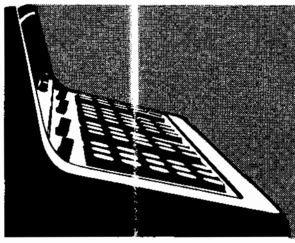
July, 1970

9100 B PROGRAM LIBRARY

ERRATA SHEET

LIBRARY	PROGRAM	PRESENT	CHANGE
MATHEMATICS			
A & B	09100-70001	Register 0 Step 5	CONTINUE to ARC
A & B	09100-70008 (UI* 2nd line)	PRESS: GO TO (5)(a)	PRESS: GO TO (5) (0)
A & B	09100-70008 (1st page, 4th line from bottom)	Y_o'' is obtained	Y_o''' is obtained......
A & B	09100-70008 (Step Page 1)	Page 1 Part #09100-70008	Example Part No. 09100-70008
A & B	09100-70009 (Cover Page)	1st Formula $ax^2 + bx + c = 0$	$aX^2 + bX + c = 0$
A & B	09100-70010 (1st Equation)	$Y = Y_o \cdots (u+1)(u(u-1)$	$Y = Y_o \cdots (u+1)(u)(u-1)$
A & B	09100-70014 (UI 2nd line)	PRESS: GO TO (0)(0)	PRESS: GO TO (6)(0)
A & B	09100-70024 (Prog. 1, pp 3, Register 7, Step 6)	Step Key 6 · (Code 21)	Step Key 6 Roll (Code 22)
A & B	09100-70412	Register -8 Step C	F to e



September 1, 1969

9100B MATHEMATICS PROGRAM LISTING

70001 - n!
Calculates n! for positive integer n. ($n < 70$)

70002 - POLYNOMIAL EVALUATION
Evaluates polynomials of the form:

$$f(z) = C_n z^n + C_{n-1} z^{n-1} + \ldots + C_1 z + C_0$$

for complex C_i, i=1, . . . , n and complex z.

70003 - NUMERICAL INTEGRATION USING SIMPSON'S ONE-THIRD RULE
Uses Simpson's rule to obtain the area under a curve. The equation used is:

$$A = \frac{h}{3}(Y_0 + 4Y_1 + Y_2)$$

70006 - 1st ORDER DIFFERENTIAL EQUATIONS
Solves differential equations of the form:

$$y' = f(x, y)$$

70007 - RAISING A NUMBER TO A POWER
Solves the equation:

$$Z_3 = {Z_1}^{Z_2} \text{ where } Z_i = X_i + jY_i \quad i = 1, 2, 3$$

70008 - 2nd ORDER DIFFERENTIAL EQUATIONS
Solves differential equations of the form:

$$y'' = f(x, y, y')$$

70009 - QUADRATIC EQUATION

Solves $ax^2 + bx + c = 0$ for the roots.

70010 - FINITE DIFFERENCE INTERPOLATION USING GAUSS'S BACKWARD FORMULA
Uses Gauss's backward formula for interpolation in tabular data with equal abscissa spacing. The program fits a cubic equation through the tabular data.

70011 - CUBIC EQUATION

Solves $x^3 + px^2 + qx + r = 0$ for the real and complex roots.

70013 - FACTORS OF n
Gives all factors of an integer n.

70014 - REAL ROOTS OF f(x)
Calculates real roots of f(x) by starting from x_0 and incrementing until f(x) changes sign, then converges on the root. f(x) is programmed in by the user.

70016 - SIMULTANEOUS SOLUTION OF TWO EQUATIONS IN TWO UNKNOWNS
The program solves two independent equations of the form:

$$ax + by = e$$
$$cx + dy = f$$

x and y are the unknowns to be found.

70017 - n! ($n < 10^{12}$)
Calculates n! for positive integer n.

70021 - POLYNOMIAL EVALUATION ($1 \leq n \leq 10$)
Repeatedly evaluates for a given x, a real polynomial of the form:

$$f(x) = A_n x^n + A_{n-1} x^{n-1} \ldots A_1 x + A_0 \quad \text{for } 1 \leq n \leq 10$$

70022 - 3 X 3 MATRIX INVERSION OR SIMULTANEOUS SOLUTION OF THREE EQUATIONS IN THREE UNKNOWNS
Solves three linear independent equations in three unknowns simultaneously or inverts a 3 x 3 matrix.

70023 - FOURIER SERIES
Calculates the Fourier Series coefficients that represent a periodic time function f(t) with period T. The specific f(t) is programmed into the calculator by the user.

70024 - GAMMA FUNCTION
Evaluates the gamma function $\Gamma(\nu)$ for $0 \leq \nu \leq 10^9$ where $\Gamma(\nu) = \int_0^{\infty} e^{-t} t^{\nu-1} dt$

70025 - BESSEL FUNCTION
Calculates the value of the Bessel function $J_n(x)$ of the first kind of integer order n where

$$J_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(-\frac{x^2}{4}\right)^k}{k!\,(n+k)!}$$

9100B ONLY

70401 - HYPERGEOMETRIC SERIES EXPANSION
Given a, b, and c, this program determines the coefficients of the hypergeometric series F(a, b, c; X). This program is useful in solving Gauss's differential equation.

70402 - (3 X 3) MATRIX MULTIPLICATION
Given two (3 x 3) matrices A and B, this program determines the product matrix $C = A \cdot B$.

70403 - ROOTS OF A 4th DEGREE POLYNOMIAL
This program determines the roots (real and complex) of a 4th degree polynomial of the form

$$X^4 + a_1 X^3 + a_2 X^2 + a_3 X + a_4$$

where a_i is real.

70404 - NUMERICAL SOLUTION OF TWO FIRST ORDER DIFFERENTIAL EQUATIONS
This program may be used to solve a wide variety of pairs of first order differential equations of the form

$$\frac{dy}{dx} = f(X, Y, Z),$$

$$\frac{dz}{dx} = g(X, Y, Z).$$

70405 - ROOTS OF A 6th DEGREE POLYNOMIAL
The program determines the roots (real and complex) of a 6th degree polynomial of the form

$$X^6 + a_1 X^5 + a_2 X^4 + a_3 X^3 + a_4 X^2 + a_5 X + a_6$$

where a_i is real.

70406 - CHARACTERISTIC EQUATION OF A (3 X 3) MATRIX AND EIGENVALUE DETERMINATION
Given a (3 x 3) matrix A, this program computes the characteristic equation

$$\lambda^3 + p\,\lambda^2 + q\,\lambda + r = 0$$

and then determines the eigenvalues by using Program 09100-70011 as a Sub Program.

70407 - SIMULTANEOUS SOLUTION OF FOUR LINEAR EQUATIONS IN FOUR UNKNOWNS
Given a system of four linear equations in four unknowns defined by the matrix equation

$$[A_{ij}]\ [X_i] = [P_i],$$

this program uses Cholewski's method to determine the X_i's.

70408 - CONVOLUTION INTEGRAL WITH PLOT
This program evaluates and plots y(t), the convolution of e(t) and h(t). Mathematically

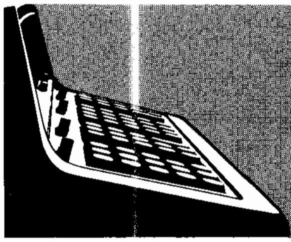
$$y(t) = \int_0^t e(\tau)\,h(t-\tau)\,d\tau\ .$$

70409 - NUMERICAL INTEGRATION USING SIMPSON'S RULE WHEN f(x) IS KNOWN
The specific f(x) is programmed into the calculator by the user and is then used by the general solution to evaluate the integral. Execution time is dependent on the number of panels. Note f(x) should not have any singularities in the integration interval.

70410 - INTEGRAL OF THE FORM: $F(x) = \int_A^x f(u)\,du$ WITH PLOT
This program calculates the integral of a known function f(u) between any lower limit A and a successively incremented upper limit X. Simpson's rule is used to perform the integration. A special application of this program is when f(u) is a probability density function. F(x) then represents the cumulative distribution function.

70411 - MAX - MIN OF Z = Z(X, Y)
This program determines the approximate range of a function Z of two independent variables X and Y given a range for X and Y. This program can be used in conjunction with Program 09100-70412, PLOT OF Z = Z(X, Y).

70412 - PLOT OF Z = Z(X, Y)
Given a function Z of two independent variables X and Y, this program creates a three dimensional plot over a prescribed range of X and Y.



n!

PART NO.
09100-70001

This program calculates n! for integer n where $0 \le n \le 69$.

$$n! = n(n-1)\cdots 3\cdot 2\cdot 1$$

EXAMPLES

0! = 1

6! = 720

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

ENTER DATA: n ⟶ X

PRESS: CONTINUE

DISPLAY

0	—	Z
n!	—	Y
n	—	X

JULY, 1969
PART NO. 09100-70001

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	N	0	0	ENTER N					
	2	x→( )	23									
	3	f	15									
	4	IF x=y	50	IF N IS ZERO ENTER 1								
▲	5	arc ▾	72									
	6	1	01									
	7	↑	27									
	8	↑	27									
	9	1	01	DECREMENT N								
	a	−	34									
	b	IF x>y	53	BRANCH WHEN N REACHES ZERO								
	c	1	01									
	d	6	06									
1	0	ROLL ↓	31									
	1	×	36									
	2	ROLL ↑	22									
	3	GO TO ( ) ( )	44									
	4	0	00									
	5	a	13									
	6	0	00									
	7	ROLL ↓	31									
	8	f	15									
	9	END	46	N	N!	0	DISPLAY					
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

▲ Denotes Revision

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
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b											
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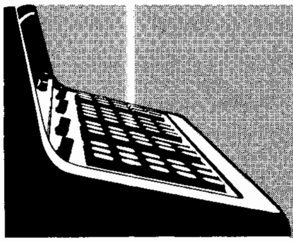

PART NO.
09100-70002

POLYNOMIAL EVALUATION

This program evaluates polynomials of the form

$$f(Z)=C_n Z^n + C_{n-1} Z^{n-1} + \cdots + C_1 Z + C_0$$

for complex C_i, $i = 0,1,\cdots,n$ and complex Z.

EXAMPLES

$P(Z) = (3 + 4i)Z^2 + (-2 + i)Z^1 + (1 - i)$

for $Z = 2 + i$ $ReZ = 2$ $ImZ = 1$

$P(2 + i) = -11 + 23i$

$P(Z) = 49.6Z^4 + 18Z^3 + 52.4Z^2 + 8Z + 12.8$

for $Z = i$ $ReZ = 0$ $ImZ = 1$

$P(i) = 10 - 10i$

GENERAL FORM

$$f(Z) = C_n Z^n + C_{n-1} Z^{n-1} + \dots + C_1 Z + C_o$$

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: n → Z, Im Z → Y,
Re Z → X

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
n_i	——	Z

n_i is an indicator or coefficient entry number.
When $n_i = 0$ enter C_o.

ENTER DATA: $ImCn_i$ → Y,
$ReCn_i$ → X

PRESS: CONTINUE

DISPLAY

0	——	Z
ImP(Z)	——	Y
ReP(Z)	——	X

Step		Key	Code	Display x	Display y	Display z	Storage
0	0	CLEAR	20				
	1	1	01	DISPLAY 1 TO INDICATE FIRST DATA ENTRY			
	2	STOP	41	ReZ	ImZ	n	ENTER Z AND n [THE DEGREE OF P(Z)]
	3	TO POLAR	62				
	4	$x\to()$	23	STORE DATA (steps 4–a)			
	5	d	17				
	6	$y\to()$	40				
	7	c	16				
	8	ROLL ↑	22				
	9	$x\to()$	23				
	a	b	14				
	b	CLEAR	20				
	c	b	14	RECALL n IN THE X REGISTER TO INDICATE COEFFICIENT TO BE ENTERED			
	d	STOP	41	ReCn	ImCn	0	ENTER Cn
1	0	TO POLAR	62				
	1	$x\to()$	23				
	2	a	13				
	3	$y\to()$	40				
	4	9	11				
	5	b	14	DECREMENT n_i (steps 5–8); FROM 3d			
	6	$x\rightleftarrows y$	30				
	7	1	01				
	8	−	34				
	9	↓	25				
	a	IF $x<y$	52	BRANCH WHEN $n_i=-1$ (steps a–c)			
	b	4	04				
	c	0	00				
	d	$x\to()$	23				
2	0	b	14	RECALL n_i IN THE X REGISTER TO INDICATE COEFFICIENT TO BE ENTERED			
	1	STOP	41	$ReCn_i$	$ImCn_i$	0	ENTER Cn_i
	2	ACC +	60				
	3	c	16				
	4	↑	27				
	5	d	17				
	6	↑	27				
	7	a	13				
	8	×	36				
	9	$x\rightleftarrows y$	30				
	a	$y\rightleftarrows()$	24				
	b	9	11				
	c	ROLL ↓	31				
	d	+	33				

FROM 1c

CALCULATE P(Z) BY USE OF THE EQUATION
$P(Z) = \cdots \{[(C_nZ + C_{n-1})Z + C_{n-2}]Z + C_{n-3}\} + \cdots + C_0$

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	↓	25									
	1	x⇄y	30									
	2	TO RECT	66	CALCULATE P(Z) BY USE OF THE EQUATION								
	3	ACC +	60									
	4	RECALL	61									
	5	TO POLAR	62									
	6	x→( )	23									
	7	a	13									
	8	y→( )	40									
	9	9	11									
	a	CLEAR	20									
	b	GO TO ( )( )	44									
	c	1	01									
	d	5	05									
4	0	y⇄( )	24									
	1	9	11	RECALL P(Z)								
	2	a	13									
	3	TO RECT	66									
	4	END	46	ReP(Z)	ImP(Z)	0	DISPLAY					
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
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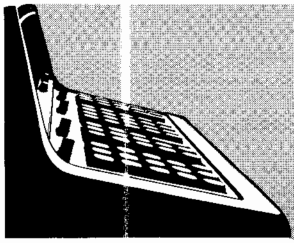
	8											
	9											
	a											
	b											
	c											
	d											



PART NO.
09100-70003

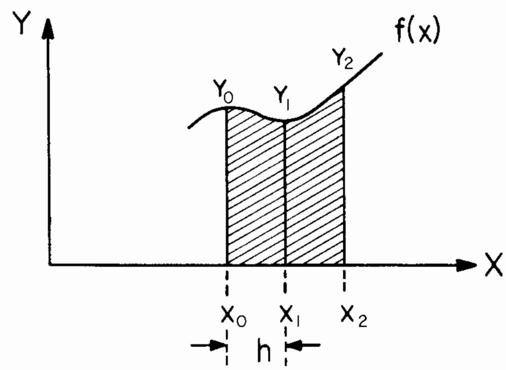
NUMERICAL INTEGRATION USING SIMPSON'S ONE-THIRD RULE

This program evaluates the area under a curve represented by discrete points. The equation used is Simpson's One-Third Rule ie.,

$$\int_{X_0}^{X_2} f(x)dx = \frac{h}{3}(Y_0 + 4Y_1 + Y_2) - \frac{1}{90} h^5 f^{(4)}(\xi)$$

where $X_0 < \xi < X_2$

Graphically the integration is performed over two panels (each of width h) as shown below



The application of Simpson's Rule over 2n panels between $X_0 = a$, and $X_{2n} = b$ gives

$$\int_a^b f(x)dx = \frac{h}{3}(Y_0 + 4Y_1 + 2Y_2 + 4Y_3 + 2Y_4 + \cdots + 4Y_{n-1} + Y_n)$$

$$- \frac{1}{180}(b-a) h^4 f^{(4)}(\xi)$$

where $a < \xi < b$ and $f^{(4)}(\xi)$ is the fourth derivative of $f(x)$ evaluated at ξ .

Thus to use the program divide the abscissa into 2n panels. (The method requires an even number of panels of width h.) Since the error term is neglected, choose h (the distance between points) such that the error term is small. Usually $h << 1$ when $f^{(4)}(\xi)$ is unknown.

Reference: Numerical Analysis
by Kaiser S. Kunz

McGraw - Hill Book Co., Inc. 1957

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: h → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: X_0 → X

PRESS: CONTINUE

→ DISPLAY

A_{i-1}	Z
X_i	Y
0	X

Note: Subsequent X_i's are calculated from X_0 and h and are not entered.

A - the area shows every other time starting after third y entry

ENTER DATA: Y_i → X (Corresponds to calculated X_i displayed in Y)

PRESS: CONTINUE (To restart a new problem PRESS: END, PRESS: CONTINUE)

To Change Increment (new h)

This can only be done if area or Z register is blank

ENTER DATA: Y_i → X

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: new h → X

PRESS: CONTINUE (Program branches to area display and ordinate entry. Proceed as before.)

EXAMPLES

INCREMENT CONSTANT

X	Y
0	2
.25	2.8
.50	3.8
.75	5.2
1.00	7.0
1.25	9.2
1.50	12.1
1.75	15.6
2.00	20

h = .25

$\int_0^2 f(x)dx = 16.58$

INCREMENT CHANGE

X	Y	
0	2	h = .25
.25	2.8	
.50	3.8	
.75	5.2	
1.00	7.0	
1.50	12.1	h = .5
2.00	20	

$\int_0^2 f(x)dx = 16.62$

Step		Key	Code	Display x	Display y	Display z	Storage / Remarks
0	0	CLEAR	20				
	1	1	01	DISPLAY 1 TO INDICATE FIRST DATA ENTRY			(FROM 4-4)
	2	STOP	41	h	0	0	ENTER INCREMENT
	3	$x \to ( )$	23				
	4	d	17				
	5	IF FLAG	43	BRANCH WHEN CHANGE h			
	6	1	01				
	7	6	06				
	8	CLEAR	20				
	9	2	02	DISPLAY 2 TO INDICATE SECOND DATA ENTRY			
	a	STOP	41	X_0	0	0	ENTER X_0
	b	$x \to ( )$	23				
	c	f	15				
	d	↑	27				
1	0	0	00				
	1	↑	27				
	2	ROLL ↓	31				DISPLAY X_0
	3	STOP	41	Y_0	X_0	0	ENTER Y_0 CORRESPONDING TO X_0
	4	$x \to ( )$	23				
	5	c	16				
	6	CLEAR x	37				
	7	↑	27				
	8	d	17	CALCULATE $X_i = X_{i-1} + h$			(FROM 4-7)
	9	ACC +	60				
	a	RECALL	61				
	b	↑	27				
	c	CLEAR x	37				DISPLAY $AREA_{i-1}$, X_i
	d	STOP	41	Y_i	X_i	$AREA_{i-1}$	ENTER Y_i CORRESPONDING TO X_i
2	0	$x \to ( )$	23				
	1	b	14				
	2	CLEAR x	37				
	3	↑	27				
	4	d	17	CALCULATE $X_{i+1} = X_i + h$			
	5	ACC +	60				
	6	f	15				
	7	↑	27				
	8	CLEAR x	37				DISPLAY X_{i+1}
	9	STOP	41	Y_{i+1}	X_{i+1}	0	ENTER Y_{i+1} CORRESPONDING TO X_{i+1}
	a	↑	27				
	b	$y \rightleftarrows ( )$	24				
	c	c	16				
	d	+	33				

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	b	14									
	1	↑	27									
	2	4	04									
	3	×	36									
	4	0	00	CALCULATE $\left(\frac{Y_{i-1}+4Y_i+Y_{i+1}}{3}\right)h = A$								
	5	ROLL ↓	31									
	6	+	33									
	7	3	03									
	8	÷	35									
	9	d	17									
	a	×	36									
	b	0	00									
	c	ACC +	60	SUM A TO PREVIOUS AREA								
	d	ROLL ↓	31									
4	0	IF FLAG	43									
	1	SET FLAG	54									
	2	GO TO ( )( )	44	CHECK FOR FLAG, IF SET BRANCH TO h INPUT								
	3	0	00									
	4	1	01									
	5	GO TO ( )( )	44									
	6	1	01	BRANCH TO ENTER NEW ORDINATE AT SAME INCREMENT								
	7	8	10									
	8	END	46									
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
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	a											
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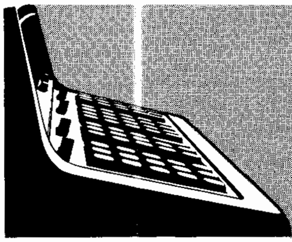
	c											
	d											



PART NO.
09100-70006

1st ORDER DIFFERENTIAL EQUATIONS

This program may be used to solve a wide variety of first order (homogenous or non-homogenous, linear or non-linear) differential equations of the form

$$Y' = f(X,Y)$$

The solution is a numerical solution which calculates Y_i for X_i. The X values are closely spaced with increment h over the desired range. Specifically the solution used in this program is a Runge-Kutta Method (third-order) which uses the equations,

$$Y_{i+1} = Y_i + \frac{1}{6}(p+2q+2r+s)$$

where

$$p = hf(X_i, Y_i)$$

$$q = hf\left(X_i + \frac{h}{2}, Y_i + \frac{p}{2}\right)$$

$$r = hf\left(X_i + \frac{h}{2}, Y_i + \frac{q}{2}\right)$$

$$s = hf(X_i + h, Y_i + r)$$

$$h = X_{i+1} - X_i$$

Reference: Numerical Analysis
by Kaiser S. Kunz

McGraw-Hill Book Co. Inc. 1957

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: (GO TO) (4) (a)

Place mode switch to PROGRAM

Starting at 4-a, enter the program steps which take the independent variable from the X register, the dependent variable from the Y register and calculate f(X, Y). Place f(X, Y) in the Y register and exit to location 8-b. Note there is a maximum of 57 steps (4-a through 8-a) available for storing and positioning f(X, Y).

Place mode switch to RUN

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

ENTER DATA: (Initial conditions and increment)

h → Z, Y_o → Y, X_o → X

PRESS: CONTINUE

The Calculator will display answers at every increment of the independent variable in the form

h	Z
Y_i	Y
X_i	X

To stop the solution at the next increment depress PAUSE until display. To restart press CONTINUE.

To start from a new set of initial conditions

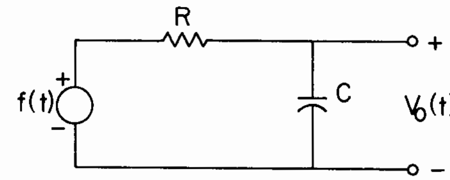
PRESS: STOP

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

Enter initial conditions and increment as before.

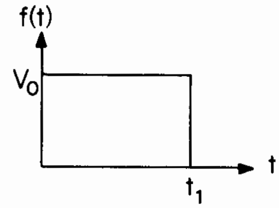
EXAMPLES

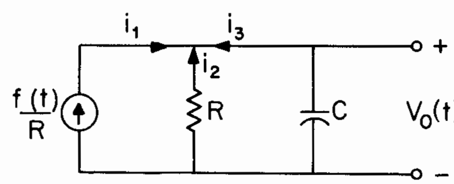


$V_o(0) = 0$

$t_0 = 0$

$h = .01$





$$i_1 + i_2 + i_3 = 0$$

$$\frac{f(t)}{R} - \frac{V_0(t)}{R} - C\frac{dV_0(t)}{dt} = 0$$

Initial Conditions: $t_0 = 0$, $V_0(0) = 0$

$$\frac{dV_0(t)}{dt} = -\frac{V_0(t)}{CR} + \frac{f(t)}{CR}$$

Increment h = .01

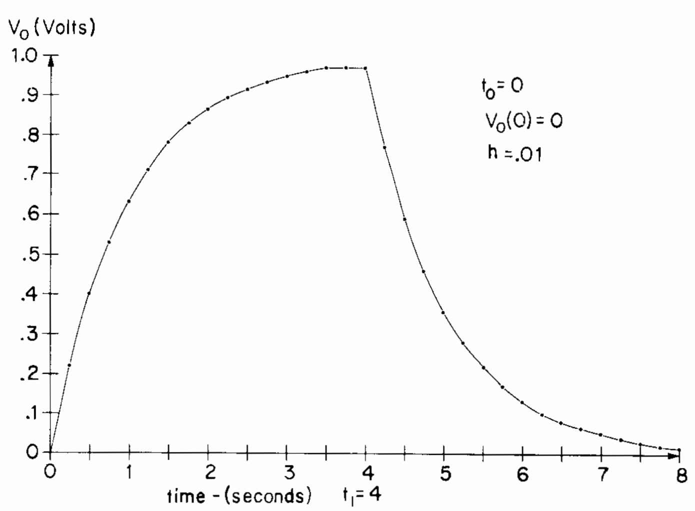
Let $V_0 = 1$ V, C = 1 F, R = 1 Ω and $t_1 = 4$ sec.

The equation becomes

$$\frac{dV_0(t)}{dt} = -V_0(t) + 1; \quad t \leq 4$$

$$\frac{dV_0(t)}{dt} = -V_0(t); \quad t > 4$$

The program steps that form $V_0' = f(t, V_0)$ appear on Page 1. See User Instructions.



Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a	Notes
4 0												
1												
2												
3												
4												
5												
6												
7												
8												
9			t	V_0								
a	↑	27		t	V_0							
b	4	04	4									COMPARE t TO 4 AND BRANCH TO APPROPRIATE DIFFERENTIAL EQUATION
c	IF $x < y$	52										
d	5	05										
5 0	9	11										
1	ROLL ↑	22	V_0	4	t							$t \leq 4$
2	CHG SIGN	32	$-V_0$									
3	↑	27		$-V_0$	4							$\frac{dV_0(t)}{dt} = -V_0(t) + 1$
4	1	01	1									
5	+	33		$1-V_0$								
6	GO TO ( )( )	44										
7	8	10										EXIT
8	b	14										
9	ROLL ↑	22	V_0	4	t							$t > 4$
a	CHG SIGN	32	$-V_0$									
b	↑	27		$-V_0$	4							$\frac{dV_0(t)}{dt} = -V_0(t)$
c	GO TO ( )( )	44										
d	8	10										EXIT
6 0	b	14										
1												
2												
3												
4												
5												
6												
7												
8												
9												
a												
b												
c												
d												

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	$x \to ()$	23									
	2	b	14									
	3	STOP	41	X_0	Y_0	h	ENTER INITIAL CONDITIONS AND INCREMENT					
	4	$x \to ()$	23									
	5	f	15									
	6	$y \to ()$	40									
	7	d	17									
	8	ROLL ↑	22									
	9	$x \to ()$	23									
	a	c	16									
	b	ROLL ↓	31									
	c	PAUSE	57	X_i	Y_i	h	DISPLAY AT EACH INCREMENT					
	d	GO TO ( )( )	44									
1	0	4	04	CALCULATE $p = hf(X_i, Y_i)$								
	1	a	13									
	2	↓	25									
	3	2	02									
	4	÷	35									
	5	d	17									
	6	+	33									
	7	c	16	CALCULATE $X_i + \frac{h}{2}$, $Y_i + \frac{p}{2}$								
	8	↑	27									
	9	2	02	OR $X_i + \frac{h}{2}$, $Y_i + \frac{q}{2}$								
	a	÷	35	IN THE X AND Y REGISTERS RESPECTIVELY								
	b	f	15									
	c	+	33									
	d	↓	25									
2	0	IF FLAG	43									
	1	CONTINUE	47									
	2	SET FLAG	54									
	3	GO TO ( )( )	44									
	4	4	04	CALCULATE q OR r								
	5	a	13									
	6	↓	25									
	7	d	17									
	8	+	33									
	9	c	16									
	a	↑	27	CALCULATE $X_i + h$ IN THE X REGISTER AND $Y_i + r$ IN THE Y REGISTER								
	b	f	15									
	c	+	33									
	d	↓	25									

Margin arrows: FROM 4-9 (to step 0b); FROM a-c (to step 12); FROM 9-d (to step 25).

