normal equations which represent the least square plane (approximating plane) formed by the data points.

$$\sum Z = a_o n + a_1 \sum X + a_2 \sum Y$$

$$\sum XZ = a_o \sum X + a_1 \sum X^2 + a_2 \sum XY$$

$$\sum YZ = a_o \sum Y + a_1 \sum XY + a_2 \sum Y^2$$

In the program the constant a_2 is found from solving the equations by matrix algebra. Therefore,

$$a_2 = \frac{n(\sum X^2 \sum YZ - \sum XZ \sum XY) - \sum X(\sum X \sum YZ - \sum Y \sum XZ) + \sum Z(\sum X \sum XY - \sum Y \sum X^2)}{D}$$

Where D (the determinant) = $\begin{vmatrix} n & \sum X & \sum Y \\ \sum X & \sum X^2 & \sum XY \\ \sum Y & \sum XY & \sum Y^2 \end{vmatrix}$

After finding a_2, the solution is reduced to two equations in two unknowns which are:

$$M = a_o n + a_1 \sum X$$

$$N = a_o \sum X + a_1 \sum X^2$$

where $M = (\sum Z - a_2 \sum Y)$ $\quad$ $N = (\sum XZ - a_2 \sum XY)$

These two equations are then solved for a_o and a_1.

Reference:

Introduction to the Theory of Statistics
Mood and Graybill
McGraw-Hill, 1963

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

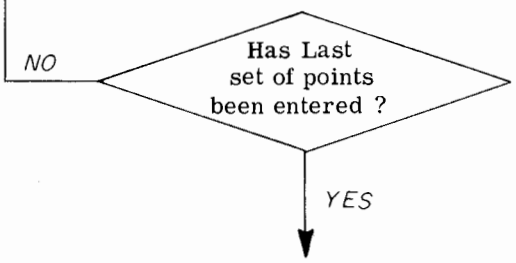
PRESS: CONTINUE

DISPLAY

i	Z
0	Y
0	X

ENTER DATA: $Z_i \rightarrow Z$, $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

a_2	Z
a_1	Y
a_0	X

To reset problem, PRESS: CONTINUE

EXAMPLES

(A) Equation of the form:

$$Z = a_o + a_1 X + a_2 Y$$

Input data:

X	Y	Z
1	0	3
0	1	4
1	1	6
3	4	19
2	2	11

Solution:

$$Z = 1 + 2X + 3Y$$

(B) Equation of the form:

$$Z = a_o + a_1 (\log X) + a_2 (\log Y)$$

Note to enter data ;

ENTER: $Z_i \rightarrow X$
PRESS: ↑
ENTER: $Y_i \rightarrow X$
PRESS: log X
PRESS: ↑
ENTER: $X_i \rightarrow X$
PRESS: log X

Input data:

X	Y	Z
1	2	4.6505
1	1	4.5
4	3.63	5.9841
10	5	6.8495
8	16	6.9082
13	7	7.1504
3	10	5.9542

Solution:

$$Z = 4.5 + 2 (\log X) + .4999 (\log Y)$$

Step	Key	Code	Display x	Display y	Display z
0	CLEAR	20			
(+) 1	x→( )	23			
2	a	13			
3	x→( )	23			
4	b	14			
5	x→( )	23			
6	c	16			
7	x→( )	23			
8	9	11			
9	x→( )	23			
a	−	34			
b	f	15			
c	x→( )	23			
d	−	34			
1 0	e	12			
(+) 1	1	01			
2	x→( )	23			
3	d	17			
4	ROLL ↓	31	ENTER		
5	STOP	41	X_i	Y_i	Z_i
6	IF FLAG	43			
7	6	06			
8	5	05			
9	y→( )	40			
a	−	34			
b	d	17			
c	×	36			
d	x⇄y	30			
2 0	y⇄( )	24			
(+) 1	a	13			
2	+	33			
3	y⇄( )	24			
4	a	13			
5	ROLL ↓	31			
6	ACC +	60			
7	↑	27			
8	×	36			
9	x⇄y	30			
a	y⇄( )	24			
b	−	34			
c	e	12			
d	+	33			

Step	Key	Code	Display x	Display y	Display z
3 0	CONT	47			
(+) 1	y⇄( )	24			
2	−	34			
3	e	12			
4	↓	25			
5	×	36			
6	c	16			
7	+	33			
8	y→( )	40			
9	c	16			
a	ROLL ↑	22			
b	y⇄( )	24			
c	−	34			
d	d	17			
4 0	x⇄y	30			
(+) 1	×	36			
2	↑	27			
3	×	36			
4	ROLL ↑	22			
5	y⇄( )	24			
6	9	11			
7	+	33			
8	y⇄( )	24			
9	9	11			
a	b	14			
b	+	33			
c	y→( )	40			
d	b	14			
5 0	ROLL ↑	22			
(+) 1	y⇄( )	24			
2	−	34			
3	f	15			
4	+	33			
5	y→( )	40			
6	−	34			
7	f	15			
8	1	01			
9	↑	27			
a	d	17			
b	+	33			
c	y→( )	40			
d	d	17			

Step	Key	Code	Display x	Display y	Display z
6 0	CLEAR x	37			
(+) 1	↑	27			
2	GO TO ( ) ( )	44			
3	1	01			
4	5	05			
5	y⇄( )	24			
6	−	34			
7	f	15			
8	d	17			
9	↑	27			
a	1	01			
b	−	34			
c	y→( )	40			
d	d	17			
7 0	↓	25			
(+) 1	f	15			
2	×	36			
3	CHG SIGN	32			
4	×	36			
5	ROLL ↑	22			
6	y⇄( )	24			
7	−	34			
8	e	12			
9	GO TO ( ) ( )	44			
a	−	34			
b	0	00			
c	0	00			
d					

Storage

	+	−
f	ΣX	ΣY^2
e	ΣZ	ΣX^2
d	n	ΣY
c	ΣXZ	
b	ΣY	
a	ΣXY	
9	ΣYZ	
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
0 0	y→( )	40			
(–) 1	–	34			
2	e	12			
3	×	36			
4	d	17			
5	×	36			
6	ROLL ↓	31			
7	+	33			
8	a	13			
9	↑	27			
a	×	36			
b	d	17			
c	×	36			
d	↓	25			
1 0	–	34			
(–) 1	↑	27			
2	y⇄( )	24			
3	–	34			
4	e	12			
5	y→( )	40			
6	–	34			
7	e	12			
8	b	14			
9	×	36			
a	×	36			
b	↓	25			
c	–	34			
d	b	14			
2 0	↑	27			
(–) 1	f	15			
2	×	36			
3	a	13			
4	×	36			
5	2	02			
6	×	36			
7	↓	25			
8	+	33			
9	y→( )	40			
a	–	34			
b	f	15			
c	y⇄( )	24			
d	9	11			

Step	Key	Code	Display x	Display y	Display z
3 0	↑	27			
(–) 1	↓	25			
2	f	15			
3	×	36			
4	CHG SIGN	32			
5	×	36			
6	ROLL ↑	22			
7	x⇄y	30			
8	d	17			
9	×	36			
a	x←( )	67			
b	–	34			
c	e	12			
d	×	36			
4 0	ROLL ↓	31			
(–) 1	+	33			
2	ROLL ↑	22			
3	x⇄y	30			
4	b	14			
5	×	36			
6	e	12			
7	×	36			
8	ROLL ↓	31			
9	–	34			
a	f	15			
b	ROLL ↑	22			
c	×	36			
d	a	13			
5 0	×	36			
(–) 1	ROLL ↓	31			
2	+	33			
3	d	17			
4	ROLL ↑	22			
5	×	36			
6	c	16			
7	×	36			
8	ROLL ↓	31			
9	–	34			
a	f	15			
b	ROLL ↑	22			
c	×	36			
d	b	14			

Step	Key	Code	Display x	Display y	Display z
6 0	×	36			
(–) 1	↓	25			
2	+	33			
3	x←( )	67			
4	–	34			
5	f	15			
6	÷	35			
7	y→( )	40			
8	9	11			
9	↑	27			
a	↓	25			
b	b	14			
c	×	36			
d	e	12			
7 0	x⇄y	30			
(–) 1	–	34			
2	a	13			
3	y→( )	40			
4	a	13			
5	ROLL ↑	22			
6	×	36			
7	c	16			
8	x⇄y	30			
9	–	34			
a	y→( )	40			
b	c	16			
c	f	15			
d	↑	27			

Storage

	7	6	5	4	3	2	1	0
F								
E								
D								
C								
B								
A								
9								
8								
7								
6								
5								
4								
3								
2								
1								
0								

Step	Key	Code	Display x	Display y	Display z
8 0	d	17			
(–) 1	y→( )	40			
2	d	17			
3	y⇄( )	24			
4	–	34			
5	e	12			
6	y→( )	40			
7	–	34			
8	f	15			
9	x→( )	23			
a	b	14			
b	CLEAR	20			
c	y⇄( )	24			
d	–	34			
9 0	f	15			
(–) 1	c	16			
2	↑	27			
3	d	17			
4	÷	35			
5	ROLL ↑	22			
6	x⇄y	30			
7	÷	35			
8	↓	25			
9	IF FLAG	43			
a	a	13			
b	9	11			
c	ACC +	60			
d	d	17			
a 0	y⇄( )	24			
(–) 1	b	14			
2	↑	27			
3	a	13			
4	ROLL ↑	22			
5	SET FLAG	54			
6	GO TO ( ) ( )	44			
7	9	11			
8	4	04			
9	ACC –	63			
a	x⇄y	30			
b	y⇄( )	24			
c	e	12			
d	f	15			