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a	Notes
3 0	$x\to()$	23										
1	f	15										
2	GO TO ( )( )	44										
3	4	04										CALCULATE S
4	a	13										
5	e	12										FROM a-6
6	↑	27										
7	6	06										
8	÷	35										CALCULATE $Y_i + \frac{1}{6}(p+2q+2r+s)$
9	d	17										
a	+	33										
b	f	15										
c	$y\to()$	40										
d	d	17										
4 0	↑	27										
1	0	00										
2	$x\to()$	23										
3	b	14										REINITIALIZE POINTS FOR STARTING NEXT INCREMENT
4	$x\to()$	23										
5	e	12										
6	c	16										
7	IF FLAG	43										
8	0	00										
9	b	14	X	Y								
a	CONTINUE	47										FROM 1-1, 2-5, 3-4
b												
c												
d												
5 0												
1												
2												
3												
4												
5												
6												
7												
8												
9												
a												
b												
c												
d												

ENTER SPECIFIC DIFFERENTIAL EQUATION HERE. NOTE LOCATION 4-9 CONTAINS X IN THE X REGISTER AND Y IN THE Y REGISTER. USE THESE VARIABLES TO PROGRAM THE SPECIFIC EQUATION $Y' = f(X,Y)$. AFTER FORMING Y', PLACE IT IN THE Y REGISTER IN LOCATION 8-b.

FILL THE REMAINING AREA WITH CONTINUES TO ELIMINATE THE POSSIBILITY OF AN END AND THUS RECORDING AND ENTERING PROBLEMS.

SEE OPERATING MANUAL

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
7	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
8	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
FROM 4-a thru 8-a →	b	c	16		Y'	NOTE: Y' MUST BE IN THE Y REGISTER AT THIS TIME.						
	c	×	36									
	d	0	00									

Step		Key	Code	Comments
9	0	IF FLAG	43	CALCULATE $p+2q+2r+s$ IN THE e REGISTER
	1	ACC +	60	
	2	SET FLAG	54	
	3	ACC +	60	
	4	SET FLAG	54	
	5	1	01	INCREMENT CONTENTS OF STORAGE REGISTER b
	6	↑	27	
	7	b	14	
	8	+	33	
	9	$y \to (\,)$	40	
	a	b	14	
	b	3	03	
	c	IF $x>y$	53	
	d	a	13	
a-	0	a	13	
	1	IF $x=y$	50	
	2	a	13	
	3	7	07	
	4	GO TO ( )( )	44	BRANCH TO CALCULATE Y_{i+1} AND REINITIALIZE ROUTINE
	5	3	03	
	6	5	05	
	7	IF FLAG	43	ZEROS FLAG AND BRANCHES TO CALCULATE S
	8	2	02	
	9	6	06	
	a	GO TO ( )( )	44	BRANCH TO CALCULATE q AND r
	b	1	01	
	c	2	02	
	d	END	46	

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

RAISING A NUMBER TO A POWER

This program solves the equation $Z_3 = Z_1^{Z_2}$

where Z_1 and Z_2 are given. Z_i may be real or complex ie. $Z_i = X_i + jY_i \qquad i = 1,2,3$

The definitions $Z_1^{Z_2} = e^{Z_2 \ln Z_1} \qquad Z_1 \neq 0$

and $$\ln Z = \ln(\sqrt{X^2+Y^2}) + j\theta \qquad (-\pi < \theta \leq \pi, \sqrt{X^2+Y^2} > 0)$$

$$\theta = \mathrm{TAN}^{-1} \frac{Y}{X}$$

are used.

The following equations are programmed

$$Z_3 = e^{Z_2 \ln Z_1}$$

$$= e^{(X_2 + jY_2)(\ln\sqrt{X_1^2+Y_1^2} + j\theta_1)}$$

$$= e^{X_2 \ln\sqrt{X_1^2+Y_1^2} - Y_2\theta_1} \left\{ \mathrm{COS}(Y_2 \ln\sqrt{X_1^2+Y_1^2} + X_2\theta_1) + \right.$$

$$\left. j\mathrm{SIN}(Y_2 \ln\sqrt{X_1^2+Y_1^2} + X_2\theta_1) \right\}$$

Reference: Complex Variable and Applications
by Churchill

McGraw-Hill 1960

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

SET: [ ] RADIANS

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $Y_1 \rightarrow Y$, $X_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: $Y_2 \rightarrow Y$, $X_2 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
Y_3	Y
X_3	X

RESULT: $Z_3 = X_3 + jY_3$

EXAMPLES

$Z_3 = (X_1 + jY_1)^{(X_2 + jY_2)}$

1) $Z_3 = e^{(2 + j2)}$

$X_1 = e$ $\quad Y_1 = 0$

$X_2 = 2$ $\quad Y_2 = 2$

$Z_3 = -3.075 + j6.719$

2) $Z_3 = (2 + j3)^{.5}$

$X_1 = 2$ $\quad Y_1 = 3$

$X_2 = .5$ $\quad Y_2 = 0$

$Z_3 = 1.674 + j.896$

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	2	STOP	41	X_1	Y_1	0	ENTER X_1 AND Y_1					
	3	x→( )	23									
	4	a	13	STORE Z_1								
	5	y→( )	40									
	6	b	14									
	7	↓	25									
	8	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY								
	9	STOP	41	X_2	Y_2	0	ENTER X_2 AND Y_2					
	a	x→( )	23									
	b	c	16	STORE Z_2								
	c	y→( )	40									
	d	d	17									
1	0	b	14									
	1	↑	27									
	2	a	13	CALCULATE $\ln\sqrt{X_1^2+Y_1^2}$ AND θ								
	3	TO POLAR	62									
	4	ln x	65									
	5	ROLL ↑	22									
	6	c	16									
	7	y→( )	40	STORE $\ln\sqrt{X_1^2+Y_1^2}$								
	8	e	12									
	9	×	36									
	a	ROLL ↑	22									
	b	×	36	CALCULATE $X_2 \ln\sqrt{X_1^2+Y_1^2}$ AND $X_2\theta_1$								
	c	y⇄( )	24									
	d	e	12									
2	0	x⇄y	30									
	1	y⇄( )	24									
	2	d	17	CALCULATE $Y_2 \ln\sqrt{X_1^2+Y_1^2}$								
	3	x⇄y	30									
	4	×	36									
	5	y⇄( )	24									
	6	d	17	CALCULATE $Y_2\theta_1$								
	7	×	36									
	8	ROLL ↓	31									
	9	−	34	CALCULATE $X_2 \ln\sqrt{X_1^2+Y_1^2} - Y_2\theta$								
	a	y⇄( )	24									
	b	d	17									
	c	e	12									
	d	+	33									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	ROLL ↓	31									
	1	↑	27	CALCULATE $\cos(Y_2 \ln\sqrt{X_1^2+Y_1^2}+X_2\theta_1)$ AND $\sin(Y_2 \ln\sqrt{X_1^2+Y_1^2}+X_2\theta_1)$								
	2	cos x	73									
	3	ROLL ↓	31									
	4	sin x	70									
	5	x⇄y	30									
	6	d	17	CALCULATE $e^{X_2\ln\sqrt{X_1^2+Y_1^2}-Y_2\theta_1}$								
	7	e^x	74									
	8	×	36	CALCULATE X_3 AND Y_3								
	9	ROLL ↑	22									
	a	×	36									
	b	0	00									
	c	ROLL ↓	31									
	d	END	46	X_3	Y_3	0	DISPLAY					
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

2nd ORDER DIFFERENTIAL EQUATIONS

This program may be used to solve a wide variety of second order differential equations of the form

$$Y'' = f(X, Y, Y')$$

The solution is a numerical solution which calculates Y_i and Y_i' for a set of closely spaced values of X_i over the desired range. The method used employs a Taylor series around the point X_i. The equations used are

$$Y_{i+1} = Y_i + hY_i' + \frac{h^2}{6}(4Y_i'' - Y_{i-1}'')$$

where the first term of the error is $\frac{-h^4}{8} Y^{(4)}$

and

$$Y_{i+1}' = Y_i' + \frac{h}{12}(5Y_{i-2}'' - 16Y_{i-1}'' + 23Y_i'')$$

where the first term of the error is $\frac{-3h^4}{8} Y^{(5)}$

where $h = X_{i+1} - X_i$

In order to start the solution the equations $Y_{-1}'' = Y_0'' - h \cdot Y_0'''$ with error of $-\frac{1}{2}h^2 Y^{(4)}$

and $Y_{-2}'' = Y_0'' - 2h \cdot Y_0'''$ with error of $-2h^2 Y^{(4)}$ are used.

X_0, Y_0, and Y_0' are known from the boundary conditions. Y_0'' is calculated from $f(X_0, Y_0, Y_0')$. Y_0''' is obtained by differentiating $f(X, Y, Y')$ and substituting in values X_0, Y_0, Y_0', and Y_0''. These initial conditions are required input.

In cases where an increasing exponential predominates it may be necessary to reverse the direction of the independent variable. This may be done simply by making h negative.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: (GO TO) ~~(5) (0)~~ (5)(0)

PLACE MODE SWITCH TO PROGRAM

The general program has a blank section for storing your specific differential equation. Starting at 5-0, enter the program steps which take the independent variable from the b register, the dependent variable from the f register, and the derivative of the dependent variable in the e register, and calculate f(X, Y, Y′). Place f(X, Y, Y′) in the Y register and exit to location 9 - 3. Note there is a maximum of 59 steps (5 - 0 through 9 - 2) available for storing and positioning f(X, Y, Y′).

PLACE MODE SWITCH TO RUN

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: $Y_0'' \rightarrow$ Z, $Y_0''' \rightarrow$ Y, h → X

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
2	——	X

ENTER DATA: $X_0 \rightarrow$ Z, $Y_0' \rightarrow$ Y, $Y_0 \rightarrow$ X

PRESS: CONTINUE

The Calculator will display answers at every increment of the independent variable in the form

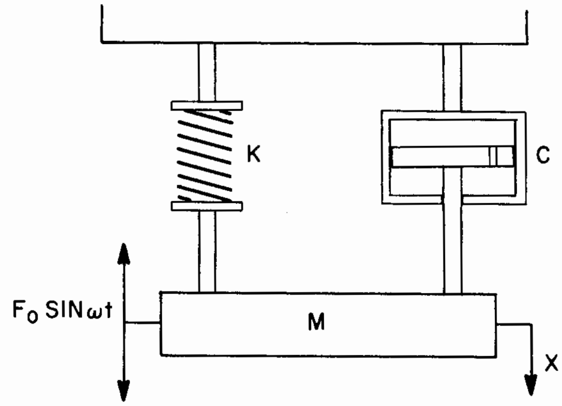
0	——	Z
Y_i	——	Y
X_i	——	X

To stop the solution at the next increment depress PAUSE until display. To restart depress CONTINUE.

To start from a new set of initial conditions

PRESS: STOP
PRESS: GO TO (0) (0) [or END]
PRESS: CONTINUE

EXAMPLES



$M\ddot{X} + C|\dot{X}|\dot{X} + KX = F_0 \text{ SIN } \omega t$

$X(0) = 10$ $C = .08$ $\omega = 4\pi$ $m = .1$

$\dot{X}(0) = 0$ $F_0 = 100$ $K = 25$ $h = .001$

$\ddot{X} = -.8|\dot{X}|\dot{X} - 250X + 1000 \text{ SIN } 4\pi t$

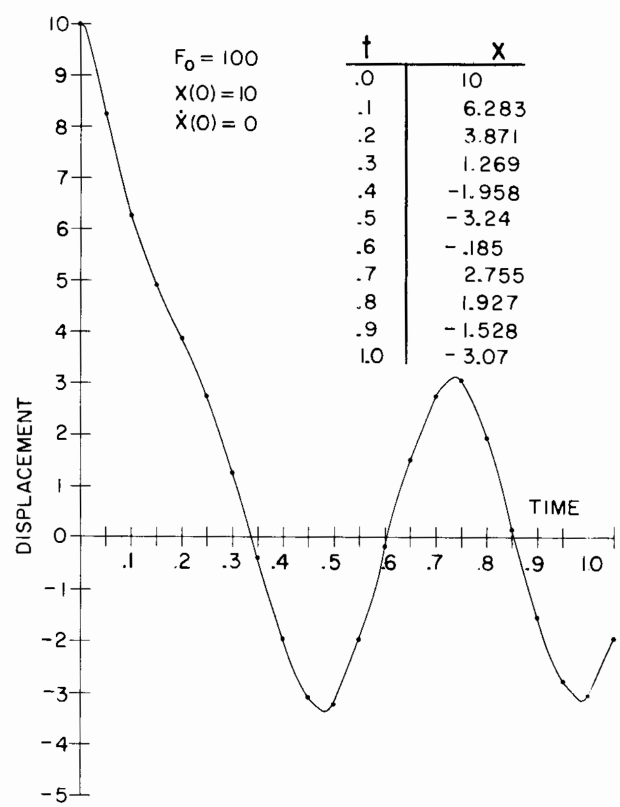
$\ddot{X}(0) = -2500$

$\dddot{X} = -1.6|\dot{X}|\ddot{X} - 250\dot{X} + 4000\pi \text{ COS } 4\pi t$

$\dddot{X}(0) = 4000\pi$

NOTE: SET [RADIANS]

The program steps that form $\ddot{X} = f(t, X, \dot{X})$ appear on Page 1. See User Instructions.



t	X
.0	10
.1	6.283
.2	3.871
.3	1.269
.4	-1.958
.5	-3.24
.6	-.185
.7	2.755
.8	1.927
.9	-1.528
1.0	-3.07

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	
5	0	e	12	$\dot{X}$			X	$\dot{X}$			t		
	1	↑	27		$\dot{X}$								
	2	|y|	55		$\|\dot{X}\|$								
	3	×	36		$\dot{X}\|\dot{X}\|$								
	4	·	21										CALCULATE $-\frac{C}{M}\|\dot{X}\|\dot{X}$
	5	8	10										
	6	CHG SIGN	32	−.8									
	7	×	36		$-.8\|\dot{X}\|\dot{X}$								
	8	f	15	X									
	9	↑	27		X	$-.8\|\dot{X}\|\dot{X}$							
	a	2	02										
	b	5	05										CALCULATE $-\frac{K}{M}X$
	c	0	00										
	d	CHG SIGN	32	−250									
6	0	×	36		−250X								
	1	↓	25	−250X	$-.8\|\dot{X}\|\dot{X}$								
	2	+	33		$-.8\|\dot{X}\|\dot{X}-250X$								
	3	4	04	4									
	4	↑	27		4	$-.8\|\dot{X}\|\dot{X}-250X$							
	5	π	56	π									
	6	×	36		4π								
	7	b	14	t									
	8	×	36		4πt								CALCULATE F_0 SIN ωt
	9	↓	25	4πt	$-.8\|\dot{X}\|\dot{X}-250X$								
	a	sin x	70	SIN 4πt									
	b	↑	27		SIN 4πt	$-.8\|\dot{X}\|\dot{X}-250X$							
	c	ENTER EXP	26										
	d	3	03	1000									
7	0	×	36		1000 SIN 4πt								
	1	↓	25	1000 SIN 4πt	$-.8\|\dot{X}\|\dot{X}-250X$								
	2	+	33		$\ddot{X}$								CALCULATE $\ddot{X}$
	3	GO TO ( ) ( )	44										
	4	9	11										EXIT TO 9-3
	5	3	03										
	6												
	7												
	8												
	9												
	a												
	b												
	c												
	d												

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	Comments
0 0	CLEAR	20			REGISTER CONTENTS →	Y_i	Y_i'	Y_i''	Y_{i-1}''	X_i	h	
1	1	01										DISPLAY 1 TO INDICATE FIRST ENTRY
2	STOP	41	h	Y_0'''	Y_0''							ENTER INCREMENT AND INITIAL CONDITIONS
3	$x\to()$	23										
4	a	13										
5	×	36										CALCULATE Y_{-1}'' (steps 0-5 to 0-b)
6	↓	25										
7	$y\to()$	40										
8	d	17										
9	−	34										
a	$y\to()$	40										
b	c	16										
c	−	34										CALCULATE Y_{-2}'' (steps 0-c to 1-0)
d	$y\to()$	40										
1 0	b	14										
1	CLEAR	20										
2	2	02										DISPLAY 2 TO INDICATE SECOND ENTRY
3	STOP	41	Y_0	Y_0'	X_0							ENTER INITIAL CONDITIONS
4	ACC +	60										
5	f	15										FROM 9-c; RECALL AND POSITION FOR DISPLAY (steps 1-5 to 1-8)
6	ROLL ↓	31										
7	CLEAR x	37										
8	ROLL ↓	31										
9	PAUSE	57	X_i	Y_i	0							DISPLAY AT EACH INCREMENT
a	$x\rightleftarrows y$	30										
b	$y\rightleftarrows()$	24										CALCULATE $\frac{h}{12}(5Y_{i-2}''-16Y_{i-1}''+23Y_i'')$ (steps 1-b to 2-d)
c	b	14										
d	5	05										
2 0	×	36										
1	c	16										
2	↑	27										
3	1	01										
4	6	06										
5	×	36										
6	↓	25										
7	−	34										
8	d	17										
9	↑	27										
a	2	02										
b	3	03										
c	×	36										
d	↓	25										

Step	Key	Code	Display x
3-0	+	33	
3-1	a	13	
3-2	×	36	
3-3	1	01	
3-4	2	02	
3-5	÷	35	
3-6	d	17	
3-7	↑	27	
3-8	4	04	
3-9	×	36	
3-a	c	16	
3-b	−	34	
3-c	a	13	
3-d	×	36	CALCULATE $hY_i' + \frac{h^2}{6}(4Y_i'' - Y_{i-1}'')$
4-0	6	06	
4-1	÷	35	
4-2	e	12	
4-3	+	33	
4-4	a	13	
4-5	×	36	
4-6	↓	25	
4-7	ACC +	60	CALCULATE Y_{i+1} AND Y_{i+1}'
4-8	y⇄( )	24	
4-9	b	14	
4-a	a	13	
4-b	+	33	INCREMENT X_i
4-c	y→( )	40	
4-d	b	14	
5-0	CONTINUE	47	ENTER SPECIFIC DIFFERENTIAL EQUATION HERE. REGISTER b CONTAINS X, REGISTER f CONTAINS Y, AND REGISTER e CONTAINS Y'. USE THESE VARIABLES TO PROGRAM THE SPECIFIC EQUATION Y''=f(X,Y,Y'). AFTER FORMING Y'' PLACE IT IN THE Y REGISTER AND EXIT TO LOCATION 9-3.
5-1	↓		
5-2			
5-3			
5-4			
5-5			FILL THE REMAINING AREA WITH CONTINUES TO ELIMINATE THE POSSIBILITY OF AN END AND THUS RECORDING AND ENTERING PROBLEMS.
5-6			
5-7			*SEE OPERATING MANUAL*
5-8			
5-9			
5-a			
5-b			
5-c			
5-d	↓		

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
7	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
8	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

FROM 5-0 THROUGH 9-2

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
9	0	CONTINUE	47									
	1											
	2				Y''	NOTE: Y'' MUST BE IN THE Y REGISTER AT THIS TIME						
	3	y⇄( )	24									
	4	d	17									
	5	y⇄( )	24									
	6	c	16									
	7	y⇄( )	24	REINITIALIZE AND BRANCH FOR NEXT INCREMENT (steps 3–c)								
	8	b	14									
	9	↑	27									
	a	GO TO ( )( )	44									
	b	1	01									
	c	5	05									
	d	END	46									
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

This program calculates the two roots X_1 and X_2 of the equation

▲ $$ax^2 + bX + c = 0 \qquad a \neq 0$$

$$aX^2 + bX + c = 0$$

The equation used to calculate the roots is:

$$X_1, X_2 = -\frac{b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$$

▲ Denotes Revision

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

ENTER DATA: a → Z, b → Y, c → X

PRESS: CONTINUE

DISPLAY

real roots:

0 — Z
X_2 — Y
X_1 — X

complex roots:

Im(X_2) — Z
Im(X_1) — Y
Re(X_1 and X_2) — X

(Note Z contains zero)

EXAMPLES

GENERAL FORM

$aX^2 + bX + c = 0$

$X^2 + X + 1.25 = 0$

$X_2 = -.5 + j1$

$X_1 = -.5 - j1$

$2X^2 + 5X + 3 = 0$

$X_2 = -1.5$

$X_1 = -1.0$

$X^2 + 4 = 0$

$X_2 = j2$

$X_1 = -j2$

Step	Key	Code	Display x	Display y	Display z	Storage / Comments
0 0	CLEAR	20				
1	STOP	41	c	b	a	ENTER a,b,c
2	ROLL ↑	22				
3	÷	35				
4	ROLL ↑	22				
5	$x \rightleftarrows y$	30				
6	÷	35				
7	2	02				
8	CHG SIGN	32				
9	÷	35	CALCULATE $-\frac{b}{2a}$ AND $\frac{b^2}{4a^2}-\frac{c}{a}$			
a	↓	25				
b	↑	27				
c	×	36				
d	ROLL ↓	31				
1 0	$x \rightleftarrows y$	30				
1	−	34				
2	CLEAR x	37				
3	IF $x=y$	50				
4	3	03	IF $\frac{b^2}{4a^2}-\frac{c}{a}=0$, SOLVE FOR $X_1,X_2=-\frac{b}{2a}$			
5	4	04				
6	IF $x>y$	53				
7	2	02	IF $\frac{b^2}{4a^2}-\frac{c}{a}<0$, SOLVE FOR COMPLEX ROOTS			
8	9	11				
9	↓	25	BRANCH HERE IF $\frac{b^2}{4a^2}-\frac{c}{a}>0$			
a	$\sqrt{x}$	76				
b	↑	27				
c	CHG SIGN	32				
d	ROLL ↑	22	CALCULATE $X_1,X_2=-\frac{b}{2a}\pm\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}$			
2 0	+	33				
1	ROLL ↑	22				
2	+	33				
3	CLEAR x	37				
4	ROLL ↓	31	X_1	X_2	0	REAL ROOTS } DISPLAY
5	STOP	41	$Re(X_1 \& X_2)$	$Im(X_1)$	$Im(X_2)$	COMPLEX ROOTS } DISPLAY
6	GO TO ( )( )	44				
7	0	00				
8	0	00				
9	↓	25	BRANCH HERE IF $\frac{b^2}{4a^2}-\frac{c}{a}<0$			FROM 3-3 & 3-9
a	CHG SIGN	32				
b	$\sqrt{x}$	76				
c	↑	27				
d	CHG SIGN	32	CALCULATE $X=-\frac{b}{2a}\pm j\sqrt{\frac{c}{a}-\frac{b^2}{4a^2}}$			

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	ROLL ↑	22									
1	GO TO ( )( )	44									
2	2	02									
3	5	05									
4	↓	25	BRANCH HERE IF $\frac{b^2}{4a^2} - \frac{c}{a} = 0$								
5	CLEAR x	37									
6	ROLL ↓	31	CALCULATE $X_1, X_2 = -\frac{b}{2a}$								
7	GO TO ( )( )	44									
8	2	02									
9	5	05									
a	END	46									
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

FROM 1-5

PART NO.
09100-70010

FINITE DIFFERENCE INTERPOLATION USING GAUSS'S BACKWARD FORMULA

This program interpolates for data points in the region of tabulated data for uniformly spaced abscissas, with spacing h. The equation used is the backward-interpolation formula of Gauss which uses four pairs of data points and sets up the polynomial for cubic interpolation.

The equation used is:

$$Y = Y_o + u\,\delta Y_{-1/2} + \frac{1}{2!}\,(u+1)\quad \delta^2 Y_o + \frac{1}{3!}\,(u+1)(u)(u-1)\,\delta^3 Y_{-1/2}$$

The difference table is:

u	X	Y			
-2	X_1	Y_1			
			$Y_2 - Y_1$		
-1	X_2	Y_2		$Y_3 - 2Y_2 + Y_1$	
0	X_3	Y_3 →	$Y_3 - Y_2$ →	$Y_4 - 2Y_3 + Y_2$ →	$Y_4 - 3Y_3 + 3Y_2 - Y_1$
			$Y_4 - Y_3$		
1	X_4	Y_4			

where $\delta Y_{-1/2} = Y_3 - Y_2$

$\delta^2 Y_o = Y_4 - 2Y_3 + Y_2$

$\delta^3 Y_{-1/2} = Y_4 - 3Y_3 + 3Y_2 - Y_1$

and $u = \dfrac{X - X_3}{h}$

Reference: Introduction to Numerical Analysis
by F.B. Hildebrand

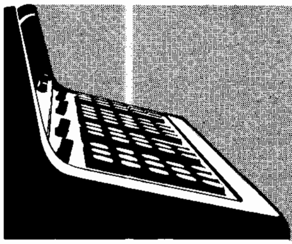
McGraw-Hill 1956

FROM 7-3 (→ 3-3)

Step	Key	Code	Comments
3 0	−	34	CALCULATE $\delta Y_{-\frac{1}{2}}$
1	y→( )	40	
2	a	13	
3	CLEAR x	37	
4	↑	27	
5	↑	27	
6	3	03	DISPLAY 3 TO INDICATE THIRD ENTRY
7	STOP	41	X 0 0 — ENTER X
8	ROLL ↓	31	CALCULATE u
9	↓	25	
a	e	12	
b	−	34	
c	f	15	
d	÷	35	
4 0	ROLL ↑	22	STORE X
1	x→( )	23	
2	9	11	
3	↓	25	CALCULATE $\frac{1}{3!}(u+1)(u)(u-1)\delta^3 Y_{-\frac{1}{2}}$
4	1	01	
5	x⇄y	30	
6	+	33	
7	×	36	
8	×	36	
9	↑	27	
a	×	36	
b	ROLL ↓	31	
c	−	34	
d	↓	25	
5 0	x⇄y	30	
1	−	34	
2	6	06	
3	÷	35	
4	b	14	
5	×	36	
6	a	13	CALCULATE $u\delta Y_{-\frac{1}{2}}$
7	ROLL ↑	22	
8	×	36	
9	ROLL ↓	31	FORM PARTIAL SUM
a	+	33	
b	1	01	
c	ROLL ↑	22	
d	+	33	

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6 0	×	36	CALCULATE $\frac{u(u+1)}{2!}\delta^2 Y_0$								
1	2	02									
2	÷	35									
3	c	16									
4	×	36									
5	CLEAR x	37	FORM PARTIAL SUM								
6	ROLL ↓	31									
7	+	33									
8	d	17	CALCULATE FINAL SUM Y								
9	+	33									
a	↓	25	POSITION DISPLAY								
b	y⇄( )	24									
c	9	11									
d	x⇄y	30									
7 0	STOP	41	X	Y(X)	0	DISPLAY					
1	GO TO ( )( )	44	BRANCH FOR X ENTRY								
2	3	03									
3	3	03									
4	END	46									

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

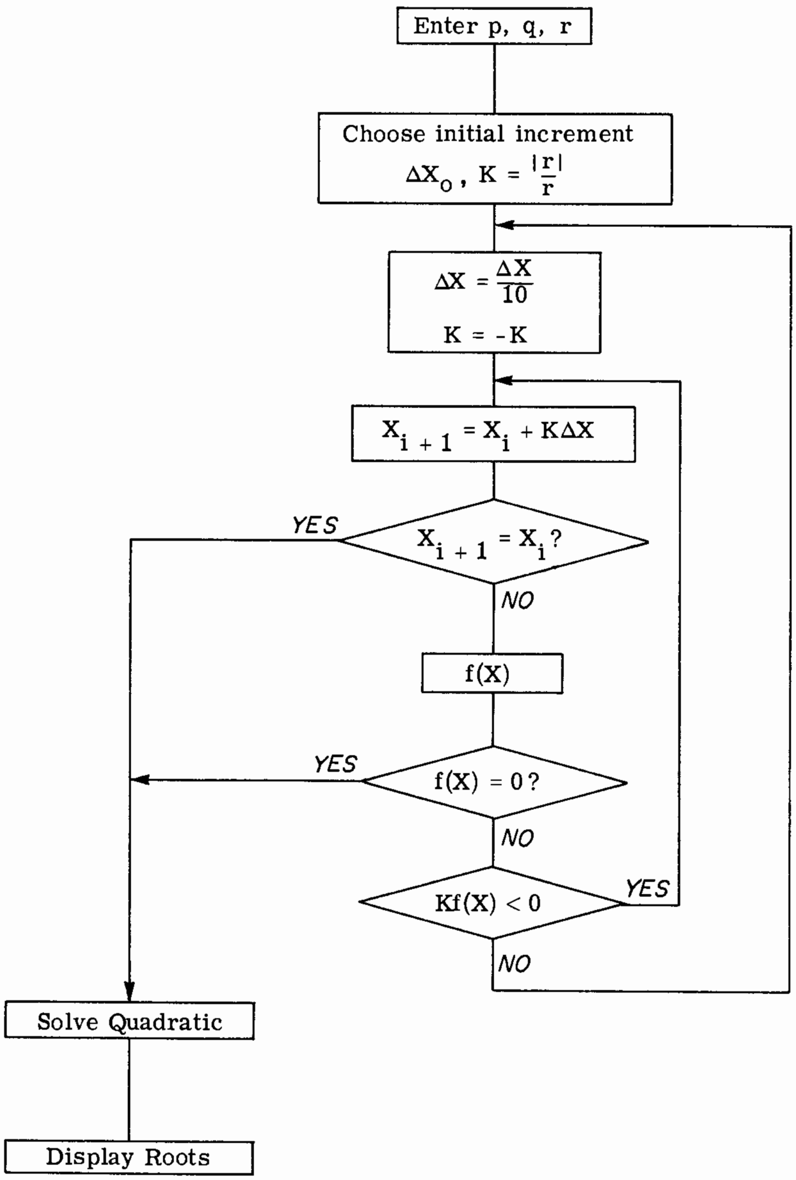


CUBIC EQUATION

This program solves

$$X^3 + pX^2 + qX + r = 0$$

for real and complex roots.



USER INSTRUCTIONS	EXAMPLES

USER INSTRUCTIONS

ENTER PROGRAM (Starting address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

ENTER DATA: p → Z, q → Y, r → X

PRESS: CONTINUE

DISPLAY:

3 real roots:

0	Z
0	Y
3	X

1 real 2 complex roots:

0	Z
0	Y
1	X

PRESS: CONTINUE

DISPLAY:

real roots:

X_3	Z
X_2	Y
X_1	X

complex roots:

(X_3)	Z
Im(X_1 and X_2)	Y
Re(X_1 and X_2)	X

EXAMPLES

General Form

$$X^3 + pX^2 + qX + r = 0$$

Example 1

$$X^3 + 3X^2 + 3X + 1$$

$X_3 = -1$

$X_2 = -1$

$X_1 = -1$

Example 2

$$X^3 - X^2 + X - 1$$

$X_3 = 1$

$X_1, X_2 = \pm i$

PART NO. 09100-70011

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	STOP	41	r	q	p	ENTER p,q,r					
2	ACC +	60									
3	↓	25									
4	y→( )	40	STORE DATA								
5	d	17									
6	|y|	55									
7	f	15									
8	x⇄y	30									
9	|y|	55									
a	+	33									
b	ENTER EXP	26									
c	CHG SIGN	32									
d	2	02									
1 0	↑	27									
1	1	01									
2	0	00									
3	×	36									
4	↓	25	STORE INITIAL INCREMENT ΔX_0								
5	IF x<y	52									
6	1	01									
7	0	00									
8	x→( )	23									
9	c	16									
a	CLEAR x	37									
b	↑	27									
c	f	15									
d	↑	27	CALCULATE K								
2 0	|y|	55									
1	÷	35									
2	y→( )	40									
3	b	14									
4 (FROM 5-8)	c	16									
5	x⇄y	30									
6	.	21	CALCULATE $\frac{\Delta X}{10}$								
7	1	01									
8	×	36									
9	y→( )	40									
a	c	16									
b	b	14	CALCULATE −K								
c	CHG SIGN	32									
d	x→( )	23									

Step	Key	Code	Display
3 0	b	14	
1	↓	25	CALCULATE $X_{i+1} = X_i + K\Delta X$
2	c	16	
3	↑	27	
4	b	14	
5	×	36	
6	↓	25	
7	+	33	
8	↓	25	
9	IF x = y	50	BRANCH IF $X_{i+1} = X_i$
a	5	05	
b	9	11	
c	↑	27	
d	↑	27	
4 0	d	17	CALCULATE f(X)
1	+	33	
2	↓	25	
3	×	36	
4	e	12	
5	+	33	
6	↓	25	
7	×	36	
8	f	15	
9	+	33	
a	CLEAR x	37	BRANCH IF f(X)=0
b	IF x = y	50	
c	5	05	
d	9	11	
5 0	b	14	COMPARE Kf(X) TO ZERO
1	×	36	
2	CLEAR x	37	
3	IF x > y	53	
4	3	03	
5	1	01	
6	GO TO ( )( )	44	BRANCH TO INCREMENT CHANGE
7	2	02	
8	4	04	
9	↓	25	
a	y→( )	40	
b	b	14	
c	d	17	
d	ROLL ↑	22	

Step	Key	Code
6 0	+	33
1	↑	27
2	↓	25
3	×	36
4	e	12
5	+	33
6	↓	25
7	CHG SIGN	32
8	ROLL ↓	31
9	2	02
a	CHG SIGN	32
b	÷	35
c	↓	25
d	↑	27
7 0	×	36
1	ROLL ↓	31
2	+	33
3	ENTER EXP	26
4	CHG SIGN	32
5	1	01
6	0	00
7	ROLL ↑	22
8	×	36
9	|y|	55
a	ROLL ↓	31
b	CHG SIGN	32
c	IF x > y	53
d	9	11
8 0	1	01
1	SET FLAG	54
2	CHG SIGN	32
3	IF x > y	53
4	CLEAR x	37
5	×	36
6	↓	25
7	√x	76
8	−	34
9	ROLL ↑	22
a	+	33
b	↓	25
c	GO TO ( )( )	44
d	9	11

SOLVE REDUCED QUADRATIC EQUATION

FROM 8-0

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9	0	4	04									
	1	↓	25									
	2	CHG SIGN	32									
	3	$\sqrt{x}$	76									
	4	$x\to( )$	23									
	5	e	12									
	6	$y\to( )$	40									
	7	f	15									
	8	CLEAR x	37									
	9	↑	27									
	a	↑	27									
	b	1	01	ENTER 1 FOR COMPLEX DISPLAY								
	c	IF FLAG	43									
	d	CLEAR x	37	FLAG IS SET FOR REAL ROOTS								
a	0	3	03									
	1	STOP	41	1 or 3	0	0	1 FOR ONE REAL AND TWO COMPLEX ROOTS, 3 FOR THREE REAL ROOTS					
	2	b	14									
	3	ROLL ↓	31	POSITION DISPLAY								
	4	RCL	61									
	5	END	46	$Re_{1,2}$	$Im_{1,2}$	X_3	DISPLAY					
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

PART NO.
09100-70013

FACTORS OF n

This program gives the factors of any positive integer n.

All repeated factors are given.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	—	Z
0	—	Y
0	—	X

ENTER DATA: No. to be factored → X

DISPLAY

0	—	Z
0	—	Y
n	—	X

PRESS: CONTINUE

DISPLAY

n	—	Z
remaining factor	—	Y
Factor	—	X

FINAL DISPLAY

n	—	Z
1	—	Y
Factor	—	X

EXAMPLES

(A)

n = 50

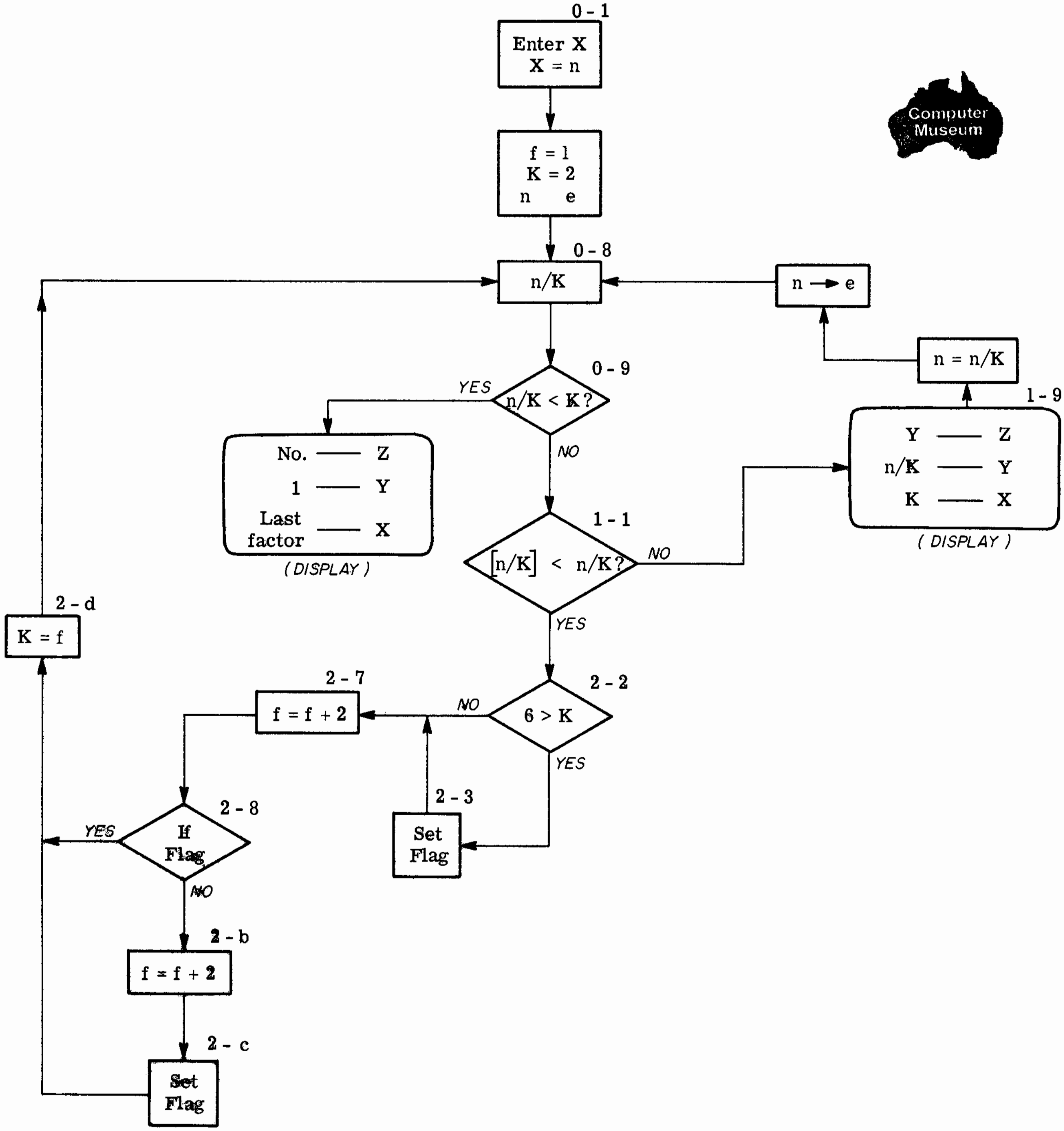
Factors are: 2, 5, 5

(B)

$n = 2.9393939 \times 10^{7}$

Factors are: 2.9393939×10^{7}

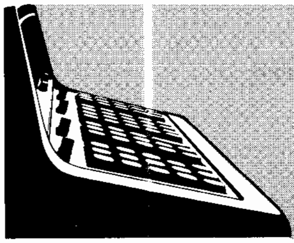
(n is a prime number)


0 - 1
Enter X
X = n
f = 1
K = 2
n e
0 - 8
n/K
n → e
n = n/K
0 - 9
n/K < K?
YES
NO
No. —— Z
1 —— Y
Last factor —— X
(DISPLAY)
1 - 9
Y —— Z
n/K —— Y
K —— X
(DISPLAY)
1 - 1
[n/K] < n/K?
NO
YES
2 - d
K = f
2 - 7
f = f + 2
2 - 2
6 > K
NO
YES
2 - 3
Set Flag
2 - 8
If Flag
YES
NO
2 - b
f = f + 2
2 - c
Set Flag

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	X	0	0						
	2	$x \to (\ )$	23									
	3	d	17	STORE X								
	4	↑	27									
	5	1	01	STORE I AND X								
	6	ACC +	60									
	7	2	02									
	8	÷	35	CALCULATE X/K								
	9	IF $x > y$	53									
	a	3	03	BRANCH FOR FINAL DISPLAY								
	b	3	03									
	c	$x \rightleftarrows y$	30									
	d	↑	27									
1	0	int x	64	COMPARE $\left[\frac{n}{K}\right]$ TO K AND BRANCH IF								
	1	IF $x < y$	52	K IS NOT A FACTOR								
	2	1	01									
	3	d	17									
	4	ROLL ↓	31									
	5	d	17									
	6	ROLL ↓	31	POSITION DISPLAY AND STORE UNFACTORED PORTION								
	7	$y \to (\ )$	40									
	8	e	12									
	9	STOP	41	K	n/K	n						
	a	GO TO ( )( )	44									
	b	0	00	BRANCH TO CONTINUE FACTORING								
	c	8	10									
	d	CLEAR x	37									
2	0	ROLL ↓	31									
	1	6	06									
	2	IF $x > y$	53									
	3	SET FLAG	54	SET FLAG IF $6 > K$								
	4	CLEAR x	37									
	5	↓	25									
	6	2	02	CALCULATE $f+2$								
	7	ACC +	60									
	8	IF FLAG	43									
	9	2	02	BRANCH WHEN FLAG IS SET								
	a	d	17									
	b	ACC +	60									
	c	SET FLAG	54	CALCULATE $f+2$ AND SET FLAG								
	d	RECALL	61	SET $K=f$								

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3	0	GO TO ( ) ( )	44	BRANCH TO CONTINUE FACTORING								
	1	0	00									
	2	8	10									
	3	d	17	POSITION DISPLAY AND LAST FACTOR								
	4	↑	27									
	5	1	01									
	6	↑	27									
	7	e	12									
	8	END	46	LAST FACTOR	1	X	DISPLAY					

FROM 0-b



REAL ROOTS OF f(x)

This program finds the real roots of equations of the form $f(x) = 0$. The solution evaluates $f(x)$ at a specified starting point x_0 and at successive x's by replacing x with $x + \Delta x$ (the initial Δx is also specified by the user). When f(x) changes sign, Δx is replaced with $-\Delta x/10$.

The search continues until f(x) is driven to zero. The value of x such that $f(x) = 0$ is a root of the equation. f(x) must change sign for the technique to converge.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (6) (0) [or END] *(handwritten correction: (6) (0) over (0) (0))*

SET: | PROGRAM | |

Starting at (6) (0) enter the program steps which take the independent variable from the X register and program f(x), (x is also contained in the d register). Place f(x) is the X register and exit to location 0-6. The last four steps of the f(x) subroutine should be:

GO TO
0
6
END

Note: Registers 6, 7, 8, 9, a, b, c, e, and f are available for programming and storage of f(x). Register d and the flag are unavailable.

SET: | | RUN |

PRESS: END

Store initial conditions (when applicable).

ENTER: $x_i \rightarrow X$ (x_i is initial x from which search is to begin.)

PRESS: CONTINUE

DISPLAY

$\pm f(x_{i-1}) \cdot f(x_i)$	Z
$f(x_i)$	Y
x_i	X

Note: $f(x_{i-1})$ is random the first pass and product is meaningless

Is x_i Satisfactory ? — NO: return to PRESS: END; YES: continue

ENTER: $\Delta x \rightarrow x$. (Δx is the searching increment. $+\Delta x$ searches right, $-\Delta x$ searches left)

USER INSTRUCTIONS (con't)

PRESS: CONTINUE

The real roots of f(x) program successively evaluates f(x).

PAUSE DISPLAY

$\pm f(x_{i-1}) \cdot f(x_i)$	Z
$f(x_i)$	Y
x_i	X

The calculation stops when x_i is a root, x_i is successively replaced by $x_i \pm \Delta x$, (See flowchart). If calculation is not converging, press: PAUSE, press: END and enter a new estimate as before.

TO ENTER A NEW PROBLEM:

PRESS: END

PRESS: GO TO (0) (0)

ENTER NEW DATA AS BEFORE

EXAMPLES

1. FIND ROOT OF x = cos x

SET: | | RADIANS |

ENTER DATA: $x_i = 0 \rightarrow X$
PRESS: END
PRESS: CONTINUE
DISPLAY

(JUNK)	Z	
-1.000	Y	(Y contains f(x))
0	X	(X contains x)

ENTER DATA: $x = 0.1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z	
0	Y	
0.7390851332	X	(X contains root)

2.

$$y = \frac{\pi^2 \sqrt{x}}{1 + \cos \frac{\pi}{2} x}$$

Given the initial x condition y = 4, find x.

Rewrite equation as

$$f(x) = y(1 + \cos \frac{\pi}{2} x) - \pi^2 \sqrt{x}$$

User subroutine assumes y is in a (it will be manually entered into a as an initial condition)

SET: [ ] RADIANS

ENTER DATA: y = 4 → X

PRESS: $x \rightarrow (a)$ to satisfy initial condition that y is in a.

PRESS: END

ENTER DATA: Initial x_i = 0 X

PRESS: CONTINUE

DISPLAY

(JUNK) —— Z		
8.00 —— Y	f (x, 4) in Y	
0 —— X	(x in X)	

Instead of entering Δ x, evaluate f(x) at a new value x_i

PRESS: END

ENTER DATA: x_i = 1 → X

PRESS: CONTINUE

DISPLAY

(JUNK) —— Z	
-5.87 —— Y	f (x, 4) in Y
1.0 —— X	(x in X)

A root has been crossed between f(0) and f(1) because f(x) changed sign. Begin search to the left by entering

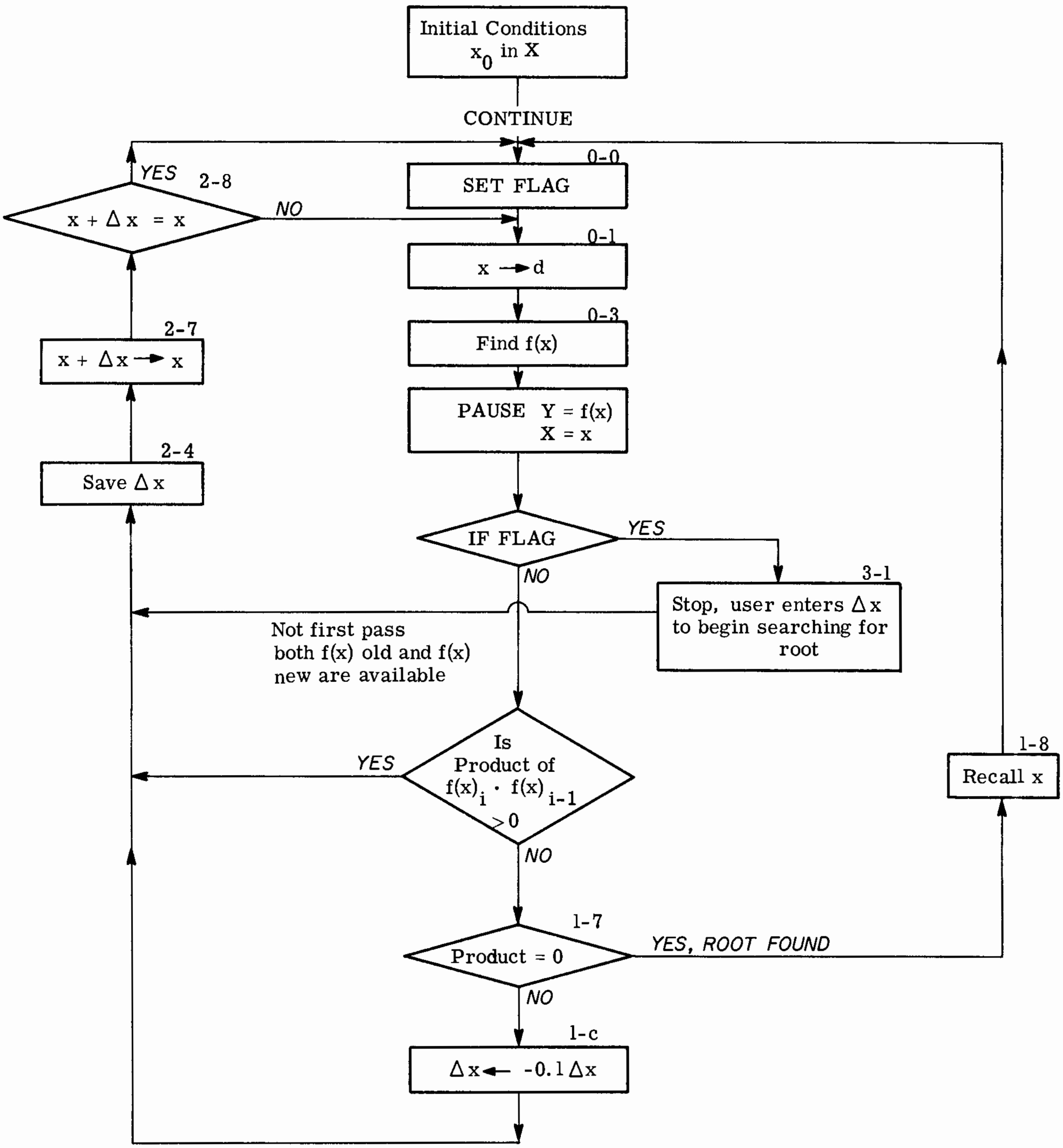
x = -0.1 → X

PRESS: CONTINUE

EXAMPLES (con't)

DISPLAY

4×10^{-22}		
-2	-11	f(x, 4) in Y
4.868292372	-01	X contains root


Initial Conditions
x0 in X
CONTINUE
0-0
SET FLAG
YES
2-8
x + Δx = x
NO
0-1
x → d
0-3
Find f(x)
2-7
x + Δx → x
PAUSE Y = f(x)
X = x
2-4
Save Δx
IF FLAG
YES
NO
3-1
Stop, user enters Δx to begin searching for root
Not first pass
both f(x) old and f(x) new are available
YES
Is Product of f(x)i · f(x)i-1 > 0
1-8
Recall x
NO
1-7
Product = 0
YES, ROOT FOUND
NO
1-c
Δx ← -0.1Δx

EXAMPLE 1 PAGE 1 PART NO. 09100-70014

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	↑	27	x	x							
	1	cos x	73	cos(x)								
	2	−	34		x − cos(x) = f(x)							
	3	↓	25	f(x)								
	4	GO TO ( ) ( )	44									
	5	0	00									
	6	6	06									
	7	END	46									
	8											
	9											
	a											
	b											
	c											
	d	EXAMPLE 2										

EXAMPLE 1 FINDS THE ROOT OF $x = \cos x$. REWRITE THE EQUATION AS $f(x) = x - \cos x$.

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	↑	27	x	x							
	1	π	56	π								
	2	×	36		πx							
	3	2	02	2								
	4	÷	35		$\pi x/2$							
	5	1	01	1								
	6	$x \rightleftarrows y$	30	$\pi x/2$	1							
	7	cos x	73	$\cos(\pi x/2)$								
	8	+	33		$1+\cos(\pi x/2)$							
	9	a	13	y								
	a	×	36		$y[1+\cos(\pi x/2)]$							
	b	d	17	x								
	c	$\sqrt{x}$	76	$\sqrt{x}$								
	d	↑	27		$\sqrt{x}$	$y[1+\cos(\pi x/2)]$						
7	0	π	56	π								
	1	×	36		$\pi\sqrt{x}$							
	2	×	36		$\pi^2\sqrt{x}$							
	3	↓	25	$\pi^2\sqrt{x}$	$y[1+\cos(\pi x/2)]$							
	4	−	34		f(x,y)							
	5	↓	25	f(x,y)								
	6	GO TO ( ) ( )	44									
	7	0	00									
	8	6	06									
	9	END	46									
	a											
	b											
	c											
	d											

EXAMPLE 2 FINDS THE ROOT OF $y = \dfrac{\pi^2\sqrt{x}}{1+\cos\frac{\pi}{2}x}$. GIVEN THE INITIAL CONDITION THAT $y = 4$, FIND x. REWRITE THE EQUATION AS $f(x) = y\left(1+\cos\frac{\pi x}{2}\right) - \pi^2\sqrt{x}$.