Step	Key	Code	Display x	Display y	Display z
b 0	÷	35			
(–) 1	e	12			
2	x⇄y	30			
3	×	36			
4	ROLL ↓	31			
5	–	34			
6	ROLL ↑	22			
7	y⇄( )	24			
8	9	11			
9	ROLL ↑	22	DISPLAY		
a	STOP	41	a_0	a_1	a_2
b	CONT	47			
c	END	46			
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

HEWLETT · PACKARD

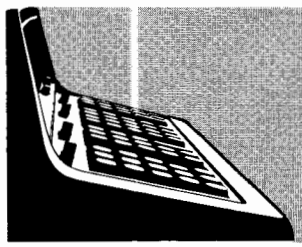
HEWLETT · PACKARD

Step	Key	Code	x	y	z	Step	Key	Code	x	y	z
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD



WEIBULL DISTRIBUTION PARAMETER CALCULATION FOR FAILURE DATA

9100B ONLY
PART NO.
09100-70902

The Weibull probability density function is given by

$$f(X) = \frac{bX^{(b-1)}}{\theta^b} e^{-(\frac{X}{\theta})^b}$$

and the cummulative distribution function is given by

$$F(X) = 1 - e^{-(\frac{X}{\theta})^b}$$

For a set of data, the Weibull parameters b and θ are to be calculated for these functions.

A common application is to use Weibull analysis for failure data where all samples are tested to failure. To use the program, list the items in order of increasing time to failure. The number of items and times to failure are entered. The parameters b, θ, and r are displayed. r is a correlation coefficient indicating goodness of fit. The time required for 10% (B_{10}) to fail is displayed and times to other failure percentages ($B\%$) may be requested.

The Median Rank (M.R.) is calculated by the equation

$$M.R. = \frac{j-.3}{N+4}$$

where j = failure order number

N = number of samples tested

This is an approximation of $F(X)$.

The commulative distribution function is linearized into the form

$$bLnX - bLn\theta = LnLn\left(\frac{1}{1-F(X)}\right)$$

A least squares fit is performed which calculates the slope, intercept, and correlation coefficient. The solution is similar to the linear regression program 09100-70803. Thus estimates of b and θ are obtained.

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

ENTER DATA: N → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
j	X

(j indicates point to be entered)

ENTER DATA: t_j → X (Data must be ordered)

NO — Last Entry? — YES

PRESS: CONTINUE

DISPLAY

r	Z
θ	Y
b	X

PRESS: CONTINUE

DISPLAY

0	Z
B%	Y
%	X

(first time will be 10%)

ENTER DATA: % → X

TO RESTART A NEW PROBLEM

PRESS: END

EXAMPLES

TEST DATA

Hours to failure (must be ordered)

34
60
75
95
119
158

N = 6 (number of samples)

r = .999

θ = 104.091

b = 1.953

B_{10} = 32.887

B_{90} = 159.539

PART NO. 09100-70902 9100B ONLY

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
(+)1	x→( )	23			
2	d	17			
3	x→( )	23			
4	c	16			
5	x→( )	23			
6	b	14	ENTER		
7	STOP	41	N	0	0
8	x→( )	23			
9	8	10			
a	↑	27			
b	•	21			
c	4	04			
d	+	33			
10	↓	25			
(+)1	x→( )	23			
2	9	11			
3	1	01			
4	x→( )	23			
5	a	13	ENTER		
6	STOP	41	T_j	0	0
7	ln x	65			
8	↑	27			
9	a	13			
a	↑	27			
b	•	21			
c	3	03			
d	−	34			
20	x←( )	67			
(+)1	9	11			
2	÷	35			
3	1	01			
4	x⇄y	30			
5	−	34			
6	1	01			
7	x⇄y	30			
8	÷	35			
9	↓	25			
a	ln x	65			
b	ln x	65			
c	x⇄y	30			
d	ACC +	60			

Step	Key	Code	Display x	Display y	Display z
30	↑	27			
(+)1	×	36			
2	x⇄y	30			
3	y⇄( )	24			
4	d	17			
5	+	33			
6	y⇄( )	24			
7	d	17			
8	CONT	47			
9	↓	25			
a	×	36			
b	b	14			
c	+	33			
d	y→( )	40			
40	b	14			
(+)1	ROLL ↑	22			
2	↑	27			
3	×	36			
4	c	16			
5	+	33			
6	y→( )	40			
7	c	16			
8	a	13			
9	↑	27			
a	1	01			
b	+	33			
c	x←( )	67			
d	8	10			
50	x⇄y	30			
(+)1	↑	27			
2	0	00			
3	ROLL ↓	31			
4	IF x>y	53			
5	6	06			
6	0	00			
7	x→( )	23			
8	a	13			
9	x⇄y	30			
a	↓	25			
b	GO TO ( )	44			
c	1	01			
d	6	06			

Step	Key	Code	Display x	Display y	Display z
60	e	12			
(+)1	↑	27			
2	a	13			
3	÷	35			
4	y⇄( )	24			
5	f	15			
6	÷	35			
7	y→( )	40			
8	e	12			
9	×	36			
a	e	12			
b	×	36			
c	d	17			
d	x⇄y	30			
70	−	34			
(+)1	y→( )	40			
2	d	17			
3	c	16			
4	↑	27			
5	f	15			
6	↑	27			
7	×	36			
8	a	13			
9	×	36			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			

Storage

	+	−
f		
e		
d		
c		
b		
a		
9	N + 4	
8	N	
7		
6		
5		
4		
3		
2		
1		
0		

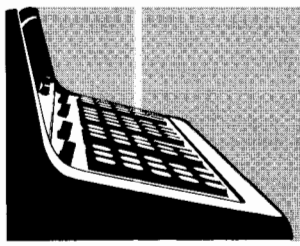
Step	Key	Code	Display x	Display y	Display z
0 0	↓	25			
(–) 1	–	34			
2	y→( )	40			
3	c	16			
4	b	14			
5	↑	27			
6	f	15			
7	↑	27			
8	e	12			
9	×	36			
a	a	13			
b	×	36			
c	↓	25			
d	–	34			
1 0	d	17			
(–) 1	↑	27			
2	↓	25			
3	√x	76			
4	÷	35			
5	c	16			
6	√x	76			
7	÷	35			
8	d	17			
9	ROLL ↑	22			
a	x⇄y	30			
b	÷	35			
c	y⇄( )	24			
d	e	12			
2 0	e	12			
(–) 1	×	36			
2	f	15			
3	x⇄y	30			
4	–	34			
5	e	12			
6	CHG SIGN	32			
7	÷	35			
8	x⇄y	30			
9	e^x	74			
a	x⇄y	30			
b	CHG SIGN	32	DISPLAY		
c	STOP	41	b	θ	r
d	CONT	47			

Step	Key	Code	Display x	Display y	Display z
3 0	↑	27			
(–) 1	1	01			
2	x⇄y	30			
3	÷	35			
4	y→( )	40			
5	f	15			
6	ROLL ↑	22			
7	ln x	65			
8	x→( )	23			
9	e	12			
a	1	01			
b	0	00			
c	x→( )	23			
d	d	17			
4 0	↑	27			
(–) 1	ENTER EXP	26			
2	2	02			
3	÷	35			
4	1	01			
5	–	34			
6	x⇄y	30			
7	CHG SIGN	32			
8	÷	35			
9	↓	25			
a	ln x	65			
b	ln x	65			
c	↑	27			
d	f	15			
5 0	×	36			
(–) 1	e	12			
2	+	33			
3	0	00			
4	ROLL ↓	31			
5	e^x	74			
6	x⇄y	30			
7	d	17	DISPLAY		
8	STOP	41	%	B%	0
9	CONT	47			
a	GO TO( )	44			
b	3	03			
c	c	16			
d	END	46			

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



Development:

The least square parabola approximating the set of points $(X_1, Y_1) \ldots\ldots (X_i, Y_i)$ has the equation:

$$Y = a_o + a_1 x + a_2 x^2$$

where the constants a_o, a_1, and a_2 are determined by solving simultaneously the following normal equations:

$$\Sigma Y = a_o n + a_1 \Sigma X + a_2 \Sigma X^2$$
$$\Sigma XY = a_o \Sigma X + a_1 \Sigma X^2 + a_2 \Sigma X^3$$
$$\Sigma X^2 Y = a_o \Sigma X^2 + a_1 \Sigma X^3 + a_2 \Sigma X^4$$

In the program the constant a_2 is found by matrix algebra; the determinate (D) involved in the solution is:

$$D = \begin{vmatrix} n & \Sigma X & \Sigma X^2 \\ \Sigma X & \Sigma X^2 & \Sigma X^3 \\ \Sigma X^2 & \Sigma X^3 & \Sigma X^4 \end{vmatrix}$$

The equation for a_2 is therefore:

$$a_2 = \frac{\left(n(\Sigma X^2 \Sigma X^2 Y - \Sigma X^3 \Sigma XY) - \Sigma X(\Sigma X \Sigma X^2 Y - \Sigma X^2 \Sigma XY) + \Sigma Y \left[\Sigma X \Sigma X^3 - (\Sigma X^2)^2\right]\right)}{D}$$

After finding a_2 the solution is reduced to two equations in two unknowns which are:

$$N = a_o n + a_1 \Sigma X$$
$$M = a_o \Sigma X + a_1 \Sigma X^2$$

where $M = \Sigma XY - a_2 \Sigma X^3$ and $N = \Sigma Y - a_2 \Sigma X^2$

These equations are then solved for a_o and a_1.