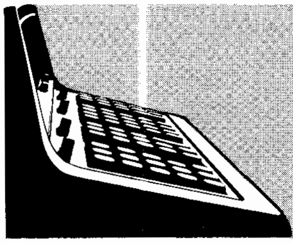
Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0	0	SET FLAG	54									
	1	$x \to ( )$	23									
	2	d	17									
	3	GO TO ( )( )	44									
	4	6	06									
	5	0	00									
	6	↑	27									
	7	$y \rightleftarrows ( )$	24									
	8	4	04									
	9	×	36									
	a	↑	27	NEGATIVE PRODUCT MEANS ROOT JUST PASSED, ZERO PRODUCT MEANS ROOT FOUND								
	b	d	17									
	c	PAUSE	57	x_i	$f(x_i)$	$f(x_{i-1}) \cdot f(x_i)$						
	d	IF FLAG	43									
1	0	3	03									
	1	1	01									
	2	ROLL ↓	31									
	3	CLEAR x	37									
	4	IF $x < y$	52									
	5	2	02									
	6	0	00									
	7	IF $x = y$	50									
	8	ROLL ↑	22									
	9	GO TO ( )( )	44	RECALCULATE f(x) SET FLAG AT (0)-(0) TO STOP AT (2)-(c) ON NEXT PASS								
	a	0	00									
	b	0	00									
	c	CHG SIGN	32									
	d	·	21									
2	0	1	01									
	1	$y \rightleftarrows ( )$	24									
	2	5	05									
	3	×	36									
	4	$y \to ( )$	40									
	5	5	05									
	6	d	17									
	7	+	33									
	8	IF $x = y$	50	IF Xnew = Xold THEN ROOT HAS BEEN FOUND AS NEAR AS POSSIBLE								
	9	0	00									
	a	0	00									
	b	↓	25									
	c	GO TO ( )( )	44									
	d	0	00									

FROM 3-0 (to step 0-1)

FROM 3-4 (to step 2-4)

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	1	01									
	1	STOP	41	x_i	$f(x_i)$	$f(x_i)\cdot f(x_{i-1})$						
	2	↑	27									
	3	GO TO ( ) ( )	44									
	4	2	02									
	5	4	04									
	6	CONTINUE	47									
4	0	CONTINUE	47									
5	0	CONTINUE	47									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	END	46	x					x			
	1			ENTER SPECIFIC f(x) STARTING AT 6-0. THE X AND d								
	2			REGISTER CONTAIN x. USE THIS VARIABLE TO PROGRAM								
	3			f(x); LEAVE f(x) IN X AND EXIT TO LOCATION 0-6								
	4			FOLLOWED BY END.								
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



SIMULTANEOUS SOLUTION OF TWO EQUATIONS IN TWO UNKNOWNS

PART NO.
09100-70016

This program solves two independent equations simultaneously for the two unknowns.

The two equations are of the following form:

$$ax + by = e$$
$$cx + dy = f$$

where x and y are the two unknowns to be found.

The solution is obtained by normalizing the coefficient of x to 1, yielding the equations:

$$x = e/a - (b/a)y \qquad (1)$$

$$x = f/c - (d/c)y \qquad (2)$$

Substracting gives the solution for y:

$$y = \frac{(e/a - f/c)}{(b/a - d/c)}$$

The value of x is then found by substituting the value of y back into the second equation.

Reference:

Numerical Analysis
by
Kaiser S. Kunz

McGraw-Hill, 1st edition
(1957)

USER INSTRUCTIONS	EXAMPLE

ENTER PROGRAM: (Starting Address is (0) (0))

PRESS: GO TO (0) (0) or [END]

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: a → Z, b → Y, e → X

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: c → Z, d → Y, f → X

PRESS: CONTINUE

DISPLAY

0	———	Z
y	———	Y
x	———	X

TO RESET PROBLEM:

Solve the following equations simultaneously for x and y.

(1) $2x + y = 6$

(2) $2x + 3y = 1$

Data to be entered:

2	→	Z	
1	→	Y	(1)
6	→	X	
2	→	Z	
3	→	Y	(2)
1	→	X	

Solution:

$x = 4.25$
$y = -2.50$

General Form:

$ax + by = e$
$cx + dy = f$

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	SET FLAG	54									
	2	ACC +	60	CLEAR OUT STORAGE AND SET THE FLAG								
	3	CLEAR x	37									
	4	↑	27									
	5	↑	27									
	6	STOP	41	e	b	a	ENTER DATA: (COEFFICIENTS AND CONSTANT OF FIRST EQUATION)					
	7	ROLL ↑	22									
	8	÷	35									
	9	ROLL ↑	22	CALCULATE e/a AND b/a ON THE FIRST PASS; f/c AND d/c ON THE SECOND PASS								
	a	$x \rightleftarrows y$	30									
	b	÷	35									
	c	↓	25									
	d	IF FLAG	43	BRANCH TO STORE e/a AND b/a, AND ENTER COEFFICIENTS AND CONSTANT OF SECOND EQUATION								
1	0	0	00									
	1	2	02									
	2	ACC −	63	CALCULATE (b/a − d/c) AND (e/a − f/c)								
	3	$x \rightleftarrows y$	30									
	4	$y \rightleftarrows ()$	24									
	5	e	12	CALCULATE y								
	6	f	15									
	7	÷	35									
	8	e	12									
	9	$x \rightleftarrows y$	30									
	a	×	36	CALCULATE x								
	b	ROLL ↓	31									
	c	−	34									
	d	CLEAR x	37									
2	0	ROLL ↓	31									
	1	END	46	x	y	0						
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
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4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

n! ($n<10^{12}$)

PART NO.
09100-70017

This program calculates n! for integer n where $0 \le n < 10^{12}$

$$n! = n\ (n-1)\ \ldots\ 3 \cdot 2 \cdot 1$$

No Reference

USER INSTRUCTIONS	EXAMPLES

ENTER PROGRAM: (Starting Address is (0) (0))

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA:

n → X

PRESS: CONTINUE

DISPLAY

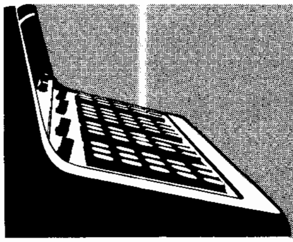
Exponent of 10	Z
Decimal Number of n!	Y
n	X

TO RESET PROBLEM:

	DATA	SOLUTION	TIME
A)	n = 0	n! = 1	
B)	n = 7	n! = 5.040×10^{3}	1 sec.
C)	n = 1000	n! = 4.0239×10^{2567}	16 sec.
D)	n = 10^4	n! = 2.8463×10^{35659}	2 min. 53 sec.

Step	Key	Code	Display x	Display y	Display z	Storage / Remarks
0 0	CLEAR	20				
1	STOP	41	n	0	0	ENTER n
2	x→( )	23	STORE n			
3	d	17				
4	↑	27				
5	1	01				
6	x⇄y	30				
7	ROLL ↑	22				
8	ACC +	60				
9	IF x = y	50	BRANCH IF n=0 TO SET n!=1			
a	2	02				
b	c	16				
c	↓	25	FORM n·(n−1)·(n−2)··· 3.2.1			
d	×	36				
1 0	ENTER EXP	26				
1	9	11				
2	0	00				
3	IF x > y	53	BRANCH IF n! (PARTIAL) < 10^{90} TO DECREMENT (n−k) BY 1			
4	2	02				
5	3	03				
6	ENTER EXP	26	REDUCE EXPONENT OF n! (PARTIAL) BY 90's AND SUM THE VALUE OF THE EXPONENT OVERFLOW IN f			
7	CHG SIGN	32				
8	9	11				
9	0	00				
a	×	36				
b	0	00				
c	↑	27				
d	9	11				
2 0	0	00				
1	ACC +	60				
2	↓	25	DECREMENT (n−k) BY 1			
3	1	01				
4	↑	27				
5	CLEAR x	37				
6	ACC −	63				
7	RCL	61				
8	CLEAR x	37				
9	GO TO ( )( )	44	BRANCH TO MULTIPLY THE PARTIAL FACTORIAL BY (n−k−1)			
a	0	00				
b	9	11				
c	↓	25				
d (FROM 3-c)	ENTER EXP	26				

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3 0	9	11									
1	IF x > y	53									
2	3	03									
3	d	17	REDUCE EXPONENT OF n! BY 9's AND SUM THE VALUE OF EXPONENT OVERFLOW IN f								
4	ENTER EXP	26									
5	CHG SIGN	32									
6	9	11									
7	×	36									
8	9	11									
9	ACC +	60									
a	GO TO ( ) ( )	44									
b	2	02	BRANCH TO COMPARE n! TO 10^9								
c	d	17									
d	1	01									
4 0	0	00									
1	IF x > y	53	BRANCH IF ARGUMENT (n!) < 10^{12} TO RECALL RESULTANT EXPONENT AND DISPLAY								
2	4	04									
3	c	16									
4	.	21									
5	1	01									
6	×	36	REDUCE EXPONENT OF n! BY 1's AND SUM THE VALUE OF EXPONENT OVERFLOW IN f								
7	1	01									
8	ACC +	60									
9	GO TO ( ) ( )	44									
a	3	03	BRANCH TO COMPARE n! TO 10^9								
b	d	17									
c	f	15									
d	x ⇄ y	30	RECALL n AND EXPONENT OF n!								
5 0	↑	27									
1	d	17									
2	END	46	n	DEC. NO. (n!)	EXP. OF 10	DISPLAY					
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70021

POLYNOMIAL EVALUATION ($0 \leq n \leq 10$)

This program will evaluate a polynomial of the form:

$$f(x) = A_n x^n + A_{n-1} x^{n-1} \ldots A_1 x + A_0$$

$$\text{for} \quad 1 \leq n \leq 10$$

Input data is a_n, $A_{n-1} \ldots A_0$, and the value of x for which the polynomial is to be evaluated. Any coefficient (A_i) may be zero. If any term(s) of the polynomial is absent, then the coefficient of that term(s) must be entered as zero.

The value of f(x) for a new x may be obtained by entering the new x after the last f(x) has been calculated. To enter new coefficients for a different f(x), the program card must be re-entered and the user instructions repeated. A_i's and x's must be real numbers.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) or [END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $A_{10} \rightarrow Z$, $A_9 \rightarrow Y$, $A_8 \rightarrow X$

ENTER 0 if term does not exist.

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $A_7 \rightarrow Z$, $A_6 \rightarrow Y$, $A_5 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $A_4 \rightarrow Z$, $A_3 \rightarrow Y$, $A_2 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $A_1 \rightarrow Y$, $A_o \rightarrow X$

PRESS: CONTINUE

0	Z
0	Y
2	X

USER INSTRUCTIONS

ENTER DATA: $x \rightarrow X$

PRESS: CONTINUE

DISPLAY

x	Z
f(x)	Y
0	X

Wish to enter a new x ?

YES (return to ENTER DATA) / NO (return to ENTER PROGRAM)

EXAMPLES

(A) Polynomial: $x + 10$ ($A_1 = 1$, $A_0 = 10$)

x	f(x)
1	11
2	12
3	13

(B) Polynomial: $x^9 - 2x^8 + 4x^7 + x^2 + x + 100$

($A_9=1$, $A_8=-2$, $A_7=4$, $A_2=1$, $A_1=1$, $A_0=100$)

x	f(x)
1	105
2	618
.1	100.11000

General Form:

$f(x) = A_n x^n + A_{n-1} x^{n-1} \ldots A_1 x + A_0 \quad 1 \leq n \leq 10$

PART NO 09100-70021

Step	Key	Code	Display x	Display y	Display z	Storage / Notes
00	CLEAR	20				
01	1	01				
02	STOP	41	A_8	A_9	A_{10}	ENTER DATA
03	x→( )	23				
04	a	13				
05	y→( )	40				
06	8	10				STORE A_{10}, A_9, AND A_8 (steps 03–0a)
07	ROLL ↓	31				
08	y→( )	40				
09	9	11				
0a	CLEAR	20				
0b	STOP	41	A_5	A_6	A_7	ENTER DATA
0c	x→( )	23				
0d	b	14				
10	y→( )	40				
11	c	16				STORE A_7, A_6, AND A_5 (steps 0c–14)
12	ROLL ↓	31				
13	y→( )	40				
14	d	17				
15	CLEAR	20				
16	STOP	41	A_2	A_3	A_4	ENTER DATA
17	x→( )	23				
18	0	00				
19	ROLL ↓	31				
1a	x→( )	23				STORE A_4, A_3, A_2 (steps 17–1d)
1b	7	07				
1c	y→( )	40				
1d	1	01				
20	CLEAR	20				
21	STOP	41	A_0	A_1	0	ENTER DATA
22	ACC +	60				STORE A_0, AND A_1
23	↓	25				
24	2	02				
25	STOP	41	X 0	0 f(X)	0 X	ENTER DATA AND DISPLAY (FROM-6-C)
26	y⇄( )	24				
27	9	11				
28	y→( )	40				
29	9	11				
2a	×	36				
2b	ROLL ↓	31				
2c	y⇄( )	24				
2d	8	10				

 PART NO 09100-70021

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	y→( )	40									
	1	8	10									
	2	+	33									
	3	↓	25									
	4	×	36									
	5	a	13									
	6	+	33									
	7	↓	25									
	8	×	36									
	9	d	17									
	a	+	33									
	b	↓	25									
	c	×	36									
	d	c	16									
4	0	+	33									
	1	↓	25									
	2	×	36									
	3	b	14									
	4	+	33									
	5	↓	25									
	6	×	36	CALCULATE f (x)								
	7	↓	25									
	8	y⇄( )	24									
	9	1	01									
	a	y→( )	40									
	b	1	01									
	c	+	33									
	d	↓	25									
5	0	×	36									
	1	↓	25									
	2	y⇄( )	24									
	3	7	07									
	4	y→( )	40									
	5	7	07									
	6	+	33									
	7	↓	25									
	8	×	36									
	9	↓	25									
	a	y⇄( )	24									
	b	0	00									
	c	y→( )	40									
	d	0	00									

Step	Key	Code	Display x	y	z	Storage f	e	d	c	b	a
6 0	+	33									
1	↓	25									
2	×	36									
3	e	12									
4	+	33									
5	↓	25									
6	×	36									
7	f	15									
8	+	33									
9	CLEAR x	37									
a	GO TO ( ) ( )	44	BRANCH TO DISPLAY f(x) AND ENTER A NEW x IF DESIRED								
b	2	02									
c	5	05									
d	END	46									

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

PART NO.
09100-70022

3x3 MATRIX INVERSION OR SIMULTANEOUS SOLUTION OF THREE EQUATIONS IN THREE UNKNOWNS

This program inverts a 3x3 matrix of the form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

and calculates:

$$A^{-1} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

where

$$A \cdot A^{-1} = I$$

where

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

or solves three independent equations of the form:

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = K_1$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 = K_2$$

$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = K_3$$

for x_1, x_2, and x_3.

Reference: Elementary Matrix Algebra
Franz E. Hohn
1958

USER INSTRUCTIONS

This program is a destructive program

ENTER PROGRAM: (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0 ——	Z
0 ——	Y
0 ——	X

ENTER DATA*:

a_{11} → Z
a_{21} → Y
a_{31} → X

PRESS: CONTINUE

DISPLAY

0 ——	Z
0 ——	Y
0 ——	X

ENTER DATA:

a_{12} → Z
a_{22} → Y
a_{32} → X

PRESS: CONTINUE

DISPLAY

0 ——	Z
0 ——	Y
0 ——	X

*If the program for either mode should turn on the RED light in the RUN mode, interchange the second and third rows of the matrix or the second and third equations of the set and run the program again. If the RED light still comes on, the equations are dependent or the matrix is singular and a solution cannot be obtained. If the matrix or equation set is ill-conditioned, an answer will be obtained but the accuracy of the solution will suffer.

USER INSTRUCTIONS (con't)

ENTER DATA:

a_{13} → Z
a_{23} → Y
a_{33} → X

Wish to solve three equations simultaneously?

NO → [Wish to invert the input matrix]

YES ↓

PRESS: CONTINUE

DISPLAY

b_{11} ——	Z
b_{21} ——	Y
b_{31} ——	X

PRESS: CONTINUE

DISPLAY

b_{12} ——	Z
b_{22} ——	Y
b_{32} ——	X

PRESS: CONTINUE

DISPLAY

b_{13} ——	Z
b_{23} ——	Y
b_{33} ——	X

Desire check?

NO ↓ (to end)

YES ↓

ENTER DATA:

b_{11} → Z
0 → Y
0 → X

PRESS: CONTINUE

DISPLAY

a_{11} ——	Z
a_{21} ——	Y
a_{31} ——	X

(Wish to solve three equations simultaneously?) YES ↓

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

——	Z
——	Y
1 ——	X

ENTER DATA:
(Do not disturb Y and Z registers when K_i's (i = 1, 2, 3) are being entered) K_1 → X

PRESS: CONTINUE

DISPLAY

——	Z
——	Y
0 ——	X

ENTER DATA: K_3 → X

PRESS: CONTINUE

DISPLAY

——	Z
——	Y
0 ——	X

ENTER DATA: K_2 → X

PRESS: CONTINUE

USER INSTRUCTIONS (con't)

PRESS: CONTINUE

DISPLAY

a_{21} — Z
a_{22} — Y
a_{32} — X

PRESS: CONTINUE

DISPLAY

a_{31} — Z
a_{32} — Y
a_{33} — X

TO RESET PROBLEM:
re-enter program and repeat USER INSTRUCTIONS.

DISPLAY

x_1 — Z
x_2 — Y
x_3 — X

TO RESET PROBLEM:
re-enter program and repeat USER INSTRUCTIONS.

EXAMPLES

(A) Find the inverse of the following matrix (A).
General form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & 4 & 2 \\ 2 & 8 & 4 \\ 1 & 2 & 8 \end{bmatrix}$$

Solution:

$$A^{-1} = \begin{bmatrix} .1429 & -.0714 & 0 \\ -.0306 & .1582 & -.0714 \\ -.0102 & -.0306 & .1429 \end{bmatrix}$$

(B) Solve the following three equations simultaneously.
General form:

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = K_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = K_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = K_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 + 2x_3 = 3 \\ 4x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 6x_3 = 4 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} x_1 = -3 \\ x_2 = 4 \\ x_3 = 1 \end{bmatrix}$$

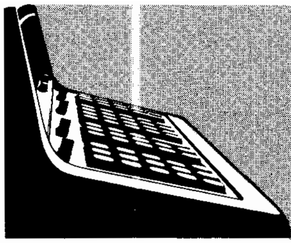
STORAGE ALLOCATION OF THE $\{aij\}$ or $\{bij\}$ IS AS FOLLOWS:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ OR } \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

	Step	Key	Code				Remarks
3	1	1	01				
	2	IF FLAG	43				
	3	STOP	41	K_1	0	0	ENTER K_1
	4	SET FLAG	54	CALCULATE K_1/a_{11}			
	5	$x \rightleftarrows y$	30				
	6	÷	35				
	7	$y \rightleftarrows ()$	24				
	8	d	17				
	9	$y \rightleftarrows ()$	24	REPOSITION COUNTER			
	a	0	00				
	b	$y \rightleftarrows ()$	24				
	c	a	13				
	d	c	16				
4	0	$x \rightleftarrows y$	30	CALCULATE AND STORE $a_{33} - \frac{a_{31}a_{12}}{a_{11}}$			
	1	×	36				
	2	ROLL ↓	31				
	3	−	34				
	4	$y \rightleftarrows ()$	24				
	5	b	14				
	6	$y \rightleftarrows ()$	24	CALCULATE AND STORE $a_{33} - \frac{a_{31}a_{13}}{a_{11}}$			
	7	0	00				
	8	f	15				
	9	ROLL ↑	22				
	a	×	36				
	b	ROLL ↓	31				
	c	−	34				
	d	$y \rightleftarrows ()$	24				
5	0	e	12				
	1	d	17	CALCULATE AND STORE $K_3 - \frac{a_{31}K_1}{a_{11}}$			
	2	ROLL ↑	22				
	3	CHG SIGN	32				
	4	×	36				
	5	CLEAR x	37				
	6	IF FLAG	43				
	7	STOP	41				
	8	SET FLAG	54				
	9	+	33				
	a	$y \rightleftarrows ()$	24				
	b	1	01				
	c	$y \rightleftarrows ()$	24				
	d	0	00				

Step		Key	Code	Display x	Display y	Display z	Remarks / Storage
6	0	c	16				
	1	x⇄y	30	CALCULATE $a_{22} - \frac{a_{21}a_{12}}{a_{11}}$			
	2	×	36				
	3	ROLL ↓	31				
	4	−	34				
	5	f	15				
	6	ROLL ↑	22				
	7	×	36				
	8	x⇄y	30				
	9	y⇄( )	24	CALCULATE AND STORE $a_{23} - \frac{a_{21}a_{13}}{a_{11}}$			
	a	0	00				
	b	−	34				
	c	y⇄( )	24				
	d	0	00				
7	0	d	17				
	1	CHG SIGN	32				
	2	×	36	CALCULATE $K_2 - \frac{a_{21}K_1}{a_{11}}$			
	3	CLEAR x	37				
	4	IF FLAG	43				
	5	CLEAR x	37				
	6	STOP	41	K_2	0	0	ENTER K_2
	7	+	33				
	8	y⇄( )	24				
	9	a	13	INCREMENT COUNTER			
	a	1	01				
	b	+	33				
	c	3	03				
	d	IF x > y	53	BRANCH WHEN 3 IS GREATER THAN			
8	0	2	02	COUNTER AND REPEAT CALCULATION			
	1	2	02				
	2	b	14				
	3	x⇄y	30	RECALL AND POSITION SOLUTION OR			b_{11}, b_{21}, b_{31}
	4	c	16				
	5	STOP	41	b_{31}	b_{21}	b_{11}	DISPLAY SOLUTION OR b_{11}, b_{21}, b_{31}
	6	y⇄( )	24				
	7	0	00				
	8	y→( )	40				
	9	0	00	RECALL AND POSITION DISPLAY			
	a	e	12				
	b	↑	27				
	c	f	15				
	d	STOP	41	b_{32}	b_{22}	b_{12}	DISPLAY

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9	0	y⇄( )	24	RECALL AND POSITION DISPLAY								
	1	1	01									
	2	y→( )	40									
	3	1	01									
	4	d	17									
	5	↑	27									
	6	a	13									
	7	ROLL ↓	31									
	8	STOP	41	b_{33}	b_{23}	b_{13}	DISPLAY					
	9	GOTO( )( )	44	BRANCH TO DISPLAY COEFFICIENTS OF ORIGINAL MATRIX IF CHECK IS DESIRED								
	a	2	02									
	b	2	02									
	c	END	46									
	d											



PART NO.
09100-70023

FOURIER SERIES
when f(t) is known

This program calculates the Fourier Series coefficients a_n and b_n that represent a periodic time function f(t) with period T. The Fourier Series representation of f(t) is:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi nt}{T} + b_n \sin\frac{2\pi nt}{T}$$

where

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos\frac{2\pi nt}{T}\,dt \qquad n = 0, 1, 2, \ldots.$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin\frac{2\pi nt}{T}\,dt \qquad n = 1, 2, 3, \ldots.$$

The program evaluates the coefficients by numerically integrating using Simpson's Rule which is the same procedure used in program 09100-70015.

The specific f(t) is programmed into the Calculator by the user and is then used by the general solution to evaluate the coefficients. Execution time is dependent on the number of panels.

Reference: Signals Systems and Communications
B. P. Lathi
Wiley (1965)

USER INSTRUCTIONS

SET: | RADIANS | |

ENTER PROGRAM: (Starting Address is 0 - 0)

PRESS: GO TO ( ) ( )

PRESS: 6

PRESS: 4

SET: | PROGRAM | |

ENTER: Steps to take t from the X register (t is also stored in the e register) and form f(t). Leave f(t) in the Y register and exit to location 8-4. Steps 6-4 through 8-3 are available for forming f(t). The FLAG is not available.

SET: | | RUN |

PRESS: GO TO ( ) ( )

PRESS: 9

PRESS: 9

SET: | PROGRAM | |

PRESS: sin X or cos X depending on desired sine or cosine coefficients. If series involves both sine and cosine, then initialize program to cos X.

SET: | | RUN |

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
1	———	X

(n is set to 0)

ENTER DATA:

Number of panels must be an even number → Z

$t_0 + T$ ——→ Y

t_0 ——→ X

Note: n is contained in the a register.

→PRESS: CONTINUE

DISPLAY

$t_0 + T$	———	Z
a_n or b_n	———	Y
n	———	X

To change from cosine series to sine series--

USER INSTRUCTIONS (con't)

PRESS: GO TO ( ) ( )

PRESS: 9

PRESS: 9

SET: | PROGRAM | |

PRESS: sin X

SET: | | RUN |

PRESS: CLEAR

PRESS: X →( )

PRESS: a

(n is set to 1)

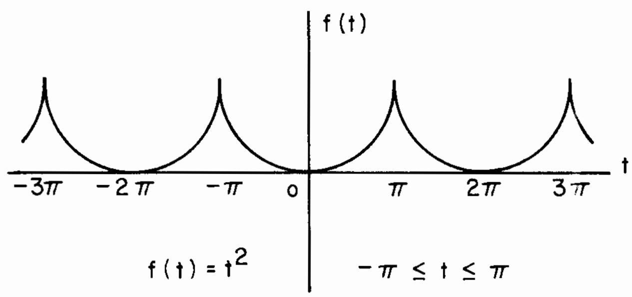
PRESS: GO TO ( ) ( )

PRESS: 5

PRESS: 7

EXAMPLES

Example #1



$$t^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$$

Data: Number of panels = 100

$T = 2\pi$

$t_0 = -\pi$

Solution:

$t_0 + T = 3.14159$

$a_0 = 6.57974$

$a_1 = -4.00000$

$a_2 = 1.00000$

$a_3 = -.44444$

$a_4 = .24998$

$a_5 = -.15997$

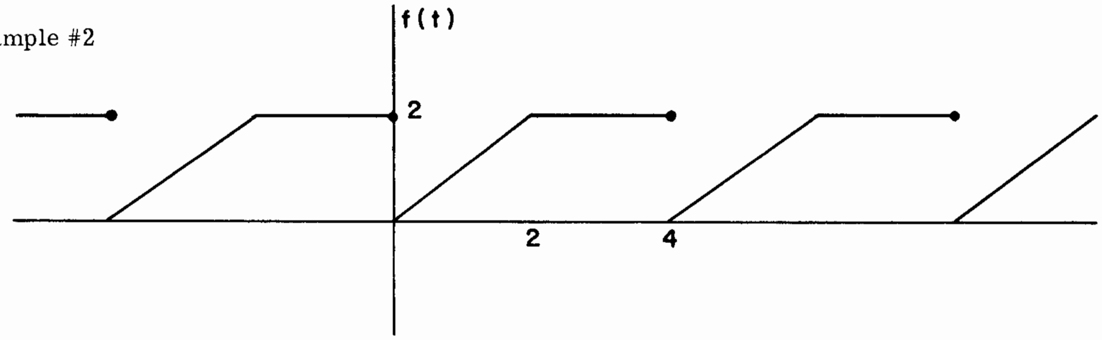
$a_6 = .11107$

$a_7 = -.08158$

$a_8 = .06243$

$b_n = 0$; $f(t) = f(-t)$

Example #2



$f(t) = t \qquad 0 < t \leq 2$

$f(t) = 2 \qquad 2 \leq t \leq 4$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T}\right)$$

Data:

$T = 4$

$t_0 = 0$

Number of panels = 50

Solution:

$t_0 + T = 4.0$

$a_0 = 3.00053$	$b_1 = -.63662$
$a_1 = -.40582$	$b_2 = -.31832$
$a_2 = .00054$	$b_3 = -.21223$
$a_3 = -.04557$	$b_4 = -.15921$
$a_4 = .00057$	$b_5 = -.12744$
$a_5 = -.01676$	$b_6 = -.10631$
$a_6 = .00062$	$b_7 = -.09128$
$a_7 = -.00883$	$b_8 = -.08009$
$a_8 = .00069$	

Note: See step page 1 for steps needed to execute examples 1 and 2.