NOTE: Curves with the following equations may also be fitted with this program:

$$Y = a_o + a_1 X \quad (1)$$
$$\log Y = a_o + a_1 X \quad (2)$$
$$\log Y = a_o + a_1 X + a_2 X^2 \quad (3)$$
$$Y = a_o + a_1 \log X \quad (4)$$
$$\log Y = a_o + a_1 \log X \quad (5)$$
$$\log Y = a_o + a_1 (\log X) + a_2 (\log X)^2 \quad (6)$$

An equation of the form of (6) is solved in the examples.

The general form, representing all of these equations, which can be fitted is:

$$f(Y) = a_o + a_1 f(X) + a_2 f^2(X)$$

Reference:

Publisher -- McGraw-Hill
Authors -- Alexander M. Mood & Franklin A. Graybill
Introduction into the Theory of Statistics -- 2nd Edition (1961)

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: END

PRESS: CONTINUE

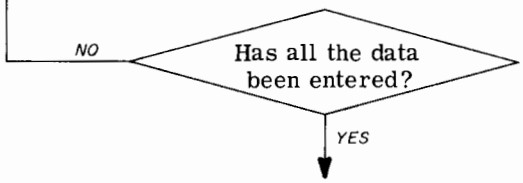
DISPLAY

i	———	Z
0	———	Y
0	———	X

(i indicates pair of points to be entered)

ENTER DATA: $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

a_2	———	Z
a_1	———	Y
a_0	———	X

To calculate coefficients for new data:

PRESS: END

EXAMPLES

(A) Equation of the form: $Y = a_0 + a_1 X + a_2 X^2$

Data:

X	Y
3	29
0	2
5	67
2	16
1.5	11
4	46
1	7

Solution: $Y = 2 + 3X + 2X^2$

(B) Equation of the form:
$\log Y = a_0 + a_1 \log X + a_2 (\log X)^2$

Note: Data to be entered is $\log Y_i$, $\log X_i$; therefore to enter data sets:

ENTER: $Y_i \rightarrow X$
PRESS: log X
PRESS: ↑
ENTER: $X_i \rightarrow Y$
PRESS: log X

Data:

X	Y
1	2.7183
2	35.1595
3	245.3746
4	1188.7946
5	4530.5750

Solution: $\log Y = .43 + 3.0 \log X + 2.30 (\log X)^2$

PART NO. 09100-70903 9100 B ONLY

HEWLETT·PACKARD

HEWLETT·PACKARD

HEWLETT·PACKARD

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	$x\rightarrow()$	23			
2	c	16			
3	$x\rightarrow()$	23			
4	b	14			
5	$x\rightarrow()$	23			
6	a	13			
7	$x\rightarrow()$	23			
8	−	34			
9	f	15			
a	$x\rightarrow()$	23			
b	−	34			
c	e	12			
d	1	01			
1 0	$x\rightarrow()$	23			
(+) 1	d	17			
2	ROLL ↓	31	ENTER		
3	STOP	41	X_i	Y_i	i
4	IF FLAG	43			
5	5	05			
6	5	05			
7	ACC +	60			
8	×	36			
9	$x\rightleftarrows y$	30			
a	$y\rightleftarrows()$	24			
b	c	16			
c	+	33			
d	$y\rightleftarrows()$	24			
2 0	c	16			
(+) 1	↑	27			
2	↓	25			
3	×	36			
4	↓	25			
5	$y\rightleftarrows()$	24			
6	−	34			
7	f	15			
8	+	33			
9	$y\rightleftarrows()$	24			
a	−	34			
b	f	15			
c	↓	25			
d	×	36			

Step	Key	Code	Display x	Display y	Display z
3 0	$x\rightleftarrows y$	30			
(+) 1	$y\rightleftarrows()$	24			
2	b	14			
3	+	33			
4	$y\rightleftarrows()$	24			
5	b	14			
6	×	36			
7	a	13			
8	$x\rightleftarrows y$	30			
9	+	33			
a	$y\rightarrow()$	40			
b	a	13			
c	ROLL ↑	22			
d	×	36			
4 0	$x\rightleftarrows y$	30			
(+) 1	$y\rightleftarrows()$	24			
2	−	34			
3	e	12			
4	+	33			
5	$y\rightarrow()$	40			
6	−	34			
7	e	12			
8	1	01			
9	↑	27			
a	d	17			
b	+	33			
c	$y\rightarrow()$	40			
d	d	17			
5 0	CLEAR x	37			
(+) 1	↑	27			
2	GO TO ( )	44			
3	1	01			
4	3	03			
5	$x\leftarrow()$	67			
6	−	34			
7	e	12			
8	↑	27			
9	f	15			
a	↑	27			
b	×	36			
c	b	14			
d	ROLL ↑	22			

Step	Key	Code	Display x	Display y	Display z
6 0	×	36			
(+) 1	ROLL ↑	22			
2	CHG SIGN	32			
3	×	36			
4	$y\rightleftarrows()$	24			
5	d	17			
6	1	01			
7	−	34			
8	$y\rightleftarrows()$	24			
9	d	17			
a	d	17			
b	ROLL ↑	22			
c	×	36			
d	↓	25			
7 0	+	33			
(+) 1	a	13			
2	↑	27			
3	×	36			
4	d	17			
5	×	36			
6	↓	25			
7	−	34			
8	b	14			
9	↑	27			
a	×	36			
b	×	36			
c	ROLL ↓	31			
d	−	34			

Storage

	+	−
f	$\sum Y$	$\sum X^2 Y$
e	$\sum X$	$\sum X^4$
d	n	
c	$\sum XY$	
b	$\sum X^2$	
a	$\sum X^3$	
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
8 0	f	15			
(+) 1	ROLL ↑	22			
2	×	36			
3	2	02			
4	×	36			
5	a	13			
6	×	36			
7	↓	25			
8	+	33			
9	y⇄( )	24			
a	−	34			
b	f	15			
c	↑	27			
d	↓	25			
9 0	b	14			
(+) 1	×	36			
2	d	17			
3	×	36			
4	f	15			
5	ROLL ↑	22			
6	x⇄y	30			
7	×	36			
8	×	36			
9	ROLL ↓	31			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			
0 0	−	34			
(−) 1	e	12			
2	ROLL ↑	22			
3	×	36			
4	a	13			
5	×	36			
6	ROLL ↓	31			
7	+	33			
8	d	17			
9	ROLL ↑	22			
a	×	36			
b	c	6			
c	×	36			
d	ROLL ↓	31			

Step	Key	Code	Display x	Display y	Display z
1 0	−	34			
(−) 1	f	15			
2	x→( )	23			
3	−	34			
4	e	12			
5	ROLL ↑	22			
6	×	36			
7	b	14			
8	×	36			
9	ROLL ↓	31			
a	+	33			
b	↓	25			
c	ROLL ↓	31			
d	×	36			
2 0	e	12			
(−) 1	×	36			
2	ROLL ↓	31			
3	−	34			
4	ROLL ↑	22			
5	y⇄( )	24			
6	−	34			
7	f	15			
8	ROLL ↓	31			
9	÷	35			
a	b	14			
b	x⇄y	30			
c	×	36			
d	ROLL ↓	31			
3 0	−	34			
(−) 1	y⇄( )	24			
2	c	16			
3	a	13			
4	ROLL ↑	22			
5	x→( )	23			
6	−	34			
7	f	15			
8	×	36			
9	ROLL ↓	31			
a	−	34			
b	y→( )	40			
c	a	13			
d	CLEAR	20			

Step	Key	Code	Display x	Display y	Display z
4 0	x←( )	67			
(−) 1	−	34			
2	e	12			
3	↑	27			
4	c	16			
5	↑	27			
6	d	17			
7	÷	35			
8	ROLL ↑	22			
9	x⇄y	30			
a	÷	35			
b	↓	25			
c	IF FLAG	43			
d	5	05			
5 0	d	17			
(−) 1	ACC +	60			
2	b	14			
3	y⇄( )	24			
4	−	34			
5	e	12			
6	↑	27			
7	a	13			
8	ROLL ↑	22			
9	SET FLAG	54			
a	GO TO ( )	44			
b	4	04			
c	7	07			
d	ACC −	63			

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
6 0	x⇄y	30			
(–) 1	y⇄( )	24			
2	e	12			
3	f	15			
4	÷	35			
5	e	12			
6	x⇄y	30			
7	×	36			
8	ROLL ↓	31			
9	–	34			
a	ROLL ↑	22			
b	y⇄( )	24			
c	–	34			
d	f	15			
7 0	ROLL ↑	22			
(–) 1	STOP	41	DISPLAY		
2	CONT	47	a_0	a_1	a_2
3	END	46			
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT·PACKARD

Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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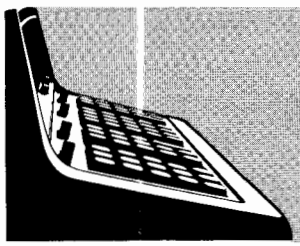


HISTOGRAM GENERATION (WITH PLOT)

This program generates and plots a histogram of ten windows given a data set of positive numbers. In addition, it determines the mean (M_x) and the variance(σ_x^2) of the raw data, and the mean (M_h) and the variance (σ_h^2) of the normalized histogram data. Since the raw data is normalized by the program to values $0 \leq h \leq 10$, the new mean and variance are given by

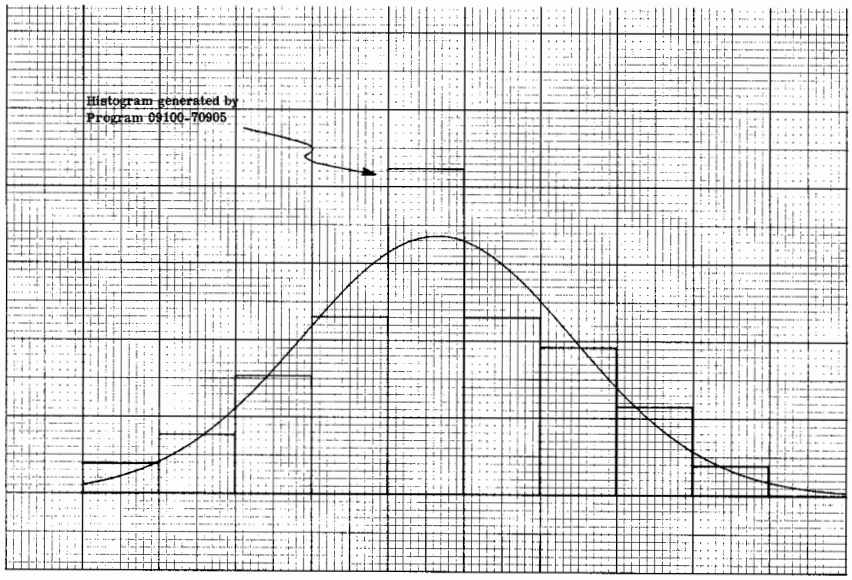
$$M_h = \frac{M_x}{W}$$

$$\sigma_h^2 = \frac{\sigma_x^2}{W^2}$$

where W is the histogram window width (normalizing factor)*. The program plots the histogram and stores M_h and σ_h^2 for use by program 09100-70904 which can be used to plot a normal curve over the histogram.

This program uses Indirect Addressing and is self-destructing of the registers +(0,0) through +(d,d). Thus, to rerun,the A side must be re-entered in the calculator.

NOTE: To generate a histogram with 1 cm. wide windows, place 2's in locations (-)(6)(c), (-)(7)(6), and (-)(8)(5).



* The window width W is chosen such that all normalized data entries X/W will lie between 0 and 10. Thus, if the data ranges from 0→200, a W of of 20 would be proper.

USER INSTRUCTIONS

PRESS: END

Using the orgin controls, locate the pen at X = 1 in., Y = 1 in.

SET: Decimal Wheel at 6 or less

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA: W → X

PRESS: CONTINUE

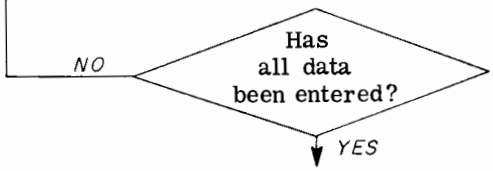
DISPLAY

N ——— Z
N ——— Y
1 ——— X

N is the number of data points entered.

ENTER DATA: X_N → X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

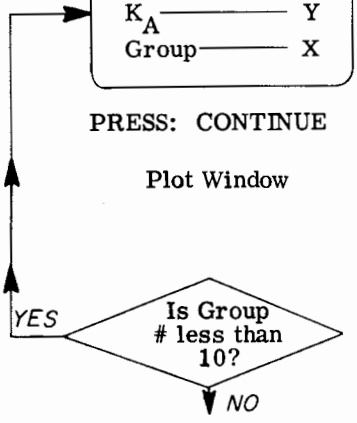
DISPLAY

% ——— Z
K_A ——— Y
Group ——— X

(% of data points in Group)
(No. of data points in Group)
(Group #)

PRESS: CONTINUE

Plot Window



USER INSTRUCTIONS (Con't)

DISPLAY

σ_x^2 ——— Z
M_x ——— Y
0 ——— X

PRESS: CONTINUE

DISPLAY

σ_h^2 ——— Z
M_h ——— Y
1 ——— X

EXAMPLE

The data set is:

104, 92, 83, 78, 58, 135, 146, 24, 74, 85, 81, 128, 140, 113, 79, 78, 53, 42, 34, 85, 96, 110, 133, 158, 171, 108, 84, 90, 73, 11, 51, 118, 68, 139, 92, 109, 89, 124, 91, 116.

The data varies between 0 and 200 so W is chosen to be 20.

Result

Group	K_A	%	
1	1	2.5	
2	2	5.	
3	4	10	
4	6	15.	
5	11	27.5	N = 40
6	7	17.5	
7	5	12.5	
8	3	7.5	
9	1	2.5	
10	0	0	

$\sigma_x^2 = 1252.644$ $\sigma_h^2 = 3.132$
$M_x = 93.575$ $M_h = 4.679$

The histogram plot is given with the normal curve superimposed. The normal curve resulted from running program 09100-70904 following completion of the Histogram Generation program.

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	x→( )	23			
2	9	11			
3	x→( )	23			
4	8	10			
5	x→( )	23			
6	7	07			
7	x→( )	23			
8	6	06			
9	x→( )	23			
a	5	05			
b	x→( )	23			
c	4	04			
d	x→( )	23			
1 0	3	03	ENTER		
(+) 1	STOP	41	W	0	0
2	x→( )	23			
3	a	13			
4	CLEAR x	37			
5	x→( )	23			
6	2	02			
7	GO TO ( )	44			
8	C	16			
9	0	00			
a	CONT	47			
b	CONT	47			
c	CONT	47			
d	CONT	47			
2 0					
(+) 1					
2					
3					
4					
5					
6	Place Continue's				
7					
8	in registers				
9					
a	2 thru b				
b					
c					
d					
e					
f					

Step	Key	Code	Display x	Display y	Display z
C 0	x→( )	23			
(+) 1	0	00			
2	x→( )	23			
3	1	01			
4	1	01			
5	CHG SIGN	32			
6	ROLL ↓	31			
7	↓	25			
8	1	01			
9	+	33			
a	↑	27			
b	↓	25			
c	CONT	47	ENTER		
d	STOP	41	X	N	N
d 0	IF FLAG	43			
(+) 1	d	17			
2	d	17			
3	↑	27			
4	×	36			
5	ACC +	60			
6	x⇄y	30			
7	a	13			
8	÷	35			
9	GO TO ( )	44			
a	−	34			
b	0	00			
c	2	02			
d	GO TO ( )	44			
0 0	3	03			
(−) 1	0	00			
2	↓	25			
3	int x	64			
4	↑	27			
5	ENTER EXP	26			
6	9	11			
7	+	33			
8	y→( )	40			
9	−	34			
a	1	01			
b	CLEAR x	37			
c	CLEAR x	37			
d	y⇄( )	24			

Step	Key	Code	Display x	Display y	Display z
1 0	0	00			
(−) 1	0	00			
2	0	00			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	0	00			
a	0	00			
b	0	00			
c	0	00			
d	0	00			
2 0	IF FLAG	43			
(−) 1	2	02			
2	a	13			
3	CLEAR x	37			
4	1	01			
5	+	33			
6	SET FLAG	54			
7	GO TO ( )	44			
8	0	00			
9	d	17			
a	GO TO ( )	44			
b	+	33			
c	C	16			
d	7	07			

Storage

	+	−
f	$\Sigma X / M_x / M_h$	
e	$\Sigma X^2 / \sigma^2_h$	
d		
c		
b	N	
a	W	
9	K_9	
8	K_8	
7	K_7	
6	K_6	
5	K_5	
4	K_4	
3	K_3	
2	K_2	
1	K_1	K_A
0	K_0	

10 HISTOGRAM WINDOWS

Step	Key	Code	Display x	Display y	Display z
3 0	y→( )	40			
(–) 1	b	14			
2	CLEAR x	37			
3	↑	27			
4	ENTER EXP	26			
5	9	11			
6	+	33			
7	y→( )	40			
8	–	34			
9	4	04			
a	–	34			
b	0	00			
c	↑	27			
d	y⇄( )	24			
4 0	0	00			
(–) 1	0	00			
2	0	00			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	0	00			
a	0	00			
b	0	00			
c	0	00			
d	0	00			
5 0	y→( )	40			
(–) 1	–	34			
2	1	01			
3	b	14			
4	÷	35			
5	ENTER EXP	26			
6	2	02			
7	×	36			
8	1	01			
9	ROLL ↑	22			
a	+	33			
b	x←( )	67			
c	–	34			
d	1	01			

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Step	Key	Code	Display x	Display y	Display z
6 0	x⇄y	30	DISPLAY		
(–) 1	STOP	41	GROUP	K_A	%
2	CONT	47			
3	ROLL ↑	22			
4	↑	27			
5	7	07			
6	5	05			
7	×	36			
8	ROLL ↑	22			
9	x⇄y	30			
a	1	01			
b	–	34			
c	5	05			
d	0	00			
7 0	0	00			
(–) 1	×	36			
2	↓	25			
3	FMT	42			
4	↓	25			
5	↑	27			
6	5	05			
7	0	00			
8	0	00			
9	+	33			
a	↓	25			
b	FMT	42			
c	↓	25			
d	x⇄y	30			
8 0	0	00			
(–) 1	x⇄y	30			
2	FMT	42			
3	↓	25			
4	↑	27			
5	5	05			
6	0	00			
7	0	00			
8	÷	35			
9	1	01			
a	0	00			
b	IF x>y	53			
c	3	03			
d	4	04			

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
9 0	0	00			
(–) 1	↑	27			
2	FMT	42			
3	↓	25			
4	5	05			
5	ENTER EXP	26			
6	3	03			
7	x⇄y	30			
8	FMT	42			
9	↓	25			
a	↑	27			
b	FMT	42			
c	↓	25			
d	RCL	61			
a 0	↑	27			
(–) 1	b	14			
2	÷	35			
3	y→( )	40			
4	f	15			
5	×	36			
6	f	15			
7	×	36			
8	↓	25			
9	–	34			
a	b	14			
b	÷	35			
c	f	15			
d	↑	27			

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
b 0	0	00	DISPLAY		
(-) 1	STOP	41	0	$\bar{M}_x$	σ_x^2
2	CONT	47			
3	a	13			
4	÷	35			
5	x⇄y	30			
6	ROLL ↓	31			
7	÷	35			
8	÷	35			
9	↓	25			
a	x⇄y	30			
b	x→( )	23			
c	f	15			
d	y→( )	40			
c 0	e	12			
(-) 1	↑	27			
2	1	01	DISPLAY		
3	STOP	41	1	M_h	σ_h^2
4	CONT	47			
5	END	46			
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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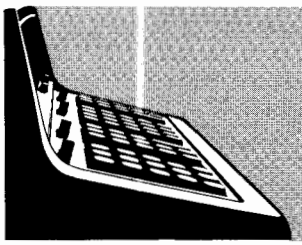
Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B ONLY
PART NO.
09100-70906

HISTOGRAM GENERATION (WITHOUT PLOT)

This program generates a histogram table of ten windows given a data set of positive numbers. In addition, it determines the mean (M_x) and the variance ($\sigma_x 2$) of the raw data, and the mean (M_h) and variance ($\sigma_h 2$) of the normalized histogram data. Since the raw data is normalized by the program to values $0 \leq h \leq 10$, the new mean and variance are given by

$$M_h = \frac{M_x}{W}$$

$$\sigma_h^2 = \frac{\sigma_x^2}{W^2}$$

where W is the histogram window width (normalization factor). The window width W is chosen such that all normalized data entries X/W will lie between 0 and 10. Thus, if the data ranges from 0 → 200, a W of 20 would be proper.

This program uses Indirect Addressing. The (+) registers are used for storage whereas the (−) registers are used for program steps.

USER INSTRUCTIONS

ENTER PROGRAM: (Starting Address is (-0)(0))

PRESS: GO TO (-) (0) (0)

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: W → X

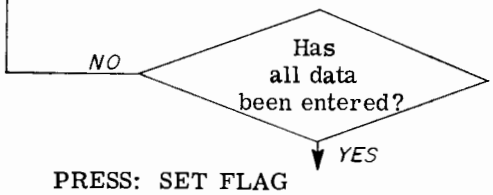
PRESS: CONTINUE

→DISPLAY

N	Z
N	Y
1	X

ENTER DATA: X → X

PRESS: CONTINUE



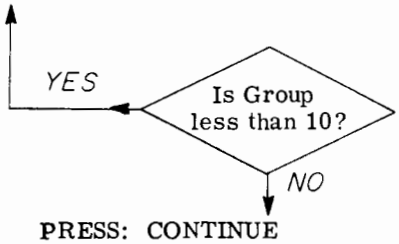
PRESS: SET FLAG

PRESS: CONTINUE

→DISPLAY

%	Z	(% of data points in Group)
K_A	Y	(Number of data points in Group)
Group	X	(Group #)

PRESS: CONTINUE



PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

σ_x^2	Z
M_x	Y
0	X

PRESS: CONTINUE

DISPLAY

σ_h^2	Z
M_h	Y
1	X

PRESS: GO TO
PRESS: —
PRESS: 0
PRESS: 0

} To consider another set of data.

EXAMPLE

The data set is:

104, 92, 83, 78, 58, 135, 146, 24, 74, 85, 81, 128, 140, 113, 79, 78, 53, 42, 34, 85, 96, 110, 133, 158, 171, 108, 84, 90, 73, 11, 51, 118, 68, 139, 92, 109, 89, 124, 91, 116.

The data varies between 0 and 200 so W is chosen to be 20.

Result

Group	K_A	%	
1	1	2.5	
2	2	5	
3	4	10	
4	6	15	
5	11	27.5	N = 40
6	7	17.5	
7	5	12.5	
8	3	7.5	
9	1	2.5	
10	0	0	

$\sigma_x^2 = 1252.644$ $\sigma_h^2 = 3.132$

$M_x = 93.575$ $M_h = 4.679$

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(–) 1	x→( )	23			
2	0	00			
3	x→( )	23			
4	1	01			
5	x→( )	23			
6	2	02			
7	x→( )	23			
8	3	03			
9	x→( )	23			
a	4	04			
b	x→( )	23			
c	5	05			
d	x→( )	23			
1 0	6	06			
(–) 1	x→( )	23			
2	7	07			
3	x→( )	23			
4	8	10			
5	x→( )	23			
6	9	11	ENTER		
7	STOP	41	W	0	0
8	x→( )	23			
9	∂	13			
a	1	01			
b	CHG SIGN	32			
c	ROLL ↓	31			
d	↓	25			
2 0	1	01			
(–) 1	+	33			
2	↑	27			
3	↓	25			
4	CONT	47	ENTER		
5	STOP	41	X	N	N
6	IF FLAG	43			
7	5	05			
8	∂	13			
9	↑	27			
a	×	36			
b	ACC +	60			
c	x⇄y	30			
d	∂	13			

Step	Key	Code	Display x	Display y	Display z
3 0	÷	35			
(–) 1	↓	25			
2	int x	64			
3	↑	27			
4	ENTER EXP	26			
5	9	11			
6	+	33			
7	y→( )	40			
8	–	34			
9	4	04			
a	CLEAR x	37			
b	CLEAR x	37			
c	CLEAR x	37			
d	y⇄( )	24			
4 0	0	00			
(–) 1	0	00			
2	0	00			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	0	00			
a	0	00			
b	0	00			
c	0	00			
d	0	00			
5 0	IF FLAG	43			
(–) 1	1	01			
2	d	17			
3	CLEAR x	37			
4	1	01			
5	+	33			
6	SET FLAG	54			
7	GO TO ( )	44			
8	3	03			
9	d	17			
a	y→( )	40			
b	b	14			
c	CLEAR x	37			
d	↑	27			

Step	Key	Code	Display x	Display y	Display z
6 0	ENTER EXP	26			
(–) 1	9	11			
2	+	33			
3	y→( )	40			
4	–	34			
5	7	07			
6	–	34			
7	0	00			
8	↑	27			
9	0	00			
a	0	00			
b	0	00			
c	0	00			
d	y⇄( )	24			
7 0	0	00			
(–) 1	0	00			
2	0	00			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	0	00			
a	0	00			
b	0	00			
c	0	00			
d	0	00			

Storage

	+	–
f	Σ X	
e	Σ X²	
d		
c	K_A	
b	N	
a	W	
9	K9	
8	K8	
7	K7	
6	K6	
5	K5	
4	K4	
3	K3	
2	K2	
1	K1	
0	K0	

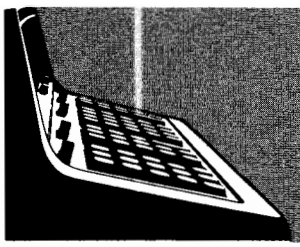
Step	Key	Code	Display x	Display y	Display z
8 0	y→( )	40			
(–) 1	c	16			
2	b	14			
3	÷	35			
4	ENTER EXP	26			
5	2	02			
6	×	36			
7	1	01			
8	ROLL ↑	22			
9	+	33			
a	x←( )	67			
b	c	16			
c	x⇄y	30	DISPLAY		
d	STOP	41	GROUP	K	%
9 0	CONT	47			
(–) 1	↑	27			
2	1	01			
3	0	00			
4	IF x>y	53			
5	6	06			
6	0	00			
7	RCL	61			
8	↑	27			
9	b	14			
a	÷	35			
b	y→( )	40			
c	f	15			
d	×	36			
a 0	f	15			
(–) 1	×	36			
2	↓	25			
3	–	34			
4	b	14			
5	÷	35			
6	f	15			
7	↑	27			
8	0	00	DISPLAY		
9	STOP	41	0	M_x	σ^2_x
a	CONT	47			
b	a	13			
c	÷	35			
d	x⇄y	30			