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6 0				*EXAMPLE 1*							
1											
2											
3			t				t				
4	↑	27	t	t							
5	×	36		$t^2 = f(t)$							
6	GO TO ( )( )	44									
7	8	10									
8	4	04									
9											
a											
b											
c											
d											
6 0				*EXAMPLE 2*							
1											
2											
3			t				t				
4	↑	27	t	t							
5	2	02	2								
6	IF $x > y$	53									
7	8	10									
8	4	04									
9	$x \rightleftarrows y$	30	t	2							
a	GO TO ( )( )	44									
b	8	10									
c	4	04									
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

Step		Key	Code	Display x	Display y	Display z	Remarks / Storage
0	0	CLEAR	20				
	1	x→( )	23				INITIALIZE n TO ZERO
	2	a	13				
	3	1	01				FROM 4-3 & 4-9
	4	STOP	41	t_0	$t_0 + T$	PANELS	ENTER DATA: NUMBER OF PANELS (MUST BE EVEN), t_0+T, t_0
	5	y→( )	40				STORE DATA
	6	d	17				
	7	x→( )	23				
	8	b	14				
	9	−	34				CALCULATE Δt
	a	ROLL ↓	31				
	b	x⇄y	30				
	c	÷	35				
	d	y→( )	40				
1	0	c	16				
	1	ROLL ↑	22				STORE t_0
	2	x→( )	23				FROM 4-9
	3	e	12				FROM 6-3
	4	GO TO ( )( )	44				BRANCH TO EVAULATE FUNCTION f(t)
	5	6	06				
	6	4	04				
	7	e	12				FROM 9-d
	8	↑	27				
	9	b	14				RECALL t_0 AND BRANCH FOR INITIAL POINT
	a	IF x = y	50				
	b	4	04				
	c	4	04				
	d	d	17				COMPARE $\lvert t-(T+t_0)\rvert$ TO 10^{-9} AND STOP INTEGRATION IF $\lvert t-(T+t_0)\rvert < 10^{-9}$
2	0	−	34				
	1	|y|	55				
	2	ENTER EXP	26				
	3	9	11				
	4	CHG SIGN	32				
	5	IF x > y	53				
	6	4	04				
	7	a	13				
	8	e	12				COMPARE t TO $T+t_0$ AND STOP INTEGRATION IF $t > T + t_0$
	9	x⇄y	30				
	a	d	17				
	b	IF x < y	52				
	c	4	04				
	d	a	13				

Storage: f, e, d, c, b, a

Step		Key	Code	Display / Remarks	Notes
3	0	c	16		
	1	ROLL ↑	22		
	2	↑	27		
	3	IF FLAG	43		
	4	3	03		
	5	b	14		
	6	SET FLAG	54		
	7	4	04	CALCULATE $4g(t_0+\Delta t)+2g(t_0+2\Delta t)$	
	8	GO TO ( )( )	44		
	9	3	03		
	a	c	16		
	b	2	02		
	c	×	36		
	d	↓	25		
4	0	ACC +	60		
	1	GO TO ( )( )	44		
	2	1	01	BRANCH TO CALCULATE ANOTHER PARTIAL AREA	
	3	3	03		FROM 1-c
	4	c	16	RECALL Δt AND CALCULATE $t+\Delta t$.	
	5	ROLL ↑	22	PLACE $t+\Delta t$ AND $g(t)$ IN THE ACCUMULATORS	
	6	ACC +	60		
	7	GO TO ( )( )	44		
	8	1	01	BRANCH FOR OTHER PANEL CALCULATION	
	9	3	03		FROM 2-7 & 2-d
	a	CLEAR x	37		
	b	ROLL ↑	22	SUM $g(t_0+T)$ TO $g(t_0)+4g(t_0+\Delta t)+2g(t_0+2\Delta t)+\cdots$	
	c	ACC +	60		
	d	RCL	61		
5	0	↑	27		
	1	c	16		
	2	×	36	MULTIPLY $\Delta t/3$ TIMES SUMMATION SERIES	
	3	3	03		
	4	÷	35		
	5	a	13	RECALL n, THE HARMONIC NUMBER	
	6	STOP	41	DISPLAY: x = n; y = a_n or b_n; z = t_0+T	
	7	CLEAR	20		
	8	a	13		
	9	↑	27		
	a	1	01	INCREMENT n	
	b	+	33		
	c	y→( )	40		
	d	a	13		

Step		Key	Code	Display x	y	z	Storage f	e	d	c	b	a
6	0	b	14	RECALL t_0								
	1	GO TO ( ) ( )	44	BRANCH TO CALCULATE COEFFICIENT								
	2	1	01									
	3	2	02									
	4	CONTINUE	47	STEPS 6-4 THROUGH 8-3 ARE AVAILABLE FOR FORMING f(t)								
	5											
	6			*WHEN ENTERING THE PROGRAM FILL THIS AREA WITH CONTINUES TO AVOID THE POSSIBILITY OF AN END STATEMENT AND THUS RECORDING AND ENTERING PROBLEMS.*								
	7											
	8											
	9											
	a											
	b											
	c											
	d											
7	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
8	0	CONTINUE	47									
	1											
	2											
	3											
	4	2	02	CALCULATE $\frac{2f(t)}{T}$								
	5	×	36									
	6	d	17									
	7	↑	27									
	8	b	14									
	9	−	34									
	a	π	56									
	b	ROLL ↓	31									
	c	÷	35									
	d	ROLL ↑	22									

FROM 1-6 (→ step 6-4)

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9	0	$x \rightleftarrows y$	30									
	1	÷	35									
	2	2	02									
	3	×	36									
	4	e	12									
	5	×	36									
	6	a	13									
	7	×	36									
	8	↓	25									
	9	cos x	73									
	a	×	36									
	b	GO TO ( )( )	44									
	c	1	01									
	d	7	07									
a	0	END	46									

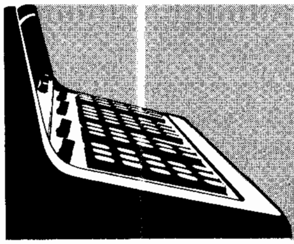
Steps 9-0 to 9-8: CALCULATE $\frac{2f(t)}{T}\cos\frac{2\pi nt}{T}$ OR $\frac{2f(t)}{T}\sin\frac{2\pi nt}{T} = g(t)$

Steps 9-9 to 9-a: AS ENTERED IN THIS STEP: COS CALCULATES a_n
SIN CALCULATES b_n

Steps 9-b to 9-d: BRANCH TO INTEGRATE AND OBTAIN COEFFICIENTS

AFTER EXECUTION DATA IS STORED IN THIS REGISTER WHICH DESTROYS THE END STATEMENT, THEREFORE, RECORD PROGRAM ON A MAGNETIC CARD BEFORE EXECUTION.

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70024

GAMMA FUNCTION

These program evaluate the Gamma Function,

where

$$\Gamma(\nu) = \int^{\infty} e^{-t} \; t^{\nu-1} \, dt$$

Program 1 evaluates $\Gamma(\nu)$ over the range $0 < \nu \leq 69.98$

Program 2 evaluates $\Gamma(\nu)$ over the range $0 < \nu \leq 10^9$

Reference: Handbook of Mathematical Functions
National Bureau of Standards
Abramowitz & Stegan
Sixth Printing, 1967

USER INSTRUCTIONS

PROGRAM #1 $0 < \nu \leq 69.95$

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0 ——	Z
0 ——	Y
0 ——	X

ENTER DATA: $\nu \rightarrow X$

PRESS: CONTINUE

DISPLAY

0 ——	Z
$\Gamma(\nu)$ ——	Y
ν ——	X

To enter new value for ν

PROGRAM #2 $0 < \nu \leq 10^9$

ENTER PROGRAM: (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0 ——	Z
0 ——	Y
0 ——	X

ENTER DATA: $\nu \rightarrow X$

PRESS: CONTINUE

DISPLAY

exponent of 10 ——	Z
decimal number —	Y
ν ——	X

To enter a new value for ν

EXAMPLE

PROGRAM #1

$\nu = 1.750$
$\Gamma(\nu) = 0.919062527$

$\nu = 51$
$\Gamma(\nu) = 3.041409320 \times 10^{64}$

PROGRAM #2

$\nu = 1.750$
$\Gamma(\nu) = 0.919062527$

$\nu = 97$
$\Gamma(\nu) = 9.916779322 \times 10^{149}$

PROGRAM 1 PAGE 1 PART NO 09100-

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	ν	0	0	ENTER $\nu \to x$					
	2	↑	27									
	3	1	01									
	4	−	34									
	5	y→( )	40									
	6	d	17									
	7	↑	27									
	8	↓	25									
	9	int x	64									
	a	−	34									
	b	CLEAR x	37									
	c	IF x > y	53									
	d	CLEAR x	37									
1	0	1	01									
	1	+	33									
	2	5	05									
	3	+	33									
	4	↓	25									
	5	↑	27									
	6	×	36									
	7	ACC +	60									
	8	2	02									
	9	x⇄y	30									
	a	÷	35									
	b	3	03									
	c	−	34									
	d	e	12									
2	0	÷	35									
	1	2	02									
	2	÷	35									
	3	+	33									
	4	e	12									
	5	÷	35									
	6	7	07									
	7	÷	35									
	8	1	01									
	9	−	34									
	a	e	12									
	b	÷	35									
	c	6	06									
	d	÷	35									

Steps 0-6 to 1-7: REPLACE ν WITH $\bar{\nu}$ WHEN $5 \le \bar{\nu} < 6$

Steps 1-8 onward: CALCULATE

$$P(\nu) = -\nu + \frac{1}{12\nu} - \frac{1}{360\nu^3} + \frac{1}{1260\nu^5} - \frac{1}{1680\nu^7} + \frac{1}{2520\nu^9}$$

$$= -\nu \ \frac{1}{60\nu}\left(5 + \frac{1}{6\nu^2}\left(-1 + \frac{1}{7\nu^2}\left(2 + \frac{1}{2\nu^2}\left(-3 + \frac{2}{\nu^2}\right)\right)\right)\right)$$

Step		Key	Code	Display x	y	z	Storage f	e	d	c	b	a
3	0	5	05									
	1	+	33									
	2	6	06									
	3	0	00									
	4	÷	35									
	5	f	15									
	6	÷	35									
	7	−	34									
	8	↑	27									
	9	.	21									
	a	5	05	CALCULATE $(\nu+\frac{1}{2})\ \ln \nu$								
	b	+	33									
	c	f	15									
	d	ln x	65									
4	0	×	36									
	1	↓	25									
	2	+	33									
	3	π	56									
	4	↑	27									
	5	+	33	CALCULATE $\ln\sqrt{2\pi}$								
	6	↓	25									
	7	ln x	65									
	8	↑	27									
	9	2	02									
	a	÷	35									
	b	↓	25									
	c	+	33									
	d	↓	25									
5	0	e^x	74									
	1	↑	27									
	2	f	15									
FROM 6-6 →	3	↑	27									
FROM 7-0 →	4	d	17									
	5	IF x > y	53									
	6	6	06									
	7	0	00									
	8	IF x < y	52									
	9	6	06									
	a	7	07									
	b	GO TO ( )( )	44									
	c	7	07	BRANCH TO DISPLAY								
	d	1	01									

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a	Notes
6-0 (FROM 5-7)	1	01										CALCULATE $\nu!$ FROM $\bar{\nu}!$
1	+	33										
2	↓	25										
3	×	36										
4	GO TO ( )( )	44										
5	5	05										
6	3	03										
7 (FROM 5-a)	↓	25										
8	÷	35										
9	↑	27										
a	1	01										
b	−	34										
c	GO TO ( )( )	44										
d	5	05										
7-0	4	04										
1 (FROM 5-d)	1	01										RECALL AND POSITION DISPLAY
2	$x \rightleftarrows y$	30										
3	d	17										
4	+	33										
5	CLEAR x	37										
6	ROLL ↓	21 22										
7	END	46	ν	$\Gamma(\nu)$	0							DISPLAY
8												
9												
a												
b												
c												
d												

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0 0	CLEAR	20									
1	STOP	41	ν	0	0	ENTER $\nu \rightarrow$ X					
2	↑	27									
3	1	01									
4	−	34									
5	y→( )	40									
6	d	17									
7	1	01									
8	7	07									
9	IF $x<y$	52									
a	3	03	OMIT LAST TERMS OF P(ν) IF $\nu > 17$								
b	4	04									
c	d	17									
d	int x	64									
1 0	IF $x=y$	50	SET FLAG IF ν IS AN INTEGER								
1	SET FLAG	54									
2	SET FLAG	54									
3	−	34									
4	CLEAR x	37									
5	IF $x>y$	53									
6	CLEAR x	37	REPLACE ν WITH $\bar{\nu}$ IF $\nu \leq 17$								
7	1	01									
8	+	33									
9	5	05									
a	+	33									
b	↓	25									
c	↑	27									
d	×	36									
2 0	ACC +	60									
1	2	02									
2	$x \rightleftarrows y$	30									
3	÷	35									
4	3	03	BEGIN CALCULATION OF P(ν)								
5	−	34									
6	e	12									
7	÷	35									
8	2	02									
9	÷	35									
a	+	33									
b	e	12									
c	÷	35									
d	7	07									

FROM 0-b

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	÷	35									
1	GO TO( )( )	44									
2	3	03									
3	9	11									
4	d	17									
5	×	36									
6	ACC +	60	PUT ν, ν^2 IN f, e IF $\nu > 17$								
7	CLEAR x	37									
8	$x \rightleftarrows y$	30									
9	1	01									
a	−	34									
b	e	12									
c	÷	35									
d	6	06									
4 0	÷	35									
1	5	05									
2	+	33	FINISH CALCULATION OF P (ν)								
3	6	06									
4	0	00									
5	÷	35									
6	f	15									
7	÷	35									
8	−	34									
9	↑	27									
a	·	21									
b	5	05	CALCULATE $(\nu + \frac{1}{2}) \ln \nu$								
c	+	33									
d	f	15									
5 0	ln x	65									
1	×	36									
2	↓	25									
3	+	33									
4	π	56									
5	↑	27									
6	+	33									
7	↓	25	CALCULATE $\ln\sqrt{2\pi}$								
8	ln x	65									
9	↑	27									
a	2	02									
b	÷	35									
c	CLEAR x	37									
d	ROLL ↓	31									

Step	Key	Code	Display x	y	z	Storage f	e	d	c	b	a
6 0	+	33									
1	3	03									
2	4	04									
3	IF x<y	52									
4	a	13									
5	4	04									
6	↓	25									
7	e^x	74									
8	↑	27									
9	f	15									
a	↑	27									
b	d	17									
c	IF x>y	53									
d	7	07									
7 0	7	07									
1	IF x<y	52									
2	8	10									
3	0	00									
4	GO TO ( )( )	44									
5	8	10									
6	8	10									
7	1	01									
8	+	33									
9	↓	25									
a	×	36									
b	GO TO ( )( )	44									
c	6	06									
d	a	13									
8 0	↓	25									
1	÷	35									
2	↑	27									
3	1	01									
4	−	34									
5	GO TO ( )( )	44									
6	6	06									
7	b	14									
8	CLEAR x	37									
9	ROLL ↑	22									
a	IF FLAG	43									
b	int x	64									
c	int x	64									
d	↑	27									

Steps 6-0 to 6-5: DON'T GO THRU. STEPS TO CONVERT $\bar{\nu}$/ TO ν/ IF ν>17 AND DON'T TAKE e^x OF ln ν WITHOUT REMOVING CHARACTERISTIC FIRST

Steps 7-4 to 7-6: BRANCH TO REMOVE INSIGNIFICANT ZEROS

Steps 6-c to 8-7: CALCULATE ν/ FROM $\bar{\nu}$/

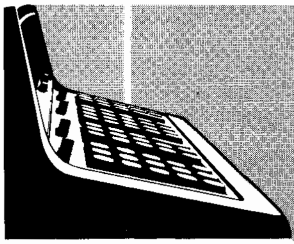
Steps 8-8 to 8-c: REMOVE INSIGNIFICANT ZEROS

Step 8-d: FROM a-3

PROGRAM 2 PAGE 7 PART NO 09100-70024

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9 0	1	01									
1	0	00									
2	IF $x > y$	53									
3	*b*	14									
4	4	04									
5	.	21									
6	1	01	CALCULATE P_{INT}								
7	×	36									
8	1	01									
9	ROLL ↑	22									
a	$x \rightleftarrows y$	30									
b	+	33									
c	$x \rightleftarrows y$	30									
d	ROLL ↓	31									
a 0	ROLL ↓	31									
1	GO TO ( )( )	44									
2	8	10									
3	*d*	17									
4 (FROM 6–5)	1	01									
5	0	00									
6	ln *x*	65									
7	÷	35									
8	$x \rightleftarrows y$	30									
9	↑	27	CONVERT ln ν! TO LOG ν!, REMOVE CHARACTERISTIC, CONVERT DECIMAL NUMBER TO ln AND TAKE EXPONENT OF 10								
a	int *x*	64									
b	−	34									
c	$x \rightleftarrows y$	30									
d	ROLL ↑	22									
b 0	×	36									
1	↓	25									
2	e^x	74									
3	↑	27									
4	$y \rightleftarrows (\,)$	24									
5	*d*	17									
6	1	01									
7	+	33	RECALL AND POSITION DISPLAY								
8	$x \rightleftarrows y$	30									
9	$y \rightleftarrows (\,)$	24									
a	*d*	17									
b	END	46	ν	DECIMAL NO.	EXP. OF 10	DISPLAY					
c											
d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



BESSEL FUNCTION

The differential equation:

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2) y = 0$$

is called Bessel's equation of order n. When n is an integer the Bessel function of the first kind of order n, $J_n(x)$, is a solution where:

$$J_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{-x^2}{4}\right)^k}{k!\ (n+k)!} \qquad x \geq 0$$

The computation is performed as follows:

$$J_n(x) = \frac{\left(\frac{x}{2}\right)^n}{n!} \left[1 - \frac{\left(\frac{x^2}{4}\right)}{1(n+1)} \cdot \left\{ 1 - \frac{\left(\frac{x^2}{4}\right)}{2(n+2)} \right\} \cdot \left\{ 1 - \frac{\left(\frac{x^2}{4}\right)}{3(n+3)} \right\} \cdot \left\{ 1 - \ldots \right\} \right]$$

Reference:

Handbook of Mathematical Functions
M. Abramowitz & I. A. Stegum
National Bureau of Standards
Sixth Printing, 1967

USER INSTRUCTIONS	EXAMPLE

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: n ⟶ Y , x ⟶ X

PRESS: CONTINUE

DISPLAY

$J_n(x)$	———	Z
n	———	Y
x	———	X

To enter new value for n and x (return to PRESS: CONTINUE)

EXAMPLE

(A)

$x = 5$
$n = 9$
$J_n(x) = 5.520283139 \times 10^{-3}$

(B)

$x = 24$
$n = 46$
$J_n(x) = 3.347208960 \times 10^{-10}$

Step		Key	Code	Display x	Display y	Display z	Comments
0	0	CLEAR	20				
	1	STOP	41	X	n	0	ENTER DATA
	2	x→( )	23				STORE DATA
	3	9	11				
	4	x→( )	23				
	5	c	16				
	6	y→( )	40				
	7	d	17				
	8	CLEAR	20				CALCULATE $\Delta = 1+9X^{\frac{1}{3}} + 3X^{\frac{1}{12}}$ TO BE ADDED TO LARGER OF n OR X TO FIND M
	9	3	03				
	a	↑	27				
	b	c	16				
	c	$\sqrt{x}$	76				
	d	$\sqrt{x}$	76				
1	0	÷	35				
	1	9	11				
	2	+	33				
	3	c	16				
	4	ln x	65				
	5	↑	27				
	6	3	03				
	7	÷	35				
	8	↓	25				
	9	e^x	74				
	a	+	33				
	b	1	01				
	c	+	33				
	d	2	02				CALCULATE 2/X AND REPLACE X IN C
2	0	↑	27				
	1	c	16				
	2	÷	35				
	3	y⇄( )	24				
	4	c	16				
	5	d	17				OMIT TEST IF $X \geq n$
	6	IF $x<y$	52				
	7	4	04				
	8	3	03				
	9	÷	35				
	a	2	02				
	b	÷	35				
	c	↓	25				
	d	↑	27				

Step	Key	Code	Notes
3 0	×	36	
1	ln x	65	
2	−	34	
3	d	17	CALCULATE $q = X^2/4n + (n+1/2)\cdot\ln(n) - n\cdot\ln(X/2) - n$
4	×	36	EXPRESSED AS $q = n\left[(X/2n)^2 - \ln(X/2n)\right]$
5	−	34	$-n + \ln(\sqrt{n})$ AND COMPARE WITH $99\ln(10)$
6	√x	76	$-\ln(\sqrt{2\pi})$, EXIT AND DISPLAY ZERO FOR
7	ln x	65	$J_n(X)$ IF LATTER IS LESS
8	+	33	
9	2	02	
a	2	02	
b	7	07	
c	IF x < y	52	
d	7	07	
4 0	d	17	
1	d	17	
2	x⇄y	30	ADD GREATER OF n OR X TO Δ
3	CLEAR x	37	
4	ROLL ↓	31	
5	+	33	
6	2	02	FORCE m TO BE EVEN
7	÷	35	
8	x⇄y	30	
9	int x	64	
a	×	36	
b	ENTER EXP	26	
c	CHG SIGN	32	
d	9	11	
5 0	9	11	
1	ROLL ↓	31	
2	y→( )	40	
3	b	14	
4	ACC +	60	
5	x⇄y	30	FROM 7-C →
6	d	17	
7	IF x = y	50	
8	↓	25	
9	y→( )	40	
a	a	13	
b	CLEAR x	37	
c	x⇄y	30	
d	f	15	

Step	Key	Code	Display x	y	z	Storage f	e	d	c	b	a
6 0	IF $x=y$	50									
1	8	10	BRANCH WHEN m= 0								
2	1	01									
3	ROLL ↓	31									
4	IF FLAG	43									
5	6	06	SKIP WHEN m IS ODD								
6	9	11									
7	ACC +	60	ACCUMULATE EVEN TERMS								
8	SET FLAG	54									
9	c	16									
a	ROLL ↑	22									
b	×	36									
c	↓	25									
d	×	36	CALCULATE T_{k-1} FROM T_k AND T_{k+1}								
7 0	↓	25									
1	$y \rightleftarrows ()$	24									
2	b	14									
3	$x \rightleftarrows y$	30									
4	−	34									
5	CLEAR x	37									
6	↑	27									
7	1	01	DECREMENT m (or k)								
8	ACC −	63									
9	f	15									
a	GO TO ( ) ( )	44									
b	5	05									
c	5	05									
d	CLEAR x	37									
8 0	$x \to ()$	23									
1	a	13									
2	ROLL ↓	31									
3	e	12									
4	+	33									
5	+	33									
6	↓	25									
7	÷	35									
8	↑	27									
9	$y \rightleftarrows ()$	24									
a	9	11	RECALL AND POSITION DISPLAY								
b	d	17									
c	$x \rightleftarrows y$	30									
d	END	46	X	n	$J_n(X)$	DISPLAY					

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

HYPERGEOMETRIC SERIES EXPANSION

The program determines the series expansion of the hypergeometric function

$$F(a, b, c; X) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \cdot \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\,\Gamma(b+n)}{\Gamma(c+n)\, n!} X^n$$

The program incorporates programs 09100-70001 (N Factorial) and 09100-70024 (Gamma Function) as subroutines. The a, b, and c must be greater than zero. The program can be used for power series expansion or for the solution of Gauss's hypergeometric differential equation.

Restrictions: (c + n), (b + n) and (c + n) must be less than 69.