Step	Key	Code	Display x	Display y	Display z
b 0	ROLL ↓	31			
(–) 1	÷	35			
2	÷	35			
3	↓	25			
4	x⇄y	30			
5	↑	27			
6	1	01	DISPLAY		
7	STOP	41	1	M_h	σ^2_h
8	CONT	47			
9	GO TO ( )	44			
a	–	34			
b	0	00			
c	0	00			
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B ONLY
PART NO.
09100-70908

F-DISTRIBUTION

This program evaluates the integral of the F distribution density function

$$Q = \int_F^{\infty} \frac{\Gamma\left(\frac{V_1+V_2}{2}\right) \quad x^{V_{1/2}-1} \quad \left(\frac{V_1}{V_2}\right)^{V_{1/2}}}{\Gamma\left(\frac{V_1}{2}\right) \quad \Gamma\left(\frac{V_2}{2}\right)\left(1+\frac{V_1}{V_2}x\right)^{\frac{V_1+V_2}{2}}} \quad dx$$

for given values of F, V_1, V_2 .

The integral is evaluated by means of the following series:

V_2 EVEN

$$Q\ (F\ /\ V_1, V_2) = 1-(1-X)^{V_{1/2}}\left[1+\frac{V_1 X}{2}+\ \ldots\ +\frac{V_1\ (V_1+2)\ldots(V_2+V_1-4)}{2\cdot 4\ldots(V_2-2)}\ X^{\frac{V_2-2}{2}}\right]$$

V_2 ODD

$$Q\ (F\ /\ V_1, V_2) = X^{V_{2/2}}\left[1+\frac{V_2}{2}\ (1-X)+\ \ldots\ +\frac{V_2(V_2+2)\ldots(V_2+V_1-4)}{2\cdot 4\ldots(V_1-2)}\ (1-X)^{\frac{V_1-2}{2}}\right]$$

V_1 and V_2 both odd

$Q(F/V_1, V_2) = 1 - A + B$

$$A = \begin{cases} \frac{2}{\pi}\left\{\theta+\sin\theta\cos\theta\left[1+2/3\cos^2\theta+\ \ldots\ +\frac{2\cdot 4\ldots(V_2-3)}{3\cdot 5\ldots(V_2-2)}\cos^{V_2-2}\theta\right]\right\} & V_2 > 1 \\ \frac{2\theta}{\pi} & V_2 = 1 \end{cases}$$

$$\theta = \text{Arc Tan}\sqrt{\frac{F}{V_2}}$$

$$B = \begin{cases} \frac{2\left(\frac{V_2-1}{2}\right)!}{\sqrt{\pi}\left(\frac{V_2-2}{2}\right)!}\ \sin\theta_1\cdot\cos^{V2}\theta_1\left\{1+\frac{V_2+1}{3}\sin^2\theta_1+\ \ldots\ +\frac{(V_2+1)(V_2+3)\ldots(V_2+V_2-4)}{(3)(5)\ldots(V_1-2)}\cdot\sin^{V_2-3}\theta_1\right\} & V_1 > 1 \\ 0 & V_1 = 1 \end{cases}$$

$$\theta_1 = \text{Arc Tan}\sqrt{\frac{V_1 F}{V_2}}$$

Reference: Handbook of Mathematical Functions, Abromowitz and Stegun, National Bureau of Standards (1964)

USER INSTRUCTIONS

SET: [ ] RADIANS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: END

PRESS: CONTINUE

ENTER DATA: F ratio → Z
V_1 for numerator → Y
V_2 for denominator → X

PRESS: CONTINUE

DISPLAY

$Q(F/ V_1,V_2)$ — Z
0 ——— Y
0 ——— X

To calculate significance level for new data.

EXAMPLE

General Form $Q(F/ V_1, V_2)$

$Q(4.21/ 7, 6) = 0.05$
$Q(11.4/ 4, 5) = 0.01$
$Q(3.79/ 7, 7) = .05$

Step	Key	Code	Display x	Display y	Display z
0	CLEAR	20	ENTER		
(+) 1	STOP	41	V_2	V_1	F
2	$x \rightarrow ( )$	23			
3	f	15			
4	$y \rightarrow ( )$	40			
5	e	12			
6	ROLL ↓	31			
7	$y \rightarrow ( )$	40			
8	b	14			
9	×	36			
a	↓	25			
b	+	33			
c	↓	25			
d	÷	35			
1	f	15			
(+) 1	↑	27			
2	1	01			
3	$x \rightarrow ( )$	23			
4	c	16			
5	2	02			
6	÷	35			
7	↓	25			
8	↑	27			
9	int x	64			
a	IF $x=y$	50			
b	3	03			
c	4	04			
d	e	12			
2	$x \rightleftarrows y$	30			
(+) 1	2	02			
2	÷	35			
3	↓	25			
4	↑	27			
5	int x	64			
6	IF $x<y$	52			
7	7	07			
8	d	17			
9	SET FLAG	54			
a	RCL	61			
b	ACC −	63			
c	$x \rightleftarrows y$	30			
d	ACC +	60			

Step	Key	Code	Display x	Display y	Display z
3 0	1	01			
(+) 1	ROLL ↑	22			
2	−	34			
3	↑	27			
4	↓	25			
5	$y \rightarrow ( )$	40			
6	d	17			
7	e	12			
8	↑	27			
9	4	04			
a	−	34			
b	f	15			
c	+	33			
d	↑	27			
4 0	2	02			
(+) 1	−	34			
2	CLEAR x	37			
3	IF $x=y$	50			
4	5	05			
5	b	14			
6	$y \rightarrow ( )$	40			
7	f	15			
8	↓	25			
9	÷	35			
a	d	17			
b	×	36			
c	c	16			
d	×	36			
5 0	1	01			
(+) 1	+	33			
2	$y \rightarrow ( )$	40			
3	c	16			
4	↓	25			
5	2	02			
6	−	34			
7	f	15			
8	GO TO ( )	44			
9	3	03			
a	d	17			
b	c	16			
c	↑	27			
d	1	01			

Step	Key	Code	Display x	Display y	Display z
6 0	↑	27			
(+) 1	d	17			
2	CHG SIGN	32			
3	+	33			
4	↓	25			
5	ln x	65			
6	↑	27			
7	e	12			
8	×	36			
9	2	02			
a	÷	35			
b	↓	25			
c	e^x	74			
d	×	36			
7 0	IF FLAG	43			
(+) 1	7	07			
2	6	06			
3	1	01			
4	$x \rightleftarrows y$	30			
5	−	34			
6	CLEAR x	37			
7	↑	27	DISPLAY		
8	STOP	41	0	0	Q
9	CONT	47			
a	GO TO ( )	44			
b	0	00			
c	0	00			
d	f	15			

Storage

	+	−
f	V_2	
e	V_1	
d		
c	F	
b		
a		
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
80	↑	27			
(+)1	2	02			
2	÷	35			
3	↓	25			
4	int x	64			
5	↑	27			
6	↑	27			
7	2	02			
8	×	36			
9	ln x	65			
a	ROLL ↑	22			
b	×	36			
c	↓	25			
d	e^x	74			
90	x→( )	23			
(+)1	c	16			
2	1	01			
3	−	34			
4	↑	27			
5	↓	25			
6	2	02			
7	−	34			
8	ROLL ↓	31			
9	×	36			
a	|y|	55			
b	ROLL ↑	22			
c	IF x<y	52			
d	9	11			
a0	7	07			
(+)1	↓	25			
2	π	56			
3	×	36			
4	c	16			
5	÷	35			
6	2	02			
7	÷	35			
8	y→( )	40			
9	c	16			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
00	y⇄( )	24			
(−)1	b	14			
2	f	15			
3	÷	35			
4	e	12			
5	×	36			
6	ROLL ↓	31			
7	$\sqrt{x}$	76			
8	arc ▾	72			
9	tan x	71			
a	↑	27			
b	sin x	70			
c	CONT	47			
d	y→( )	40			
10	−	34			
(−)1	f	15			
2	↑	27			
3	×	36			
4	y→( )	40			
5	d	17			
6	RCL	61			
7	1	01			
8	IF x=y	50			
9	7	07			
a	3	03			
b	x→( )	23			
c	b	14			
d	f	15			
20	+	33			
(−)1	4	04			
2	−	34			
3	e	12			
4	↑	27			
5	2	02			
6	−	34			
7	IF x>y	53			
8	4	04			
9	5	05			
a	y→( )	40			
b	−	34			
c	e	12			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
30	÷	35			
(−)1	d	17			
2	×	36			
3	b	14			
4	×	36			
5	1	01			
6	+	33			
7	y→( )	40			
8	b	14			
9	↓	25			
a	2	02			
b	−	34			
c	↑	27			
d	y⇄( )	24			
40	−	34			
(−)1	e	12			
2	GO TO ( )	44			
3	2	02			
4	6	06			
5	IF FLAG	43			
6	9	11			
7	5	05			
8	ROLL ↑	22			
9	x⇄y	30			
a	3	03			
b	IF x>y	53			
c	5	05			
d	c	16			

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
5 0	2	02			
(–) 1	÷	35			
2	↑	27			
3	↓	25			
4	1	01			
5	–	34			
6	ROLL ↓	31			
7	×	36			
8	ROLL ↑	22			
9	IF $x<y$	52			
a	5	05			
b	5	05			
c	ROLL ↓	31			
d	c	16			

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
6 0	÷	35			
(–) 1	1	01			
2	↑	27			
3	d	17			
4	–	34			
5	$\sqrt{x}$	76			
6	ROLL ↑	22			
7	×	36			
8	ROLL ↑	22			
9	$\sqrt{x}$	76			
a	ln x	65			
b	$x \rightleftarrows y$	30			
c	f	15			
d	×	36			

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
7 0	ROLL ↓	31			
(–) 1	e^x	74			
2	×	36			
3	b	14			
4	×	36			
5	$y\rightarrow( )$	40			
6	c	16			
7	$y \rightleftarrows ( )$	24			
8	–	34			
9	f	15			
a	$y\rightarrow( )$	40			
b	e	12			
c	↓	25			
d	cos x	73			

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
8 0	↑	27			
(–) 1	×	36			
2	$y\rightarrow( )$	40			
3	d	17			
4	f	15			
5	↑	27			
6	1	01			
7	$y\rightarrow( )$	40			
8	–	34			
9	e	12			
a	$x\rightarrow( )$	23			
b	b	14			
c	SET FLAG	54			
d	–	34			

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
9 0	IF $x<y$	52			
(–) 1	3	03			
2	a	13			
3	CLEAR x	37			
4	$x\rightarrow( )$	23			
5	b	14			
6	↑	27			
7	d	17			
8	$\sqrt{x}$	76			
9	×	36			
a	e	12			
b	sin x	70			
c	×	36			
d	e	12			

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
a 0	+	33			
(–) 1	2	02			
2	×	36			
3	π	56			
4	÷	35			
5	c	16			
6	$x \rightleftarrows y$	30			
7	–	34			
8	1	01			
9	+	33			
a	CLEAR x	37			
b	↑	27			
c	STOP	41			
d	END	46			

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

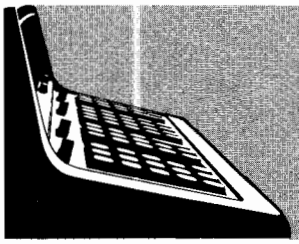
Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD



TWO WAY ANALYSIS OF VARIANCE (m x 4)

This program analyzes the total statistical variance in a table of data by separating the total variance into two parts, the variance among rows of data, and the variance between columns of data, and comparing each to the variance due to random influence. In a table of four columns and m rows it calculates the variance ratio between columns.

$$F_c = \frac{m \sum_{j=1}^{4} (\bar{X}_j - \bar{\bar{X}})^2 \Big/ 3}{\sum_{j=1}^{4} \sum_{i=1}^{m} (X_{ij} - \bar{X}_j - \bar{X}_i + \bar{\bar{X}})^2 \Big/ (m-1)(3)}$$

with $V_1 = 3$ degrees of freedom

and $V_2 = 3(m-1)$ degrees of freedom

and the variance ratio between rows:

$$F_r = \frac{4 \sum_{i=1}^{m} (\bar{X}_i - \bar{\bar{X}})^2 \Big/ (m-1)}{\sum_{j=1}^{4} \sum_{i=1}^{m} (X_{ij} - \bar{X}_j - \bar{X}_i + \bar{\bar{X}})^2 \Big/ 3(m-1)}$$

with $V_1 = m - 1$ degrees of freedom
$V_2 = 3(m-1)$ degrees of freedom

where:

$$\bar{X}_j = \frac{1}{m} \sum_{i=1}^{m} X_{ij} \qquad \bar{X}_i = \frac{1}{4} \sum_{j=1}^{4} X_{ij}$$

$$\bar{\bar{X}} = \frac{1}{4m} \sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}$$

V_1 = degrees of freedom in numerator
V_2 = degrees of freedom in denominator

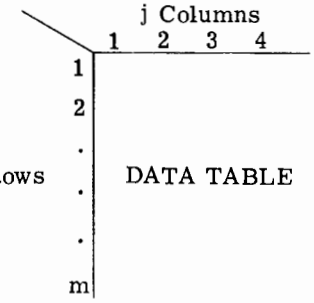
The equations used by the program are:

$$F_r = \frac{4(m-1)\left\{\frac{1}{4}\sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 - \frac{1}{4m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2\right\}}{3\left\{\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}^2 - \frac{1}{4m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{m}\sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4}\sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 + \frac{1}{2m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2\right\}}$$

$$F_c = \frac{3m\left\{\frac{1}{m}\sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2\right\}}{(m-1)\left\{\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}^2 - \frac{1}{4m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{m}\sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4}\sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 + \frac{1}{2m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2\right\}}$$

Reference: Freund, John E., Mathematical Statistics, Prentice Hall (1962)

USER INSTRUCTIONS



PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: m → X number of rows in data table

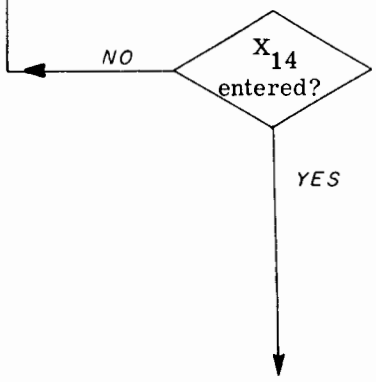
PRESS: CONTINUE

DISPLAY

0	Z
i	Y
j	X

ENTER DATA: X_{ij} → X j = column, i = row
(data entered row by row)

PRESS: CONTINUE



USER INSTRUCTIONS (Con't)

DISPLAY

F_c	Z	F ratio between columns
V_1	Y	V_1 degrees of freedom in numerator
V_2	X	V_2 degrees of freedom in denominator

PRESS: CONTINUE

DISPLAY

F_r	Z	F ratio between rows
V_1	Y	V_1 degrees of freedom in numerator
V_2	X	V_2 degrees of freedom in denominator

To re-run program:

PRESS: END

EXAMPLE

General form

	j Columns 1	2	3	4
1	X_{11}	. . .		X_{14}
2	.			
i Rows .	.	X_{ij}		
.	.			
.				
m	X_{m1}	. . .		X_{m4}

	Columns			
Rows	58.2	49.1	60.1	75.8
	56.2	54.1	70.9	58.2
	65.3	51.6	39.2	48.7

$F_c = 0.43 \quad V_1 = 3 \quad V_2 = 6$

$F_r = 0.92 \quad V_1 = 2 \quad V_2 = 6$

HEWLETT·PACKARD (#1) HEWLETT·PACKARD (#2) HEWLETT·PACKARD (#3) HEWLETT·PACKARD (#4)

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	x→( )	23			
2	a	13			
3	x→( )	23			
4	b	14			
5	x→( )	23			
6	c	16			
7	x→( )	23			
8	9	11			
9	x→( )	23			
a	8	10	ENTER		
b	STOP	41	m	0	0
c	ROLL ↑	22			
d	y→( )	40			
1 0	d	17			
(+) 1	1	01	ENTER		
2	STOP	41	X_{1j}	0	0
3	x→( )	23			
4	7	07			
5	y⇄( )	24			
6	a	13			
7	+	33			
8	y⇄( )	24			
9	a	13			
a	↑	27			
b	×	36			
c	ACC +	60			
d	CLEAR x	37			
2 0	ROLL ↓	31			
(+) 1	2	02	ENTER		
2	STOP	41	X_{2j}	0	0
3	y⇄( )	24			
4	7	07			
5	+	33			
6	y⇄( )	24			
7	7	07			
8	y⇄( )	24			
9	b	14			
a	+	33			
b	y⇄( )	24			
c	b	14			
d	↑	27			

Step	Key	Code	Display x	Display y	Display z
3 0	×	36			
(+) 1	ACC +	60			
2	CLEAR x	37			
3	ROLL ↓	31			
4	3	03	ENTER		
5	STOP	41	X_{3j}	0	0
6	y⇄( )	24			
7	7	07			
8	+	33			
9	y⇄( )	24			
a	7	07			
b	y⇄( )	24			
c	c	16			
d	+	33			
4 0	y⇄( )	24			
(+) 1	c	16			
2	↑	27			
3	×	36			
4	ACC +	60			
5	CLEAR x	37			
6	ROLL ↓	31			
7	4	04	ENTER		
8	STOP	41	X_{4j}	0	0
9	y⇄( )	24			
a	7	07			
b	+	33			
c	y⇄( )	24			
d	7	07			
5 0	y⇄( )	24			
(+) 1	8	10			
2	+	33			
3	y⇄( )	24			
4	8	10			
5	↑	27			
6	×	36			
7	ACC +	60			
8	y⇄( )	24			
9	7	07			
a	↓	25			
b	y⇄( )	24			
c	9	11			
d	↑	27			