Reference: Advanced Engineering Mathematics, E. Kreyszig, John Wiley & Sons, 1968
Handbook of Mathematical Functions, U. S. Department of Commerce, Applied Math Series 55

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: c → Z, b → Y, a → X

→ PRESS: CONTINUE

DISPLAY

0	Z
F_n	Y
n	X

To run another case:

PRESS: END

EXAMPLE

1. The differential equation (Gauss's Hyper-geometric)

$$X(1 - X)\,Y'' + \left[c - (a + b + 1)\,X\right] Y' - abY = 0$$

has solutions given by

$$Y_1(X) = F(a, b, c; X) \text{ and}$$

$$Y_2(X) = X^{1-c} F(a - c + 1,\ b - c + 1,\ 2 - c; X)$$

$$|X| < 1 \quad \text{and } c \neq 0, -1, -2, \ldots$$

Thus

$$X(1 - X)\,Y'' + (1 - 1.25X)\,Y' - .375\,Y = 0$$

is a Gauss equation with

$$a = .75,\ b = .50 \text{ and } c = 1$$

It's solutions are

$$Y_1(X) = F(.75,\ .50,\ 1; X) = Y_2(X)$$

n	F
0	1
1	.375
2	.246
3	.188
4	.154
5	.132
6	.116

Thus

$$Y_1(X) = 1 + .375\,X + .246\,X^2 + .188\,X^3 + .154\,X^4 + \ldots$$

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
01	x→( )	23			
02	—	34			
03	d	17		ENTER	
04	STOP	41	a	b	c
05	CONT	47			
06	CONT	47			
07	x→( )	23			
08	a	13			
09	y→( )	40			
0a	b	14			
0b	↓	25			
0c	y→( )	40			
0d	c	16			
10	GO TO ( )	44			
11	▲SUB▼	77			
12	8	10			
13	c	16			
14	CONT	47			
15	x→( )	23			
16	—	34			
17	f	15			
18	a	13			
19	GO TO ( )	44			
1a	▲SUB▼	77			
1b	8	10			
1c	c	16			
1d	↑	27			
20	x←( )	67			
21	—	34			
22	f	15			
23	×	36			
24	y→( )	40			
25	—	34			
26	f	15			
27	c	16			
28	GO TO ( )	44			
29	▲SUB▼	77			
2a	8	10			
2b	c	16			
2c	↑	27			
2d	x←( )	67			
30	—	34			
31	f	15			
32	÷	35			
33	y→( )	40			
34	—	34			
35	f	15			
36	a	13			
37	GO TO ( )	44			
38	▲SUB▼	77			
39	8	10			
3a	6	06			
3b	x→( )	23			
3c	—	34			
3d	e	12			
40	b	14			
41	GO TO ( )	44			
42	▲SUB▼	77			
43	8	10			
44	6	06			
45	↑	27			
46	x←( )	67			
47	—	34			
48	e	12			
49	×	36			
4a	y→( )	40			
4b	—	34			
4c	e	12			
4d	c	16			
50	GO TO ( )	44			
51	▲SUB▼	77			
52	8	10			
53	6	06			
54	↑	27			
55	x←( )	67			
56	—	34			
57	e	12			
58	x⇄y	30			
59	÷	35			
5a	x←( )	67			
5b	—	34			
5c	f	15			
5d	×	36			
60	y→( )	40			
61	—	34			
62	e	12			
63	x←( )	67			
64	—	34			
65	d	17			
66	GO TO ( )	44			
67	▲SUB▼	77			
68	—	34			
69	6	06			
6a	c	16			
6b	ROLL ↓	31			
6c	x←( )	67			
6d	—	34			
70	e	12			
71	ROLL ↑	22			
72	÷	35			
73	x←( )	67			
74	—	34			
75	d	17			
76	↑	27			
77	0	00			
78	ROLL ↓	31		DISPLAY	
79	STOP	41	n	F_n	0
7a	CONT	47			
7b	↑	27			
7c	1	01			
7d	+	33			

Storage

Register	+	—
f		
e		
d		n
c	c	
b	b	
a	a	
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

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Step	Key	Code	Display x	Display y	Display z
8 0	$y\to(\ )$	40			
(+) 1	−	34			
2	d	17			
3	GO TO ( )	44			
4	3	03			
5	6	06			
6	↑	27			
7	$x\leftarrow(\ )$	67			
8	−	34			
9	d	17			
a	+	33			
b	↓	25			
c	↑	27			
d	1	01			

Step	Key	Code	Display x	Display y	Display z
9 0	−	34			
(+) 1	$y\to(\ )$	40			
2	d	17			
3	↑	27			
4	↓	25			
5	int x	64			
6	−	34			
7	CLEAR x	37			
8	GO TO ( )	44			
9	−	34			
a	0	00			
b	0	00			
c					
d					

Step	Key	Code	Display x	Display y	Display z
0 0	IF $x>y$	53			
(−) 1	CLEAR x	37			
2	1	01			
3	+	33			
4	5	05			
5	+	33			
6	↓	25			
7	↑	27			
8	×	36			
9	ACC +	60			
a	2	02			
b	$x \rightleftarrows y$	30			
c	÷	35			
d	3	03			

Step	Key	Code	Display x	Display y	Display z
1 0	−	34			
(−) 1	e	12			
2	÷	35			
3	2	02			
4	÷	35			
5	+	33			
6	e	12			
7	÷	35			
8	7	07			
9	÷	35			
a	1	01			
b	−	34			
c	e	12			
d	÷	35			

Step	Key	Code	Display x	Display y	Display z
2 0	6	06			
(−) 1	÷	35			
2	5	05			
3	+	33			
4	6	06			
5	0	00			
6	÷	35			
7	f	15			
8	÷	35			
9	−	34			
a	↑	27			
b	•	21			
c	5	05			
d	+	33			

Step	Key	Code	Display x	Display y	Display z
3 0	f	15			
(−) 1	ln x	65			
2	×	36			
3	↓	25			
4	+	33			
5	π	56			
6	↑	27			
7	+	33			
8	↓	25			
9	ln x	65			
a	↑	27			
b	2	02			
c	÷	35			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
4 0	+	33			
(−) 1	↓	25			
2	e^x	74			
3	↑	27			
4	f	15			
5	↑	27			
6	d	17			
7	IF $x>y$	53			
8	5	05			
9	2	02			
a	IF $x<y$	52			
b	5	05			
c	9	11			
d	GO TO ( )	44			

Step	Key	Code	Display x	Display y	Display z
5 0	6	06			
(−) 1	3	03			
2	1	01			
3	+	33			
4	↓	25			
5	×	36			
6	GO TO ( )	44			
7	4	04			
8	5	05			
9	↓	25			
a	÷	35			
b	↑	27			
c	1	01			
d	−	34			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
6 0	GO TO ( )	44			
(−) 1	4	04			
2	6	06			
3	CONT	47			
4	CONT	47			
5	0	00			
6	x→( )	23			
7	e	12			
8	x→( )	23			
9	f	15			
a	ROLL ↑	22			
b	RETURN	77			
c	↑	27			
d	0	00			
7 0	x⇄y	30			
(−) 1	IF x=y	50			
2	arc ▾	72			
3	1	01			
4	↑	27			
5	↑	27			
6	1	01			
7	−	34			
8	IF x>y	53			
9	8	10			
a	3	03			
b	ROLL ↓	31			
c	×	36			
d	ROLL ↑	22			
8 0	GO TO ( )	44			
(−) 1	7	07			
2	7	07			
3	ROLL ↑	22			
4	RETURN	77			
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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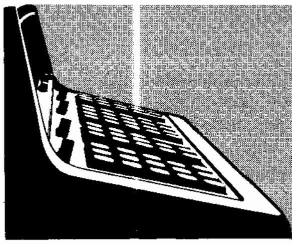
Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



(3 x 3) MATRIX MULTIPLICATION

This program calculates the product matrix C of two 3 x 3 matrices A and B of the form:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

where $$C_{ij} = \sum_{k=1}^{3} a_{ik}b_{kj} \qquad \begin{cases} i = 1, 2, 3 \\ j = 1, 2, 3 \end{cases}$$

Reference: Elementary Matrix Algebra (1958), Franz E. Hohn

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
1 —— X

(4) → ENTER: Row 1 of B
$b_{13} \to Z$, $b_{12} \to Y$, $b_{11} \to X$

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
2 —— X

ENTER: Row 2 of B
$b_{23} \to Z$, $b_{22} \to Y$, $b_{21} \to X$

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
3 —— X

ENTER: Row 3 of B
$b_{33} \to Z$ $b_{32} \to Y$, $b_{31} \to X$

(3) → PRESS: CONTINUE

DISPLAY

i —— Z
i —— Y
i —— X

i = 4 indicates C is complete → (1)

ENTER: i th Row of A
$A_{i3} \to Z$, $A_{i2} \to Y$, $A_{i1} \to X$

(2) → PRESS: CONTINUE

DISPLAY

C_{ij} —— Z
j —— Y
i —— X

(4) (3) (2) (1)

USER INSTRUCTIONS (Con't)

(4) (3) (2) (1)

j = 3 ?
NO → (2)
YES → (3)

DISPLAY

4 —— Z
4 —— Y
4 —— X

← (1)

To run another case: → (4)

PRESS: CONTINUE

EXAMPLE

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 4 & 2 & 1 \\ 1 & 4 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 4 \\ 4 & 3 & 1 \\ 1 & 6 & -2 \end{pmatrix}$$

$$C = \begin{pmatrix} 17 & 35 & -1 \\ 13 & 20 & 16 \\ 15 & 2 & 12 \end{pmatrix}$$

Step	Key	Code	Display x	Display y	Display z
0 0 (+)	CLEAR	20			
1	1	01	ENTER		
2	STOP	41	b_{11}	b_{12}	b_{13}
3	x→( )	23			
4	b	14			
5	y→( )	40			
6	a	13			
7	↓	25			
8	y→( )	40			
9	9	11			
a	CLEAR	20			
b	2	02	ENTER		
c	STOP	41	b_{21}	b_{22}	b_{23}
d	x→( )	23			
1 0 (+)	8	10			
1	y→( )	40			
2	7	07			
3	↓	25			
4	y→( )	40			
5	6	06			
6	CLEAR	20			
7	3	03	ENTER		
8	STOP	41	b_{31}	b_{32}	b_{33}
9	x→( )	23			
a	5	05			
b	y→( )	40			
c	4	04			
d	↓	25			
2 0 (+)	y→( )	40			
1	3	03			
2	1	01			
3	x→( )	23			
4	f	15			
5	x→( )	23			
6	−	34			
7	f	15			
8	↑	27			
9	↑	27			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
0 0 (−)	STOP	41	a_{11}	a_{12}	a_{13}
1	GO TO ( )	44	ENTER		
2	▲SUB▼	77			
3	1	01			
4	6	06	ENTER		
5	STOP	41	a_{21}	a_{22}	a_{23}
6	GO TO ( )	44			
7	▲SUB▼	77			
8	1	01			
9	6	06	ENTER		
a	STOP	41	a_{31}	a_{32}	a_{33}
b	GO TO ( )	44			
c	▲SUB▼	77			
d	1	01			
1 0 (−)	6	06	DISPLAY		
1	STOP	41	4	4	4
2	GO TO ( )	44			
3	+	33			
4	0	00			
5	0	00			
6	x→( )	23			
7	d	17			
8	y→( )	40			
9	c	16			
a	ROLL ↑	22			
b	x→( )	23			
c	−	34			
d	e	12			
2 0 (−)	b	14			
1	×	36			
2	e	12			
3	+	33			
4	y→( )	40			
5	e	12			
6	↓	25			
7	x←( )	67			
8	8	10			
9	×	36			
a	e	12			
b	+	33			
c	y→( )	40			
d	e	12			

Step	Key	Code	Display x	Display y	Display z
3 0 (−)	x←( )	67			
1	−	34			
2	e	12			
3	↑	27			
4	x←( )	67			
5	5	05			
6	×	36			
7	e	12			
8	+	33			
9	f	15			
a	↑	27			
b	x←( )	67			
c	−	34			
d	f	15	DISPLAY		
4 0 (−)	STOP	41	i	j	C_{ij}
1	CONT	47			
2	y⇄( )	24			
3	b	14			
4	y⇄( )	24			
5	9	11			
6	y⇄( )	24			
7	a	13			
8	y→( )	40			
9	b	14			
a	y⇄( )	24			
b	8	10			
c	y⇄( )	24			
d	6	06			

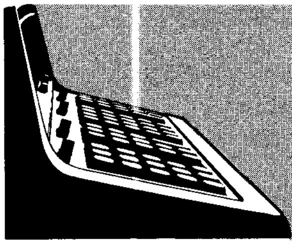
Storage

Register	+			−
f	j			i
e	c_{ij}			a_{i3}
d	a_{i1}			
c	a_{i2}			
b	b_{11}	b_{12}	b_{13}	
a	b_{12}	b_{13}	b_{11}	
9	b_{13}	b_{11}	b_{12}	
8	b_{21}	b_{22}	b_{23}	
7	b_{22}	b_{23}	b_{21}	
6	b_{23}	b_{21}	b_{22}	
5	b_{31}	b_{32}	b_{33}	
4	b_{32}	b_{33}	b_{31}	
3	b_{33}	b_{31}	b_{32}	
2				
1				
0				

Step	Key	Code	Display x	Display y	Display z
5 (−) 0	y⇄( )	24			
1	7	07			
2	y→( )	40			
3	8	10			
4	y⇄( )	24			
5	5	05			
6	y⇄( )	24			
7	3	03			
8	y⇄( )	24			
9	4	04			
a	y→( )	40			
b	5	05			
c	0	00			
d	x→( )	23			
6 (−) 0	e	12			
1	3	03			
2	↑	27			
3	f	15			
4	IF x = y	50			
5	7	07			
6	5	05			
7	↑	27			
8	1	01			
9	+	33			
a	y→( )	40			
b	f	15			
c	c	16			
d	↑	27			
7 (−) 0	d	17			
1	↑	27			
2	GO TO ( )	44			
3	2	02			
4	0	00			
5	1	01			
6	x→( )	23			
7	f	15			
8	↑	27			
9	x←( )	67			
a	−	34			
b	f	15			
c	+	33			
d	y→( )	40			
8 (−) 0	−	34			
1	f	15			
2	↓	25			
3	↑	27			
4	↑	27			
5	RETURN	77			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B ONLY
PART NO.
09100-70403

ROOTS OF 4TH DEGREE POLYNOMIAL

This program determines the real and complex roots of the fourth degree polynomial

$$f(x) = X^4 + a_1X^3 + a_2X^2 + a_3X + a_4 ,$$

where the coefficients a_i are real. The program uses the Lin-Bairstow method which determines a quadratic factor $(X^2 + rX + s)$ such that

$$f(x) = (X^2 + rX + s)(X^2 + b_1X + b_2) + RX + S$$

The variables r and s are obtained by an iteration scheme which reduces the remainder terms R and S to zero. The user can specify the remainder which he can tolerate.

The program applies the following recurrsive relationships:

$$b_1 = a_1 - r \qquad c_1 = b_1 - r$$

$$b_2 = a_2 - rb_1 - s \qquad c_2 = b_2 - rc_1$$

$$b_3 = a_3 - rb_2 - sb_1 \qquad \overline{c}_3 = -rc_2 - sc_1$$

$$b_4 = a_4 - rb_3 - sb_2 \qquad R = b_3$$

$$S = b_4 + rb_3$$

These quantities (b_i and c_i) are required for the determination of Δr and Δs in the equations:

$$c_2 \quad \Delta r + c_1 \quad \Delta s = b_3$$

$$\overline{c}_3 \quad \Delta r + c_2 \quad \Delta s = b_4$$

The terms r and s are incremented by Δ r and Δ s respectively and the remainder terms are tested against the tolerance. If the remainders are small enough to pass the test, then the two quadratics $(X^2 + rX + s)$ and $(X^2 + b_1X + b_2)$ are solved by the quadratic formula. If the remainder is too large, the iteration is repeated and the test repeated.

Locations + (5 - 6) through + (5 - 9) are used for storing the tolerance on $|R|$ and $|S|$.

Reference: Numerical Analysis, Kaiser S. Kunz, 1957, McGraw-Hill Book Company

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $a_3 \rightarrow Z$, $a_2 \rightarrow Y$, $a_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

1	Z
1	Y
1	X

ENTER DATA: $a_4 \rightarrow Z$, $s \rightarrow Y$, $r \rightarrow X$ *

PRESS CONTINUE

DISPLAY

$\lvert S \rvert$	Z
$\lvert R \rvert$	Y
0	X

PRESS: CONTINUE

DISPLAY

s	Z
r	Y
1	X

PRESS: CONTINUE

Real Roots

0	Z
R_2	Y
R_1	X

or

Complex Roots

$R = a \pm bj$

+ Imaginary Part	Z
- Imaginary Part	Y
Real Part of R	X

PRESS: CONTINUE

* Both r and s must be non-zero

USER INSTRUCTIONS (Con't)

DISPLAY

b_2	Z
b_1	Y
2	X

PRESS: CONTINUE

Real Root

0	Z
R_4	Y
R_3	X

or

Complex Roots

$R = a \pm bj$

+ Imaginary Part	Z
- Imaginary Part	Y
Real Part of R	X

To run another case:

PRESS: END

EXAMPLE

Fourth Order Butterworth Polynomial

$F(s) = s^4 + 2.613\, s^3 + 3.414\, s^2 + 2.613\, s + 1$

$a_1 = 2.613$ $r = 1$
$a_2 = 3.414$ $s = 1$
$a_3 = 2.613$
$a_4 = 1$

Tolerance is set as .001

Solution		Actual *
$\lvert S \rvert$	= 0.00094	
$\lvert R \rvert$	= 0.00015	
s	= 1.0005	1.0
r	= 0.7661	0.7654
$root_1$	= -.3831 - j.924	
$root_2$	= -.3831 + j.924	
b_2	= 0.9985	1.0
b_1	= 1.8469	1.8478
$root_3$	= -.9234 -j.3818	
$root_4$	= -.9234 +j.3818	

* Network Analysis and Synthesis, Franklin F. Kuo, 1962, John Wiley & Sons

Step	Key	Code	Display x	Display y	Display z
0 (+) 0	CLEAR	20	DISPLAY		
1	STOP	41	a_1	a_2	a_3
2	x→( )	23			
3	6	06			
4	y→( )	40			
5	7	07			
6	↓	25			
7	y→( )	40			
8	8	10			
9	1	01			
a	↑	27			
b	↑	27	ENTER		
c	STOP	41	r	s	a_4
d	ACC +	60			
1 (+) 0	ROLL ↓	31			
1	y→( )	40			
2	9	11			
3	ROLL ↑	22			
4	↑	27			
5	x←( )	67			
6	6	06			
7	x⇄y	30			
8	−	34			
9	y→( )	40			
a	d	17			
b	×	36			
c	↓	25			
d	+	33			
2 (+) 0	↑	27			
1	x←( )	67			
2	7	07			
3	ROLL ↑	22			
4	−	34			
5	y→( )	40			
6	c	16			
7	f	15			
8	×	36			
9	e	12			
a	↑	27			
b	d	17			
c	×	36			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
3 (+) 0	+	33			
1	↑	27			
2	x←( )	67			
3	8	10			
4	ROLL ↑	22			
5	−	34			
6	y→( )	40			
7	b	14			
8	f	15			
9	×	36			
a	e	12			
b	↑	27			
c	c	16			
d	×	36			
4 (+) 0	↓	25			
1	+	33			
2	↑	27			
3	x←( )	67			
4	9	11			
5	ROLL ↑	22			
6	−	34			
7	y→( )	40			
8	a	13			
9	b	14			
a	↑	27			
b	f	15			
c	x⇄y	30			
d	×	36			
5 (+) 0	ROLL ↓	31			
1	+	33			
2	|y|	55			
3	↓	25			
4	|y|	55			
5	↑	27			
6	•	21			
7	0	00			
8	1	01			
9	0	00			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
0 (−) 0	IF x<y	52			
1	2	02			
2	d	17			
3	ROLL ↑	22			
4	IF x>y	53			
5	2	02			
6	d	17			
7	ROLL ↓	31			
8	↓	25			
9	x⇄y	30			
a	↑	27			
b	0	00	DISPLAY		
c	STOP	41	0	|R|	|S|
d	RCL	61			
1 (−) 0	↑	27			
1	1	01	DISPLAY		
2	STOP	41	1	r	s
3	x⇄y	30			
4	ROLL ↑	22			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	9	11			
8	2	02			
9	c	16			
a	↑	27			
b	d	17			
c	↑	27			
d	2	02			

Storage

	+	−
f	r	r
e	s	s
d	b_1 / c_1	
c	b_2 / c_2	
b	b_3	
a	b_4	
9	a_4	
8	a_3	
7	a_2	
6	a_1	
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
2 0	STOP	41	2	b_1	b_2
1	1	01	DISPLAY		
2	x⇄y	30			
3	ROLL ↑	22			
4	GO TO ( )	44			
5	▲SUB▼	77			
6	9	11			
7	2	02			
8	CLEAR	20			
9	STOP	41			
a	CONT	47			
b	CONT	47			
c	CONT	47			
d	d	17			
3 0	↑	27			
1	f	15			
2	−	34			
3	y→( )	40			
4	d	17			
5	×	36			
6	c	16			
7	x⇄y	30			
8	−	34			
9	y→( )	40			
a	c	16			
b	f	15			
c	×	36			
d	d	17			
4 0	↑	27			
1	e	12			
2	×	36			
3	↓	25			
4	+	33			
5	↓	25			
6	CHG SIGN	32			
7	↑	27			
8	f	15			
9	x→( )	23			
a	−	34			
b	f	15			
c	e	12			
d	x→( )	23			

Step	Key	Code	Display x	Display y	Display z
5 0	−	34			
1	e	12			
2	0	00			
3	x→( )	23			
4	e	12			
5	x→( )	23			
6	f	15			
7	c	16			
8	↑	27			
9	a	13			
a	ROLL ↑	22			
b	÷	35			
c	ROLL ↑	22			
d	x⇄y	30			
6 0	÷	35			
1	↓	25			
2	IF FLAG	43			
3	7	07			
4	1	01			
5	ACC +	60			
6	SET FLAG	54			
7	c	16			
8	↑	27			
9	d	17			
a	↑	27			
b	b	14			
c	GO TO ( )	44			
d	5	05			
7 0	a	13			
1	ACC −	63			
2	x⇄y	30			
3	y⇄( )	24			
4	e	12			
5	f	15			
6	÷	35			
7	e	12			
8	x⇄y	30			
9	×	36			
a	ROLL ↓	31			
b	−	34			
c	x←( )	67			
d	−	34			

Step	Key	Code	Display x	Display y	Display z
8 0	f	15			
1	+	33			
2	y→( )	40			
3	f	15			
4	↓	25			
5	x←( )	67			
6	−	34			
7	e	12			
8	+	33			
9	y→( )	40			
a	e	12			
b	f	15			
c	GO TO ( )	44			
d	+	33			
9 0	1	01			
1	4	04			
2	ROLL ↑	22			
3	÷	35			
4	ROLL ↑	22			
5	x⇄y	30			
6	÷	35			
7	2	02			
8	CHG SIGN	32			
9	÷	35			
a	↓	25			
b	↑	27			
c	×	36			
d	ROLL ↓	31			

Storage

	+	−
f	r	r
e	s	s
d	b_1/c_1	
c	b_2/c_2	
b	b_3	
a	b_4	
9	a_4	
8	a_3	
7	a_2	
6	a_1	
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
a 0	x⇄y	30			
(−) 1	−	34			
2	CLEAR x	37			
3	IF x = y	50			
4	c	16			
5	3	03			
6	IF x > y	53			
7	b	14			
8	8	10			
9	↓	25			
a	$\sqrt{x}$	76			
b	↑	27			
c	CHG SIGN	32			
d	ROLL ↑	22			
b 0	+	33			
(−) 1	ROLL ↑	22			
2	+	33			
3	CLEAR x	37			
4	ROLL ↓	31			
5	CONT	47	DISPLAY		
6	STOP	41	ROOTS		
7	RETURN	77			
8	↓	25			
9	CHG SIGN	32			
a	$\sqrt{x}$	76			
b	↑	27			
c	CHG SIGN	32			
d	ROLL ↑	22			
c 0	GO TO ( )	44			
(−) 1	b	14			
2	5	05			
3	↓	25			
4	CLEAR x	37			
5	ROLL ↓	31			
6	GO TO ( )	44			
7	b	14			
8	5	05			
9	END	46			
a					
b					
c					
d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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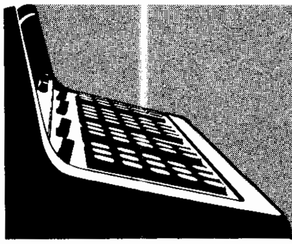
Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



PROGRAM FOR NUMERICAL SOLUTION OF TWO 1ST ORDER DIFFERENTIAL EQUATIONS

9100B ONLY
PART NO.
09100-70404

This program may be used to solve a wide variety of pairs of first order differential equations of the form:

$$\frac{dY}{dX} = f(X, Y, Z)$$

$$\frac{dZ}{dX} = g(X, Y, Z)$$

with initial conditions

$$Y_o = Y(X_o)$$
$$Z_o = Z(X_o)$$

The program will calculate successive Y_i's corresponding to equally spaced X_i's with an increment h specified by the operator. Specifically the type of solution involved is a Runge-Kutta Method which uses the equations:

$$Y_{i+1} = Y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$Z_{i+1} = Z_i + \frac{1}{6}(p_1 + 2p_2 + 2p_3 + p_4)$$

where

$$k_1 = h\, f\,(X_i,\ Y_i,\ Z_i)$$

$$k_2 = h\, f\,(X_i + \frac{h}{2},\ Y_i + \frac{k}{2}1,\ Z_i + \frac{p}{2}1)$$

$$k_3 = h\, f\,(X_i + \frac{h}{2},\ Y_i + \frac{k}{2}2,\ Z_i + \frac{p}{2}2)$$

$$k_4 = h\, f\,(X_i + h,\ Y_i + k_3,\ Z_i + p_3)$$

$$p_1 = h\, g\,(X_i,\ Y_i,\ Z_i)$$

$$p_2 = h\, g\,(X_i + \frac{h}{2},\ Y_i + \frac{k}{2}1,\ Z_i + \frac{p}{2}1)$$

$$p_3 = h\, g\,(X_i + \frac{h}{2},\ Y_i + \frac{k}{2}2,\ Z_i + \frac{p}{2}2)$$

$$p_4 = h\, g\,(X_i + h,\ Y_i + k_3,\ Z_i + p_3)$$

Reference: An Introduction to Numerical Mathematics, Eduard L. Stiefel, Academic Press, Inc. 1964.

USER INSTRUCTIONS

ENTER PROGRAM: (Starting Address is 0-0)

PRESS: GO TO ( )( )

PRESS: -

PRESS: 0

PRESS: 0

SET: PROGRAM

At this time the independent variable X is in both the X and c registers, the variable Y is in both the Y and b registers, and the variable Z is in both the Z and a registers. Starting at -0,0 enter the program steps which take the variables from their respective registers and calculate f(X, Y, Z) and g(X, Y, Z). Place f(X, Y, Z) in the Y register and g(X, Y, Z) in the Z register. The last statement must be RETURN since the main program calls for a subroutine containing the two differential equations.

Note there is a maximum of 14 registers available for storing and positioning f(X, Y, Z) and g(X, Y, Z).

Note also that the contents of the a, b and c registers must be preserved.

SET: RUN

→ PRESS: END

PRESS: CONTINUE

ENTER DATA: h → X (Increment)

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
1	———	X

ENTER DATA: $Z_0 \rightarrow Z$, $Y_0 \rightarrow Y$, $X_0 \rightarrow X$

(Initial Conditions)

PRESS: CONTINUE

The calculator will display answers at every increment of the independent variable in the form:

Z_i	———	Z
Y_i	———	Y
X_i	———	X

USER INSTRUCTIONS (Con't)

To hold the solution at the next increment, depress PAUSE until display. To restart, press CONTINUE. To execute program again, enter the new differential equations.

EXAMPLE

$$\frac{dY}{dX} = -\sin X + Z$$

$$\frac{dZ}{dX} = \cos X + 4Y$$

Initial Conditions:

$X_0 = 0$

$Y_0 = 1$

$Z_0 = 2$

Let increment h = .005 SET: RADIANS

The steps that form f(X, Y, Z) and g(X, Y, Z) appear on page 2 of the step pages.