Step	Key	Code	Display x	Display y	Display z
6 0	×	36			
(+) 1	CLEAR x	37			
2	ROLL ↓	31			
3	+	33			
4	y⇄( )	24			
5	9	11			
6	1	01			
7	−	34			
8	CLEAR x	37			
9	GO TO ( )	44			
a	−	34			
b	0	00			
c	0	00			
d					
0 0	IF x<y	52			
(−) 1	6	06			
2	3	03			
3	RCL	61			
4	ACC −	63			
5	↑	27			
6	×	36			
7	4	04			
8	÷	35			
9	d	17			
a	÷	35			
b	↓	25			
c	−	34			
d	ACC +	60			

Storage

f	
e	
d	M
c	
b	
a	ΣX_{ij}
9	
8	
7	ΣX_{ij}
6	
5	
4	
3	
2	
1	
0	

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM 1: Side A followed by Side B

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: rows $k \rightarrow Z$, columns $n \rightarrow Y$, repetitions $m \rightarrow X$

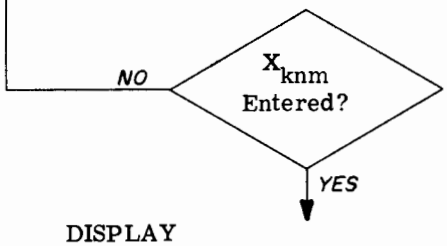
PRESS: CONTINUE

DISPLAY

i — Z		row
j — Y		column
r — X		repitition

ENTER DATA: $X_{ijr} \rightarrow X$

PRESS: CONTINUE



DISPLAY

?*)(?	Z
0	Y
0	X

ENTER PROGRAM 2: Starting Address is (–)(0)(0)

PRESS: GO TO

PRESS: —

PRESS: 0

PRESS: 0

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

F_{row}	Z
V_1	Y
V_2	X

PRESS: CONTINUE

DISPLAY

F_{column}	Z
V_1	Y
V_2	X

PRESS: CONTINUE

DISPLAY

$F_{interaction}$	Z
V_1	Y
V_2	X

EXAMPLE

n = 3 columns

k = 4 rows	58.2	56.2	65.3	
	52.6	41.2	60.8	
	49.1	54.1	51.6	
	42.8	50.5	48.4	
	60.1	70.9	39.2	m = 2
	58.3	73.2	40.7	repititions
	75.8	58.2	48.7	
	71.5	51.0	41.4	

Results:

$F_{row} = 4.42$ $V_1 = 3$ $V_2 = 12$

$F_{column} = 9.39$ $V_1 = 2$ $V_2 = 12$

$F_{interaction} = 14.93$ $V_1 = 6$ $V_2 = 12$

Program 1 Page 1 PART NO. 09100-70910 9100B ONLY

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20	ENTER		
(+) 1	STOP	41	k	n	m
2	x→( )	23			
3	d	17			
4	y→( )	40			
5	c	16			
6	↓	25			
7	y→( )	40			
8	b	14			
9	CLEAR	20			
a	x→( )	23			
b	a	13			
c	x→( )	23			
d	9	11			
1 0	x→( )	23			
(+) 1	8	10			
2	x→( )	23			
3	7	07			
4	x→( )	23			
5	6	06			
6	x→( )	23			
7	5	05			
8	x→( )	23			
9	4	04			
a	x→( )	23			
b	3	03			
c	x→( )	23			
d	−	34			
2 0	f	15			
(+) 1	x→( )	23			
2	−	34			
3	e	12			
4	x→( )	23			
5	1	01			
6	x→( )	23			
7	0	00			
8	GO TO ( )	44			
9	−	34			
a	0	00			
b	0	00			
c					
d					

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
0 0	x→( )	23			
(−) 1	2	02			
2	1	01			
3	+	33			
4	y→( )	40			
5	9	11			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	9	11			
9	d	17			
a	y⇄( )	24			
b	5	05			
c	+	33			
d	y⇄( )	24			
1 0	5	05			
(−) 1	GO TO ( )	44			
2	▲SUB▼	77			
3	9	11			
4	d	17			
5	y⇄( )	24			
6	4	04			
7	+	33			
8	y⇄( )	24			
9	4	04			
a	GO TO ( )	44			
b	▲SUB▼	77			
c	9	11			
d	d	17			
2 0	y⇄( )	24			
(−) 1	3	03			
2	+	33			
3	y⇄( )	24			
4	3	03			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	9	11			
8	d	17			
9	y⇄( )	24			
a	2	02			
b	+	33			
c	y⇄( )	24			
d	2	02			

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
3 0	GO TO ( )	44			
(−) 1	▲SUB▼	77			
2	9	11			
3	d	17			
4	y⇄( )	24			
5	8	10			
6	+	33			
7	y→( )	40			
8	8	10			
9	GO TO ( )	44			
a	▲SUB▼	77			
b	9	11			
c	d	17			
d	y⇄( )	24			
4 0	−	34			
(−) 1	e	12			
2	+	33			
3	y⇄( )	24			
4	−	34			
5	e	12			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	9	11			
9	d	17			
a	y⇄( )	24			
b	−	34			
c	f	15			
d	+	33			

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
8 0	↑	27			
(-) 1	d	17	DISPLAY		
2	STOP	41	V_1	V_2	$F_{col.}$
3	CONT	47			
4	CONT	47			
5	x←( )	67			
6	2	02			
7	↑	27			
8	x←( )	67			
9	3	03			
a	×	36			
b	b	14			
c	↑	27			
d	c	16			
9 0	×	36			
(-) 1	↓	25			
2	÷	35			
3	↑	27			
4	d	17	DISPLAY		
5	STOP	41	V_1	V_2	F_{inter}
6	CONT	47			
7	CONT	47			
8	STOP	41			
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage
f
e
d
c
b
a
9
8
7
6
5
4
3
2
1
0

Step	Key	Code	Display x	Display y	Display z
00	f	15			
(-) 01	↑	27			
02	×	36			
03	d	17			
04	÷	35			
05	c	16			
06	÷	35			
07	b	14			
08	÷	35			
09	e	12			
0a	x⇄y	30			
0b	−	34			
0c	y→( )	40			
0d	e	12			
10	x→( )	23			
(-) 11	f	15			
12	x←( )	67			
13	0	00			
14	↑	27			
15	c	16			
16	÷	35			
17	d	17			
18	÷	35			
19	f	15			
1a	−	34			
1b	y→( )	40			
1c	5	05			
1d	x←( )	67			
20	1	01			
(-) 21	↑	27			
22	b	14			
23	÷	35			
24	d	17			
25	÷	35			
26	f	15			
27	−	34			
28	y→( )	40			
29	4	04			
2a	x←( )	67			
2b	6	06			
2c	↑	27			
2d	d	17			
30	÷	35			
(-) 31	x←( )	67			
32	5	05			
33	−	34			
34	x←( )	67			
35	1	01			
36	↑	27			
37	d	17			
38	÷	35			
39	b	14			
3a	÷	35			
3b	↓	25			
3c	−	34			
3d	y→( )	40			
40	3	03			
(-) 41	RCL	61			
42	x←( )	67			
43	5	05			
44	−	34			
45	x←( )	67			
46	4	04			
47	−	34			
48	x←( )	67			
49	3	03			
4a	−	34			
4b	d	17			
4c	↑	27			
4d	1	01			
50	−	34			
(-) 51	c	16			
52	×	36			
53	b	14			
54	×	36			
55	↓	25			
56	x⇄y	30			
57	y→( )	40			
58	d	17			
59	÷	35			
5a	y→( )	40			
5b	2	02			
5c	x←( )	67			
5d	5	05			
60	×	36			
(-) 61	b	14			
62	↑	27			
63	1	01			
64	−	34			
65	y→( )	40			
66	b	14			
67	↓	25			
68	÷	35			
69	↑	27			
6a	d	17	DISPLAY		
6b	STOP	41	V_1	V_2	F_{row}
6c	CONT	47			
6d	CONT	47			
70	x←( )	67			
(-) 71	2	02			
72	↑	27			
73	x←( )	67			
74	4	04			
75	×	36			
76	c	16			
77	↑	27			
78	1	01			
79	−	34			
7a	y→( )	40			
7b	c	16			
7c	↓	25			
7d	÷	35			

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
80	↑	27			
81 (–)	d	17	DISPLAY		
82	STOP	41	V_1	V_2	$F_{col.}$
83	CONT	47			
84	CONT	47			
85	x←( )	67			
86	2	02			
87	↑	27			
88	x←( )	67			
89	3	03			
8a	×	36			
8b	b	14			
8c	↑	27			
8d	c	16			
90	×	36			
91 (–)	↓	25			
92	÷	35			
93	↑	27			
94	d	17	DISPLAY		
95	STOP	41	V_1	V_2	F_{inter}
96	CONT	47			
97	CONT	47			
98	STOP	41			

Storage

Step	Key	Code	Display x	Display y	Display z
d 0	×	36			
(−) 1	ROLL ↓	31			
2	y⇄( )	24			
3	6	06			
4	+	33			
5	y⇄( )	24			
6	6	06			
7	CLEAR x	37			
8	x→( )	23			
9	7	07			
a	x→( )	23			
b	a	13			
c	ROLL ↑	22			
d	RETURN	77			

Storage