X	Y	Z
.01	1.020	2.050
.02	1.041	2.102
.03	1.062	2.154
.04	1.083	2.207
.05	1.105	2.260
.06	1.127	2.315
.07	1.150	2.370
.08	1.174	2.427
.09	1.197	2.484
.10	1.221	2.543

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20	ENTER		
01	STOP	41	h	0	0
02	↑	27			
03	2	02			
04	÷	35			
05	↓	25			
06	x→( )	23			
07	d	17			
08	1	01	ENTER		
09	STOP	41	X_0	Y_0	Z_0
0a	x→( )	23			
0b	c	16			
0c	ROLL ↓	31			
0d	ACC +	60			
10	x→( )	23			
11	b	14			
12	y→( )	40			
13	a	13			
14	ROLL ↑	22			
15	CLEAR x	37			
16	x→( )	23			
17	−	34			
18	f	15			
19	c	16			
1a	SET FLAG	54	DISPLAY		
1b	PAUSE	57	X_i	Y_i	Z_i
1c	CONT	47			
1d	CONT	47			
20	GO TO ( )	44			
21	▲SUB▼	77			
22	−	34			
23	0	00			
24	0	00			
25	d	17			
26	×	36			
27	ROLL ↑	22			
28	×	36			
29	3	03			
2a	÷	35			
2b	ROLL ↑	22			
2c	x⇄y	30			
2d	÷	35			

Step	Key	Code	Display x	Display y	Display z
30	1	01			
31	ROLL ↓	31			
32	ACC +	60			
33	IF FLAG	43			
34	ACC −	63			
35	CONT	47			
36	ACC +	60			
37	ROLL ↑	22			
38	y⇄( )	24			
39	−	34			
3a	f	15			
3b	+	33			
3c	2	02			
3d	IF x<y	52			
40	6	06			
41	5	05			
42	IF x=y	50			
43	SET FLAG	54			
44	CONT	47			
45	3	03			
46	y⇄( )	24			
47	−	34			
48	f	15			
49	×	36			
4a	ROLL ↑	22			
4b	×	36			
4c	a	13			
4d	+	33			
50	b	14			
51	ROLL ↑	22			
52	+	33			
53	y⇄( )	24			
54	d	17			
55	c	16			
56	x⇄y	30			
57	+	33			
58	IF FLAG	43			
59	arc ▼	72			
5a	−	34			
5b	x⇄y	30			
5c	y⇄( )	24			
5d	d	17			

Step	Key	Code	Display x	Display y	Display z
60	x→( )	23			
61	c	16			
62	GO TO ( )	44			
63	2	02			
64	0	00			
65	3	03			
66	IF x<y	52			
67	6	06			
68	d	17			
69	6	06			
6a	GO TO ( )	44			
6b	4	04			
6c	6	06			
6d	y⇄( )	24			
70	−	34			
71	f	15			
72	ROLL ↓	31			
73	ACC −	63			
74	RCL	61			
75	GO TO ( )	44			
76	1	01			
77	0	00			
78	END	46			
79					
7a					
7b					
7c					
7d					

Storage

Register	Contents
f	Y_0
e	Z_0
d	h/2
c	X_0
b	Y_0
a	Z_0
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

NOTE

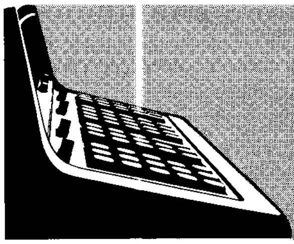
Do not destroy these registers when generating f(X Y Z) and g(X Y Z)

 PART NO. 09100-70404 9100B ONLY

	Step	Key	Code	Display x	Display y	Display z
0	0	4	04			
(–)	1	×	36			
	2	c	16			
	3	cos x	73			
	4	+	33			
	5	c	16			
	6	sin x	70			
	7	CHG SIGN	32			
	8	ROLL ↑	22			
	9	+	33			
	a	RETURN	77			
	b					
	c					
↓	d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



09100 B ONLY
PART NO.
09100-70405

ROOTS OF 6TH DEGREE POLYNOMIAL

This program determines the real and complex roots of the sixth degree polynomial.

$$f(x) = X^6 + a_1X^5 + a_2X^4 + a_3X^3 + a_4X^2 + a_5X + a_6 \quad ,$$

where the coefficients a_i are real. The program uses the Lin-Bairstow method which determines a quadratic factor $(X^2 + rX + s)$ such that

$$f(x) = (x^2 + rX + s)\ (X^4 + b_1X^3 + b_2X^2 + b_3X + b_4) + RX + S$$

The variables r and s are obtained by an iteration scheme which reduces the remainder terms R and S to zero. The user can specify the remainder which he can tolerate.

The program applies the following recurrsive relationships:

$b_1 =$	$a_1 - r$	$c_1 \equiv$	$b_1 - r$
$b_2 =$	$a_2 - rb_1 - s$	$c_2 =$	$b_2 - rc_1 - s$
$b_3 =$	$a_3 - rb_2 - sb_1$	$c_3 =$	$b_3 - rc_2 - sc_1$
$b_4 =$	$a_4 - rb_3 - sb_2$	$c_4 =$	$b_4 - rc_3 - sc_2$
$b_5 =$	$a_5 - rb_4 - sb_3$	$\overline{c}_5 =$	$-rc_4 - sc_3$
$b_6 =$	$a_6 - rb_5 - sb_4$	$R =$	b_5

$$S = b_6 + rb_5$$

These quantities (b_i and c_i) are required for the determination of Δr and Δs in the equations:

$$c_4 \Delta r + c_3 \Delta s = b_5$$

$$\overline{c}_5 \Delta r + c_4 \Delta s = b_6$$

The terms r and s are incremented by Δr and Δ s respectively and the remainder terms are tested against the tolerance. If the remainders are small enough to pass the test, then the quadratic factor $(X^2 + rX + s)$ is solved by the quadratic formula and the remaining quartic $(X^4 + b_1X^3 + b_2X^2 + b_3X + b_4)$ is solved using the equations of program 09100-70403 "Roots of 4th Degree Polynomial." If the remainders are too large, the interation and the test are repeated.

Locations — (2 - c) through — (3 - 1) are used for storing the tolerance on $|R|$ and $|S|$ for pass 1 whereas locations (4 - d) through (5 - 8) contain the tolerance for pass 2 (for the reduction of the quartic).

Reference: Numerical Analysis, Kaiser S. Kunz, 1957, McGraw-Hill Book Company

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM 1: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: s → Y, r → X *

PRESS: CONTINUE

DISPLAY

0	Z
s	Y
1	X

ENTER DATA: a_6 → Z, a_5 → Y, a_4 → X

PRESS: CONTINUE

DISPLAY

a_6	Z
a_6	Y
2	X

ENTER DATA: a_3 → Z, a_2 → Y, a_1 → X

PRESS: CONTINUE

DISPLAY

1	Z
s_1	Y
r_1	X

Terms b_1, b_2, b_3, and b_4, are stored in registers 6, 7, 8, and 9 respectively.

PRESS: GO TO

PRESS: —

PRESS: 0

PRESS: 0

* Both r and s must be non-zero.

USER INSTRUCTIONS (Con't)

ENTER PROGRAM 2: Side B followed by Side A

PRESS: CONTINUE

DISPLAY

Real Roots or Complex Roots ($R = a \pm jb$)

Real Roots			Complex Roots	
0	Z	or	+ Imaginary Part	Z
R_2	Y		- Imaginary Part	Y
R_1	X		Real Part of R	X

PRESS: CONTINUE

DISPLAY

2	Z
s_2	Y
r_2	X

PRESS: CONTINUE

DISPLAY

Real Roots or Complex Roots ($R = a \pm jb$)

Real Roots			Complex Roots	
0	Z	or	+ Imaginary Part	Z
R_2	Y		-Imaginary Part	Y
R_1	X		Real Part of R	X

PRESS: CONTINUE

DISPLAY

3	Z
s_3	Y
r_3	X

PRESS: CONTINUE

DISPLAY

Real Roots or Complex Roots ($R = a \pm jb$)

Real Roots			Complex Roots	
0	Z	or	+ Imaginary Part	Z
R_2	Y		- Imaginary Part	Y
R_1	X		Real Part of R	X

To run another case, return to the beginning of the USER INSTRUCTIONS

EXAMPLE

Sixth Order Butterworth Polynomial

$$F(s) = s^6 + 3.864\ s^5 + 7.464\ s^4 + 9.141\ s^3 + 7.464\ s^2 + 3.864\ s + 1$$

$a_1 = 3.864$ $r = 1$
$a_2 = 7.464$ $s = -1$
$a_3 = 9.141$
$a_4 = 7.464$
$a_5 = 3.864$
$a_6 = 1.$

Tolerance is set at .001

Solution		Actual *
r_1	= 1.9317	1.9318
s_1	= 1.0000	1.0000
$root_1$	= -.9658 - j.2591	
$root_2$	= -.9658 + j.2591	
r_2	= 1.4144	1.4142
s_2	= 0.9970	1.0000
$root_3$	= -.7072 - j.7049	
$root_4$	= -.7072 + j.7049	
r_3	= .5179	.5176
s_3	= 1.0019	1.0000
$root_5$	= -.2589 - j.9669	
$root_6$	= -.2589+ j.9669	

Step 0 (+)

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20		ENTER	
0 1	STOP	41	r	s	0
0 2	ACC +	60			
0 3	1	01		ENTER	
0 4	STOP	41	a_4	a_5	a_6
0 5	x→( )	23			
0 6	7	07			
0 7	y→( )	40			
0 8	8	10			
0 9	↓	25			
0 a	y→( )	40			
0 b	9	11			
0 c	2	02		ENTER	
0 d	STOP	41	a_1	a_2	a_3

Step 1 (+)

Step	Key	Code	Display x	Display y	Display z
1 0	x→( )	23			
1 1	4	04			
1 2	y→( )	40			
1 3	5	05			
1 4	ROLL ↑	22			
1 5	x→( )	23			
1 6	6	06			
1 7	f	15			
1 8	−	34			
1 9	y→( )	40			
1 a	a	13			
1 b	×	36			
1 c	e	12			
1 d	+	33			

Step 2 (+)

Step	Key	Code	Display x	Display y	Display z
2 0	↓	25			
2 1	−	34			
2 2	y→( )	40			
2 3	b	14			
2 4	a	13			
2 5	GO TO ( )	44			
2 6	▲SUB▼	77			
2 7	−	34			
2 8	d	17			
2 9	4	04			
2 a	x←( )	67			
2 b	6	06			
2 c	x⇄y	30			
2 d	−	34			

Step 3 (+)

Step	Key	Code	Display x	Display y	Display z
3 0	y→( )	40			
3 1	c	16			
3 2	b	14			
3 3	GO TO ( )	44			
3 4	▲SUB▼	77			
3 5	−	34			
3 6	d	17			
3 7	4	04			
3 8	x←( )	67			
3 9	7	07			
3 a	GO TO ( )	44			
3 b	−	34			
3 c	0	00			
3 d	0	00			

Step 0 (−)

Step	Key	Code	Display x	Display y	Display z
0 0	x⇄y	30			
0 1	−	34			
0 2	y→( )	40			
0 3	d	17			
0 4	c	16			
0 5	GO TO ( )	44			
0 6	▲SUB▼	77			
0 7	d	17			
0 8	4	04			
0 9	x←( )	67			
0 a	8	10			
0 b	x⇄y	30			
0 c	−	34			
0 d	y→( )	40			

Step 1 (−)

Step	Key	Code	Display x	Display y	Display z
1 0	−	34			
1 1	e	12			
1 2	d	17			
1 3	GO TO ( )	44			
1 4	▲SUB▼	77			
1 5	d	17			
1 6	4	04			
1 7	x←( )	67			
1 8	9	11			
1 9	x⇄y	30			
1 a	−	34			
1 b	y→( )	40			
1 c	−	34			
1 d	f	15			

Step 2 (−)

Step	Key	Code	Display x	Display y	Display z
2 0	f	15			
2 1	↑	27			
2 2	x←( )	67			
2 3	−	34			
2 4	e	12			
2 5	×	36			
2 6	ROLL ↓	31			
2 7	+	33			
2 8	|y|	55			
2 9	↓	25			
2 a	|y|	55			
2 b	↑	27			
2 c	•	21			
2 d	0	00			

Step 3 (−)

Step	Key	Code	Display x	Display y	Display z
3 0	0	00			
3 1	1	01			
3 2	IF x<y	52			
3 3	5	05			
3 4	8	10			
3 5	ROLL ↑	22			
3 6	IF x>y	53			
3 7	5	05			
3 8	8	10			
3 9	a	13			
3 a	x→( )	23			
3 b	6	06			
3 c	b	14			
3 d	x→( )	23			

Storage

Register	+	−
f	r /1	b_6
e	s /1	b_5
d	b_4 / c_4 / r final	
c	b_3 / c_3 / s final	
b	b_2 / c_2 / r	
a	b_1 / c_1 / s	
9	a_6 / b_4	
8	a_5 / b_3	
7	a_4 / b_2	
6	a_3 / b_1	
5	a_2	
4	a_1	
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
4 0	7	07			
1	c	16			
2	x→( )	23			
3	8	10			
4	d	17			
5	x→( )	23			
6	9	11			
7	RCL	61			
8	↑	27			
9	1	01			
a	x→( )	23			
b	e	12			
c	x→( )	23			
d	f	15			

Step	Key	Code	Display x	Display y	Display z
5 0	ROLL ↓	31			
1	x→( )	23			
2	d	17			
3	y→( )	40			
4	c	16	DISPLAY		
5	STOP	41	r_1	s_1	1
6	CONT	47	ENTER		
7	STOP	41	Program 2		
8	a	13			
9	↑	27			
a	f	15			
b	−	34			
c	y→( )	40			
d	a	13			

Step	Key	Code	Display x	Display y	Display z
6 0	×	36			
1	e	12			
2	+	33			
3	b	14			
4	x⇄y	30			
5	−	34			
6	y→( )	40			
7	b	14			
8	a	13			
9	GO TO ( )	44			
a	▲SUB▼	77			
b	d	17			
c	4	04			
d	c	16			

Step	Key	Code	Display x	Display y	Display z
7 0	x⇄y	30			
1	−	34			
2	y→( )	40			
3	c	16			
4	b	14			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	d	17			
8	4	04			
9	d	17			
a	x⇄y	30			
b	−	34			
c	y→( )	40			
d	d	17			

Step	Key	Code	Display x	Display y	Display z
8 0	c	16			
1	GO TO ( )	44			
2	▲SUB▼	77			
3	d	17			
4	4	04			
5	↓	25			
6	CHG SIGN	32			
7	↑	27			
8	f	15			
9	x→( )	23			
a	b	14			
b	e	12			
c	x→( )	23			
d	a	13			

Step	Key	Code	Display x	Display y	Display z
9 0	0	00			
1	x→( )	23			
2	e	12			
3	x→( )	23			
4	f	15			
5	d	17			
6	↑	27			
7	x←( )	67			
8	−	34			
9	f	15			
a	ROLL ↑	22			
b	÷	35			
c	ROLL ↑	22			
d	x⇄y	30			

Step	Key	Code	Display x	Display y	Display z
a 0	÷	35			
1	↓	25			
2	IF FLAG	43			
3	b	14			
4	3	03			
5	ACC +	60			
6	SET FLAG	54			
7	d	17			
8	↑	27			
9	c	16			
a	↑	27			
b	x←( )	67			
c	−	34			
d	e	12			

Step	Key	Code	Display x	Display y	Display z
b 0	GO TO ( )	44			
1	9	11			
2	a	13			
3	ACC −	63			
4	x⇄y	30			
5	y⇄( )	24			
6	e	12			
7	f	15			
8	÷	35			
9	x⇄y	30			
a	×	36			
b	ROLL ↓	31			
c	−	34			
d	b	14			

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
c 0	+	33			
(–) 1	y→( )	40			
2	f	15			
3	↓	25			
4	a	13			
5	+	33			
6	y→( )	40			
7	e	12			
8	x←( )	67			
9	5	05			
a	↑	27			
b	x←( )	67			
c	4	04			
d	↑	27			
d 0	GO TO ( )	44			
(–) 1	+	33			
2	1	01			
3	7	07			
4	↑	27			
5	e	12			
6	×	36			
7	ROLL ↑	22			
8	x⇄y	30			
9	f	15			
a	×	36			
b	↓	25			
c	+	33			
d	RETURN	77			
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
0 0	1	01			
(+) 1	↑	27			
2	d	17			
3	↑	27			
4	c	16			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	−	34			
8	9	11			
9	2	02			
a	RCL	61			
b	↑	27			
c	x←( )	67			
d	6	06			
1 0	x⇄y	30			
(+) 1	−	34			
2	y→( )	40			
3	d	17			
4	×	36			
5	↓	25			
6	+	33			
7	↑	27			
8	x←( )	67			
9	7	07			
a	ROLL ↑	22			
b	−	34			
c	y→( )	40			
d	c	16			
2 0	f	15			
(+) 1	×	36			
2	e	12			
3	↑	27			
4	d	17			
5	×	36			
6	↓	25			
7	+	33			
8	↑	27			
9	x←( )	67			
a	8	10			
b	ROLL ↑	22			
c	−	34			
d	y→( )	40			

Step	Key	Code	Display x	Display y	Display z
3 0	b	14			
(+) 1	f	15			
2	×	36			
3	e	12			
4	↑	27			
5	c	16			
6	×	36			
7	↓	25			
8	+	33			
9	↑	27			
a	x←( )	67			
b	9	11			
c	ROLL ↑	22			
d	−	34			
4 0	y→( )	40			
(+) 1	a	13			
2	b	14			
3	↑	27			
4	f	15			
5	x⇄y	30			
6	×	36			
7	ROLL ↓	31			
8	+	33			
9	|y|	55			
a	↓	25			
b	|y|	55			
c	↑	27			
d	.	21			
5 0	0	00			
(+) 1	0	00			
2	1	01			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	GO TO ( )	44			
a	−	34			
b	0	00			
c	0	00			
d	END	46			

Step	Key	Code	Display x	Display y	Display z
0 0	IF x<y	52			
(−) 1	2	02			
2	d	17			
3	ROLL ↑	22			
4	IF x>y	53			
5	2	02			
6	d	17			
7	RCL	61			
8	↑	27			
9	2	02			
a	ROLL ↓	31	DISPLAY		
b	STOP	41	r_2	s_2	2
c	CONT	47			
d	ROLL ↑	22			
1 0	1	01			
(−) 1	ROLL ↓	31			
2	x⇄y	30			
3	GO TO ( )	44			
4	▲SUB▼	77			
5	9	11			
6	2	02			
7	c	16			
8	↑	27			
9	d	17			
a	↑	27			
b	3	03			
c	ROLL ↓	31	DISPLAY		
d	STOP	41	r_3	s_3	3

Storage

Register	+	−
f	r(1)	
e	s(1)	
d	r_1 / b_1	
c	s_1 / b_2	
b	b_4	
a	a_4	
9	a_3	
8	a_2	
7	a_1	
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
2 0	CONT	47			
(−) 1	ROLL ↑	22			
2	1	01			
3	ROLL ↓	31			
4	x⇄y	30			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	9	11			
8	2	02			
9	CLEAR	20	DISPLAY		
a	STOP	41	0	0	0
b	CONT	47			
c	CONT	47			
d	d	17			
3 0	↑	27			
(−) 1	f	15			
2	−	34			
3	y→( )	40			
4	d	17			
5	×	36			
6	c	16			
7	x⇄y	30			
8	−	34			
9	y→( )	40			
a	c	16			
b	f	15			
c	×	36			
d	d	17			
4 0	↑	27			
(−) 1	e	12			
2	×	36			
3	↓	25			
4	+	33			
5	↓	25			
6	CHG SIGN	32			
7	↑	27			
8	f	15			
9	x→( )	23			
a	−	34			
b	f	15			
c	e	12			
d	x→( )	23			

Step	Key	Code	Display x	Display y	Display z
5 0	−	34			
(−) 1	e	12			
2	0	00			
3	x→( )	23			
4	e	12			
5	x→( )	23			
6	f	15			
7	c	16			
8	↑	27			
9	a	13			
a	ROLL ↑	22			
b	÷	35			
c	ROLL ↑	22			
d	x⇄y	30			
6 0	÷	35			
(−) 1	↓	25			
2	IF FLAG	43			
3	7	07			
4	1	01			
5	ACC +	60			
6	SET FLAG	54			
7	c	16			
8	↑	27			
9	d	17			
a	↑	27			
b	b	14			
c	GO TO ( )	44			
d	5	05			
7 0	a	13			
(−) 1	ACC −	63			
2	x⇄y	30			
3	y⇄( )	24			
4	e	12			
5	f	15			
6	÷	35			
7	e	12			
8	x⇄y	30			
9	×	36			
a	ROLL ↓	31			
b	−	34			
c	x←( )	67			
d	−	34			

Step	Key	Code	Display x	Display y	Display z
8 0	f	15			
(−) 1	+	33			
2	y→( )	40			
3	f	15			
4	↓	25			
5	x←( )	67			
6	−	34			
7	e	12			
8	+	33			
9	y→( )	40			
a	e	12			
b	f	15			
c	GO TO ( )	44			
d	+	33			
9 0	0	00			
(−) 1	b	14			
2	ROLL ↑	22			
3	÷	35			
4	ROLL ↑	22			
5	x⇄y	30			
6	÷	35			
7	2	02			
8	CHG SIGN	32			
9	÷	35			
a	↓	25			
b	↑	27			
c	×	36			
d	ROLL ↓	31			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
a 0	$x \rightleftarrows y$	30			
(−) 1	−	34			
2	CLEAR x	37			
3	IF $x=y$	50			
4	c	16			
5	3	03			
6	IF $x>y$	53			
7	b	14			
8	8	10			
9	↓	25			
a	$\sqrt{x}$	76			
b	↑	27			
c	CHG SIGN	32			
d	ROLL ↑	22			
b 0	+	33			
(−) 1	ROLL ↑	22			
2	+	33			
3	CLEAR x	37			
4	ROLL ↓	31	DISPLAY		
5	CONT	47	ROOTS		
6	STOP	41			
7	RETURN	77			
8	↓	25			
9	CHG SIGN	32			
a	$\sqrt{x}$	76			
b	↑	27			
c	CHG SIGN	32			
d	ROLL ↑	22			
c 0	GO TO ( )	44			
(−) 1	b	14			
2	5	05			
3	↓	25			
4	CLEAR x	37			
5	ROLL ↓	31			
6	GO TO ( )	44			
7	b	14			
8	5	05			
9	END	46			
a					
b					
c					
d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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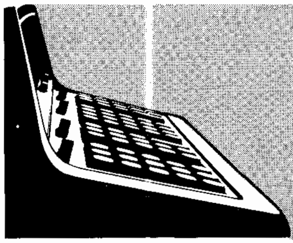
Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B ONLY
PART NO.
09100-70406

CHARACTERISTIC EQUATION OF A 3 X 3 MATRIX AND EIGENVALUE DETERMINATION

This program will calculate the coefficients of the characteristic polynomial of a matrix of the form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where the a_{ij} are real numbers.

The characteristic polynomial of A is determined by reducing the equation:

$$\text{determinant} \begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{bmatrix} = 0$$

to a polynomial of the form:

$$\lambda^3 + p\lambda^2 + q\lambda + r = 0$$

The eigenvalues are then obtained using the techniques described in Program 09100-70011.

Reference: AN INTRODUCTION TO NUMERICAL ANALYSIS, L. Fox, Oxford University Press, 1965

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 ———	Z
0 ———	Y
1 ———	X

ENTER DATA: $a_{11} \rightarrow Z$, $a_{12} \rightarrow Y$, $a_{13} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0 ———	Z
0 ———	Y
2 ———	Z

ENTER DATA: $a_{21} \rightarrow Z$, $a_{22} \rightarrow Y$, $a_{23} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0 ———	Z
0 ———	Y
3 ———	X

ENTER DATA: $a_{31} \rightarrow Z$, $a_{32} \rightarrow Y$, $a_{33} \rightarrow X$

PRESS: CONTINUE

DISPLAY

r ———	Z
q ———	Y
p ———	X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0 ———	Z	
0 ———	Y	# of real roots
1 or 3 ———	X	

PRESS: CONTINUE

DISPLAY

3 Real Roots			Complex Roots	
λ_3 ———	Z		λ_3 ———	Z
λ_2 ———	Y	or	Im 1, 2 ———	Y
λ_1 ———	X		Re 1, 2 ———	X

To run program for a new matrix:

PRESS: CONTINUE

EXAMPLE

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & -2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Coefficient Solution:

$r = -17$
$q = -15$
$p = -1$

$$\lambda^3 - 1\lambda^2 - 15\lambda - 17 = 0$$

Eigenvalues:

$\lambda_3 = 4.832$
$\lambda_2 = -1.524$
$\lambda_1 = -2.308$

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	1	01	ENTER		
2	STOP	41	a_{13}	a_{12}	a_{11}
3	CONT	47			
4	x→( )	23			
5	d	17			
6	CLEAR x	37			
7	ROLL ↓	31			
8	ACC +	60			
9	↓	25			
a	2	02	ENTER		
b	STOP	41	a_{23}	a_{22}	a_{21}
c	x→( )	23			
d	c	16			
1 0	ROLL ↓	31			
(+) 1	x→( )	23			
2	b	14			
3	y→( )	40			
4	a	13			
5	f	15			
6	×	36			
7	e	12			
8	↑	27			
9	b	14			
a	+	33			
b	y→( )	40			
c	−	34			
d	f	15			
2 0	−	34			
(+) 1	×	36			
2	CLEAR x	37			
3	ROLL ↓	31			
4	−	34			
5	y→( )	40			
6	−	34			
7	e	12			
8	↓	25			
9	3	03	ENTER		
a	STOP	41	a_{33}	a_{32}	a_{31}
b	y⇄( )	24			
c	−	34			
d	e	12			
3 0	y→( )	40			
(+) 1	−	34			
2	d	17			
3	×	36			
4	y⇄( )	24			
5	−	34			
6	f	15			
7	+	33			
8	y→( )	40			
9	−	34			
a	c	16			
b	−	34			
c	×	36			
d	d	17			
4 0	ROLL ↑	22			
(+) 1	x→( )	23			
2	−	34			
3	b	14			
4	×	36			
5	↓	25			
6	−	34			
7	↓	25			
8	y⇄( )	24			
9	−	34			
a	d	17			
b	x⇄y	30			
c	−	34			
d	x←( )	67			
5 0	−	34			
(+) 1	e	12			
2	↑	27			
3	c	16			
4	×	36			
5	ROLL ↓	31			
6	−	34			
7	y⇄( )	24			
8	e	12			
9	↓	25			
a	×	36			
b	d	17			
c	↑	27			
d	a	13			
6 0	×	36			
(+) 1	↓	25			
2	−	34			
3	↓	25			
4	y⇄( )	24			
5	−	34			
6	e	12			
7	×	36			
8	d	17			
9	ROLL ↓	31			
a	y⇄( )	24			
b	−	34			
c	f	15			
d	+	33			
7 0	y⇄( )	24			
(+) 1	b	14			
2	↓	25			
3	×	36			
4	c	16			
5	↑	27			
6	f	15			
7	×	36			
8	↓	25			
9	−	34			
a	x⇄y	30			
b	y⇄( )	24			
c	−	34			
d	b	14			

Storage

Register	+	−
f	a_{12} /r	
e	a_{11} /q	a_{32}
d	a_{13} /p	
c	a_{22}	
b	a_{22}	
a	a_{21}	
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

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Step	Key	Code	Display x	Display y	Display z
80	×	36			
(+) 81	b	14			
82	+	33			
83	↑	27			
84	y⇄( )	24			
85	−	34			
86	c	16			
87	e	12			
88	x⇄y	30			
89	CHG SIGN	32	DISPLAY		
8a	STOP	41	p	q	r
8b	CONT	47			
8c	ROLL ↓	31			
8d	x⇄y	30			
90	x→( )	23			
(+) 91	f	15			
92	y→( )	40			
93	e	12			
94	↓	25			
95	y→( )	40			
96	d	17			
97	|y|	55			
98	f	15			
99	x⇄y	30			
9a	GO TO ( )	44			
9b	−	34			
9c	0	00			
9d	0	00			
00	|y|	55			
(−) 01	+	33			
02	ENTER EXP	26			
03	CHG SIGN	32			
04	2	02			
05	↑	27			
06	1	01			
07	0	00			
08	×	36			
09	↓	25			
0a	IF x<y	52			
0b	0	00			
0c	5	05			
0d	x→( )	23			

Step	Key	Code	Display x	Display y	Display z
10	c	16			
(−) 11	CLEAR x	37			
12	↑	27			
13	f	15			
14	↑	27			
15	|y|	55			
16	÷	35			
17	y→( )	40			
18	b	14			
19	c	16			
1a	x⇄y	30			
1b	•	21			
1c	1	01			
1d	×	36			
20	y→( )	40			
(−) 21	c	16			
22	b	14			
23	CHG SIGN	32			
24	x→( )	23			
25	b	14			
26	↓	25			
27	c	16			
28	↑	27			
29	b	14			
2a	×	36			
2b	↓	25			
2c	+	33			
2d	↓	25			
30	IF x=y	50			
(−) 31	5	05			
32	0	00			
33	↑	27			
34	↑	27			
35	d	17			
36	+	33			
37	↓	25			
38	×	36			
39	e	12			
3a	+	33			
3b	↓	25			
3c	×	36			
3d	f	15			

Step	Key	Code	Display x	Display y	Display z
40	+	33			
(−) 41	CLEAR x	37			
42	IF x=y	50			
43	5	05			
44	0	00			
45	b	14			
46	×	36			
47	CLEAR x	37			
48	IF x>y	53			
49	2	02			
4a	6	06			
4b	GO TO ( )	44			
4c	1	01			
4d	9	11			
50	↓	25			
(−) 51	y→( )	40			
52	b	14			
53	d	17			
54	ROLL ↑	22			
55	+	33			
56	↑	27			
57	↓	25			
58	×	36			
59	e	12			
5a	+	33			
5b	↓	25			
5c	CHG SIGN	32			
5d	ROLL ↓	31			

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
60	2	02			
(−) 1	CHG SIGN	32			
2	÷	35			
3	↓	25			
4	↑	27			
5	×	36			
6	ROLL ↓	31			
7	+	33			
8	ENTER EXP	26			
9	CHG SIGN	32			
a	1	01			
b	0	00			
c	ROLL ↑	22			
d	×	36			
70	|y|	55			
(−) 1	ROLL ↓	31			
2	CHG SIGN	32			
3	IF x>y	53			
4	8	10			
5	6	06			
6	SET FLAG	54			
7	CHG SIGN	32			
8	IF x>y	53			
9	CLEAR x	37			
a	×	36			
b	↓	25			
c	√x	76			
d	−	34			
80	ROLL ↑	22			
(−) 1	+	33			
2	↓	25			
3	GO TO ( )	44			
4	8	10			
5	9	11			
6	↓	25			
7	CHG SIGN	32			
8	√x	76			
9	x→( )	23			
a	e	12			
b	y→( )	40			
c	f	15			
d	CLEAR x	37			
90	↑	27			
(−) 1	↑	27			
2	1	01			
3	IF FLAG	43			
4	CLEAR x	37			
5	3	03	DISPLAY		
6	STOP	41	1/3	0	0
7	CONT	47			
8	b	14			
9	ROLL ↓	31	DISPLAY		
a	RCL	61	λ_1	λ_2	λ_3
b	STOP	41	$R_{1,2}$	$I_{1,2}$	λ_3
c	CONT	47			
d	END	46			

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

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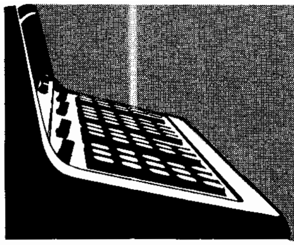
Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



PROGRAM FOR SIMULTANEOUS SOLUTION OF FOUR EQUATIONS IN FOUR UNKNOWNS

9100B ONLY
PART NO.
09100-70407

This program in its original form was written by Dr. Stefan J. Medwadowski, a Consulting Structural Engineer in San Francisco. One of his hobbies is programming the -hp- 9100A.

4x4 SYSTEM OF LINEAR ALGEBRAIC EQUATIONS

Given a system of linear algebraic equations:

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = p_1$$
$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = p_2$$
$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = p_3$$
$$a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = p_4$$

or a matrix notation:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

i.e., $\left[a_{ij}\right] \left\{x_i\right\} = \left\{p_i\right\}$ with i, j = 1, 2, 3, 4

Such systems occur frequently in the solution of boundary value problems of structural mechanics, such as those which arise in the theory of thin elastic shells or plates.

It is assumed that the solution of the system exists; i.e., that the determinant of the a_{ij} coefficient matrix does not vanish. The coefficients a_{ij} are assumed real.

NOTE: None of the determinates of the leading submatrices may be zero, or

$$\left| a_{11} \right| \neq 0$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

Should one or more of these conditions exist (and should therefore the illegal operation light come on), it may be removed by re-arranging the sequence of the equations within the system. It is always possible to do this as a consequence of the postulated existence of a unique solution.

Method of Solution: Cholewski's Method

Reference: Salvadori and Baron, Numerical Method in Engineering, Prentice-Hall, 1952.

9100B ONLY
PART NO.
09100-70407

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

→ ENTER DATA: a_{ij} (or p_i) → X

NOTE: Data is entered row by row in the following order with a CONTINUE following each entry:

$a_{11}, a_{12}, a_{13}, a_{14}$, p_1

$a_{21}, a_{22}, a_{23}, a_{24}$, p_2

$a_{31}, a_{32}, a_{33}, a_{34}$, p_3

$a_{41}, a_{42}, a_{43}, a_{44}$, p_4

DISPLAY

0	Z
x_1	Y
x_2	X

PRESS: CONTINUE

DISPLAY

0	Z
x_3	Y
x_4	X

To re-run program,

← PRESS: CONTINUE

EXAMPLE

$2x_1 + x_2 + 3x_3 + x_4 = 7$

$x_1 + 4x_2 + x_3 = 10$

$3x_1 - 5x_3 - 2x_4 = 10$

$8x_1 - x_2 + 4x_3 + 2x_4 = 22$

$x_1 = 3$ $x_3 = -1$

$x_2 = 2$ $x_4 = 2$

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20	ENTER		
01	STOP	41	a_{11}	—	—
02	ROLL ↓	31	ENTER		
03	STOP	41	a_{12}	—	—
04	ROLL ↑	22			
05	÷	35			
06	y→( )	40			
07	f	15			
08	ROLL ↓	31	ENTER		
09	STOP	41	a_{13}	—	—
0a	ROLL ↑	22			
0b	÷	35			
0c	y→( )	40			
0d	e	12			
10	ROLL ↓	31	ENTER		
11	STOP	41	a_{14}	—	—
12	ROLL ↑	22			
13	÷	35			
14	y→( )	40			
15	b	14			
16	ROLL ↓	31	ENTER		
17	STOP	41	P_1	—	—
18	ROLL ↑	22			
19	÷	35			
1a	y→( )	40			
1b	a	13	ENTER		
1c	STOP	41	a_{21}	—	—
1d	↑	27			
20	↑	27			
21	f	15			
22	×	36	ENTER		
23	STOP	41	a_{22}	—	—
24	x⇄y	30			
25	—	34			
26	y→( )	40			
27	c	16			
28	↓	25			
29	e	12			
2a	×	36	ENTER		
2b	STOP	41	a_{23}	—	—
2c	x⇄y	30			
2d	—	34			

Step	Key	Code	Display x	Display y	Display z
30	STOP	41	a_{24}	—	—
31	x→( )	23	ENTER		
32	d	17			
33	c	16			
34	÷	35			
35	y⇄( )	24			
36	d	17			
37	b	14			
38	ROLL ↑	22			
39	×	36			
3a	ROLL ↓	31			
3b	—	34			
3c	c	16			
3d	÷	35			
40	y⇄( )	24			
41	a	13			
42	y→( )	40			
43	—	34			
44	f	15			
45	↓	25			
46	×	36	ENTER		
47	STOP	41	P_2	—	—
48	x⇄y	30			
49	—	34			
4a	c	16			
4b	÷	35	ENTER		
4c	STOP	41	a_{31}	—	—
4d	ROLL ↑	22			
50	f	15			
51	x⇄y	30			
52	×	36			
53	ROLL ↓	31			
54	y→( )	40			
55	—	34			
56	e	12			
57	x⇄y	30	ENTER		
58	STOP	41	a_{32}	—	—
59	x⇄y	30			
5a	—	34			
5b	y→( )	40			
5c	c	16			
5d	d	17			

Step	Key	Code	Display x	Display y	Display z
60	×	36			
61	e	12			
62	ROLL ↑	22			
63	×	36			
64	ROLL ↓	31			
65	+	33	ENTER		
66	STOP	41	a_{33}	—	—
67	x⇄y	30			
68	—	34			
69	y⇄( )	24			
6a	a	13			
6b	y→( )	40			
6c	—	34			
6d	d	17			
70	c	16			
71	×	36			
72	b	14			
73	ROLL ↑	22			
74	×	36			
75	ROLL ↓	31			
76	+	33	ENTER		
77	STOP	41	a_{34}	—	—
78	x⇄y	30			
79	—	34			
7a	a	13			
7b	÷	35			
7c	y⇄( )	24			
7d	—	34			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	y	z
8 0 (+)	e	12			
1	y→( )	40			
2	−	34			
3	c	16			
4	c	16			
5	×	36			
6	ROLL ↑	22			
7	x⇄y	30			
8	x←( )	67			
9	−	34			
a	f	15			
b	x⇄y	30			
c	×	36			
d	ROLL ↓	31			

Step	Key	Code	Display x	y	z
9 0 (+)	+	33	ENTER		
1	STOP	41	P_3	—	—
2	x⇄y	30			
3	−	34			
4	a	13			
5	÷	35			
6	y→( )	40			
7	−	34			
8	b	14	ENTER		
9	STOP	41	a_{41}	—	—
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			

Step	Key	Code	Display x	y	z
0 0 (−)	↑	27			
1	↑	27			
2	f	15			
3	×	36	ENTER		
4	STOP	41	a_{42}	—	—
5	x⇄y	30			
6	−	34			
7	y→( )	40			
8	c	16			
9	d	17			
a	×	36			
b	e	12			
c	ROLL ↑	22			
d	×	36			

Step	Key	Code	Display x	y	z
1 0 (−)	ROLL ↓	31			
1	+	33	ENTER		
2	STOP	41	a_{43}	—	—
3	x⇄y	30			
4	−	34			
5	↓	25			
6	↑	27			
7	y⇄( )	24			
8	−	34			
9	e	12			
a	y→( )	40			
b	a	13			
c	×	36			
d	y⇄( )	24			

Step	Key	Code	Display x	y	z
2 0 (−)	−	34			
1	d	17			
2	c	16			
3	x⇄y	30			
4	×	36			
5	x⇄y	30			
6	y⇄( )	24			
7	−	34			
8	d	17			
9	+	33			
a	b	14			
b	ROLL ↑	22			
c	×	36			
d	ROLL ↓	31			

Step	Key	Code	Display x	y	z
3 0 (−)	+	33	ENTER		
1	STOP	41	a_{44}	—	—
2	x⇄y	30			
3	−	34			
4	y⇄( )	24			
5	−	34			
6	f	15	ENTER		
7	STOP	41	P_4	—	—
8	ROLL ↓	31			
9	×	36			
a	ROLL ↓	31			
b	−	34			
c	x⇄y	30			
d	y⇄( )	24			

Step	Key	Code	Display x	y	z
4 0 (−)	−	34			
1	b	14			
2	ROLL ↓	31			
3	y⇄( )	24			
4	−	34			
5	e	12			
6	×	36			
7	ROLL ↓	31			
8	−	34			
9	ROLL ↓	31			
a	y⇄( )	24			
b	−	34			
c	c	16			
d	ROLL ↑	22			

Step	Key	Code	Display x	y	z
5 0 (−)	c	16			
1	ROLL ↑	22			
2	×	36			
3	ROLL ↓	31			
4	−	34			
5	x←( )	67			
6	−	34			
7	f	15			
8	÷	35			
9	a	13			
a	y→( )	40			
b	c	16			
c	×	36			
d	x⇄y	30			

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
6 (−) 0	y⇄( )	24			
1	−	34			
2	c	16			
3	−	34			
4	y→( )	40			
5	a	13			
6	d	17			
7	×	36			
8	ROLL ↓	31			
9	−	34			
a	ROLL ↑	22			
b	y⇄( )	24			
c	−	34			
d	d	17			
7 (−) 0	c	16			
1	×	36			
2	ROLL ↓	31			
3	−	34			
4	y→( )	40			
5	d	17			
6	f	15			
7	×	36			
8	a	13			
9	↑	27			
a	e	12			
b	×	36			
c	↓	25			
d	+	33			
8 (−) 0	c	16			
1	↑	27			
2	b	14			
3	×	36			
4	↓	25			
5	+	33			
6	↓	25			
7	y⇄( )	24			
8	−	34			
9	e	12			
a	−	34			
b	d	17			
c	↑	27			
d	0	00			
9 (−) 0	ROLL ↓	31	DISPLAY		
1	STOP	41	X_2	X_1	0
2	CONT	47			
3	a	13			
4	↑	27			
5	c	16			
6	↑	27			
7	0	00			
8	ROLL ↓	31			
9	CONT	47	DISPLAY		
a	END	46	X_4	X_3	0
b					
c					
d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

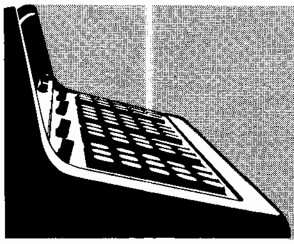
Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



CONVOLUTION WITH PLOT

This program uses the convolution integral to find the response of a system to an arbitrary forcing function. The forcing function and the impulse response of the system must be known.

Development:

The convolution integral is: $e(t) * h(t) = y(t) = \int_0^t e(\tau)\, h(t-\tau)\, d\tau$

where e (t) is the forcing function

y (t) is the system response due to the forcing function.

h (t) is the impulse response.

It is assumed that the forcing function is zero prior to time equal zero.

NOTE: X_{coef} and Y_{coef} are multiplicative scaling constants applied in the plotting subroutine.

Reference: Transform and State Variable Methods in Linear Systems --
Someshwar C. Gupta
Publisher - John Wiley and Sons, Inc.
Copyright - 1966, 1st Edition

USER INSTRUCTIONS

SET: Decimal Wheel at 6 or less

PRESS: STOP

Using the origin controls, place the pen at an appropriate initial position. A trial run with large Δt will help to determine a proper origin.

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→PRESS: GO TO

PRESS: —

PRESS: 0

PRESS: 0

Enter program steps to form $e(\tau) \cdot h(t-\tau)$ and place this quantity in the Y register. τ is in the X register and in storage register e. t is in storage register d. Registers -0 through -8 are available for generating the product.

The last step must be:

RETURN

SET: | | RUN |

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: Xcoef→X, Ycoef→Y

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: Panels/unit time (N) →Y
Time increment (Δt) →X

→PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

y(t)	Z
t	Y
0	X

NOTE: Each successive depression of the CONTINUE button increments t by the amount Δt and calculates y (t) for the incremented value of t.

y (t) is multiplied by Ycoef, X (t) is multiplied by Xcoef and the scaled values are plotted. To generate the plot free of interruptions replace the STOP at + (8 - 4) with a CONTINUE. *

To run another case

* If no plot is desired, place a RETURN in (-9, 0). To scale the plot axis with AXIS PLOT (09100-76007) use Yshift = 0, Xshift = 0, Yscale = 500/Ycoef, Xscale = 500/Xcoef, Yorigin = 0, and Xorigin = 0.

EXAMPLE

(A) Find the response of a circuit with an impulse response of e^{-t} to a forcing function of u (t) -- for $0 \leq t \leq \infty$.

Therefore: $e(\tau) = u(\tau) = 1$ for $0 \leq t \leq \infty$

$$h(t-\tau) = e^{-(t-\tau)} = e^{\tau - t}$$

Y coef = 5000

X coef = 1000

N = 10

Δt = .05

Integrating analytically, the result is:

$$\int_0^t e(\tau)\,h(t-\tau)\,d\tau = \int_0^t e^{(\tau-t)}\,d\tau = 1 - e^{-t} \text{ for } t > 0$$

Results:

t	y (t)
1	.63212
2	.86466
3	.95021
4	.98168
5	.99326

EXAMPLE (Con't)

(B) Find the response of a circuit with an impulse response of e^{-t} to a forcing function of sin 2t -- for $0 \leq t \leq \pi$.

Therefore: $e(\tau) = \sin(2\tau)$ for $0 \leq t \leq \pi$

$$h(t-\tau) = e^{-(t-\tau)} = e^{(\tau-t)}$$

SET: RADIANS

Ycoef = 4000

Xcoef = 1000

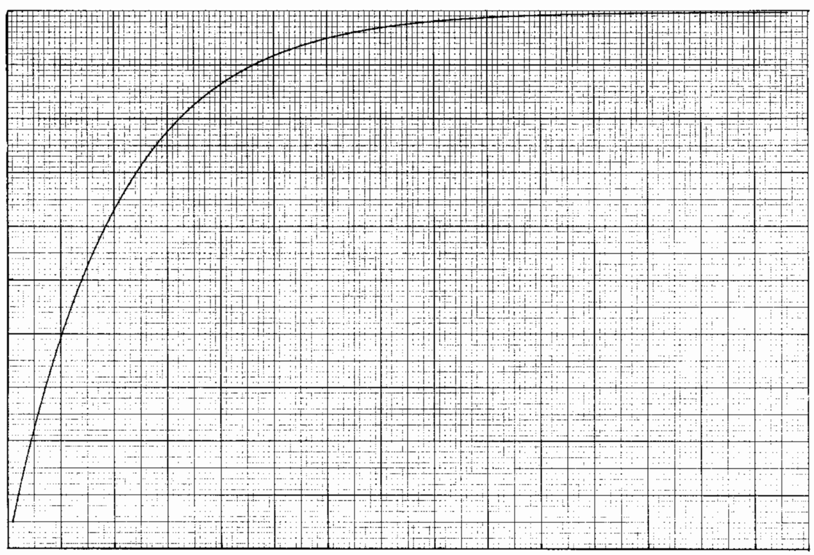
N = 20

$\Delta t = .05$

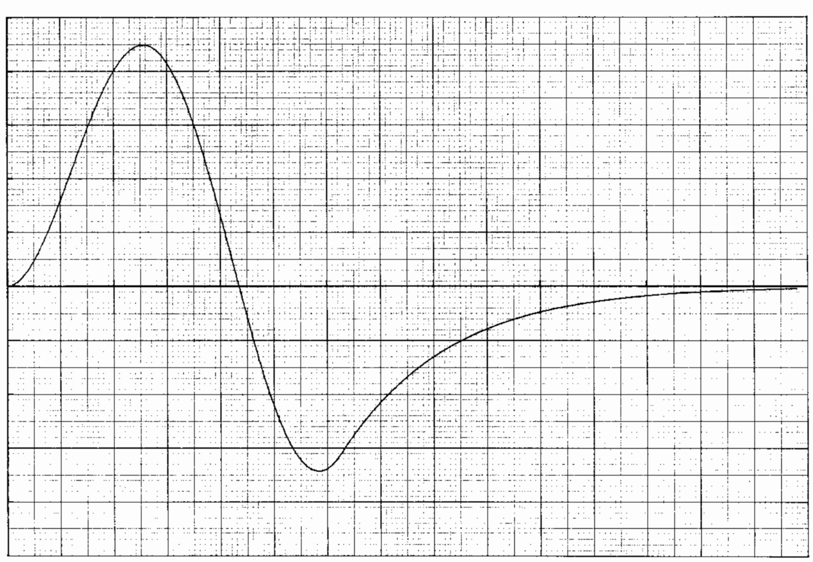
EXAMPLE (Con't)

Results:

t	y(t)
.5	.1948
1.0	.4955
1.5	.5135
2.0	.1642
2.5	-.2724
3.0	-.4200
3.5	-.2672
4.0	-.1626
4.5	-.0983
5.0	-.0596



EXAMPLE A



EXAMPLE B

Step	Key	Code	Display x	Display y	Display z
0 0 (+)	CLEAR	20			
1	FMT	42			
2	↓	25			
3	1	01	ENTER		
4	STOP	41	X_{coef}	Y_{coef}	0
5	x→( )	23			
6	−	34			
7	f	15			
8	y→( )	40			
9	−	34			
a	e	12			
b	CLEAR	20			
c	2	02	ENTER		
d	STOP	41	Δt	panels	0
1 0 (+)	y→( )	40			
1	b	14			
2	ROLL ↑	22			
3	y→( )	40			
4	a	13			
5	+	33			
6	y→( )	40			
7	d	17			
8	↓	25			
9	×	36			
a	•	21			
b	5	05			
c	+	33			
d	↓	25			
2 0 (+)	int x	64			
1	↑	27			
2	↑	27			
3	2	02			
4	÷	35			
5	↓	25			
6	↑	27			
7	int x	64			
8	IF x<y	52			
9	ROLL ↓	31			
a	1	01			
b	+	33			
c	↑	27			
d	RCL	61			

Step	Key	Code	Display x	Display y	Display z
3 0 (+)	ACC −	63			
1	d	17			
2	x⇄y	30			
3	CLEAR x	37			
4	ROLL ↓	31			
5	x⇄y	30			
6	÷	35			
7	y→( )	40			
8	c	16			
9	ROLL ↑	22			
a	x→( )	23			
b	e	12			
c	GO TO ( )	44			
d	▲SUB▼	77			
4 0 (+)	−	34			
1	0	00			
2	0	00			
3	e	12			
4	↑	27			
5	CLEAR x	37			
6	IF x=y	50			
7	7	07			
8	0	00			
9	d	17			
a	−	34			
b	|y|	55			
c	ENTER EXP	26			
d	9	11			
5 0 (+)	CHG SIGN	32			
1	IF x>y	53			
2	7	07			
3	6	06			
4	e	12			
5	x⇄y	30			
6	d	17			
7	IF x<y	52			
8	0	00			
9	0	00			
a	c	16			
b	ROLL ↑	22			
c	↑	27			
d	IF FLAG	43			

Step	Key	Code	Display x	Display y	Display z
6 0 (+)	6	06			
1	7	07			
2	SET FLAG	54			
3	4	04			
4	GO TO ( )	44			
5	6	06			
6	8	10			
7	2	02			
8	×	36			
9	↓	25			
a	ACC +	60			
b	GO TO ( )	44			
c	3	03			
d	b	14			
7 0 (+)	c	16			
1	ROLL ↑	22			
2	ACC +	60			
3	GO TO ( )	44			
4	3	03			
5	b	14			
6	CLEAR x	37			
7	ROLL ↑	22			
8	ACC +	60			
9	RCL	61			
a	↑	27			
b	c	16			
c	×	36			
d	3	03			

Storage

Register	+	−
f		X_{coef}
e	τ	Y_{coef}
d	ΣΔt	t
c	ΔX	
b	N	
a	Δt	
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
8 (+) 0	÷	35			
1	CLEAR x	37			
2	ROLL ↑	22			
3	$x \rightleftarrows y$	30	DISPLAY		
4	STOP	41	0	t	y(t)
5	CONT	47			
6	$y\to( )$	40			
7	−	34			
8	d	17			
9	GO TO ( )	44			
a	▲SUB▼	77			
b	−	34			
c	9	11			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
9 (+) 0	b	14			
1	↑	27			
2	$x\leftarrow( )$	67			
3	−	34			
4	d	17			
5	↑	27			
6	IF FLAG	43			
7	1	01			
8	4	04			
9	GO TO ( )	44			
a	0	00			
b	0	00			
c					
d					

Step	Key	Code	Display x	Display y	Display z
9 (−) 0	$x\leftarrow( )$	67			
1	−	34			
2	f	15			
3	×	36			
4	ROLL ↑	22			
5	$x \rightleftarrows y$	30			
6	$x\leftarrow( )$	67			
7	−	34			
8	e	12			
9	×	36			
a	$x \rightleftarrows y$	30			
b	ROLL ↑	22			
c	↑	27			
d	7	07			

Step	Key	Code	Display x	Display y	Display z
a (−) 0	5	05			
1	0	00			
2	0	00			
3	IF $x<y$	52			
4	$y\to( )$	40			
5	RETURN	77			
6	↓	25			
7	FMT	42			
8	↓	25			
9	RETURN	77			
a					
b					
c					
d					

0 (−): Registers (−)(0) through (−)(8) available for generation $e(\tau)\cdot h(t-\tau)$

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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EXAMPLE A

Step		Key	Code	Display x	Display y	Display z
0	0	↑	27			
(–)	1	d	17			
	2	−	34			
	3	↓	25			
	4	e^x	74			
	5	↑	27			
	6	RETURN	77			

EXAMPLE B

Step		Key	Code	Display x	Display y	Display z
0	0	↑	27			
(–)	1	d	17			
	2	−	34			
	3	↓	25			
	4	e^x	74			
	5	↑	27			
	6	π	56			
	7	↑	27			
	8	e	12			
	9	IF x>y	53			
	a	CLEAR x	37			
	b	↑	27			
	c	x⇄y	30			
	d	2	02			
1	0	×	36			
(–)	1	↓	25			
	2	sin x	70			
	3	×	36			
	4	RETURN	77			

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

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Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

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Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

9100B ONLY
PART NO.
09100-70409

NUMERICAL INTEGRATION USING SIMPSON'S RULE WHEN f(x) IS KNOWN

This program evaluates the integral of a known f(x) using Simpson's rule. The equation is:

$$\int_a^b f(X)dX \cong \frac{\Delta X}{3}\left[f(a) + 4f(a+\Delta X) + 2f(a+2\Delta X) + \ldots + 2f\left\{a+(n-2)\Delta X\right\} + 4f\left\{a+(n-1)\Delta X\right\} + f(b)\right]$$

for n panels (n must be even) where $\Delta X = \frac{b-a}{n}$

The specific f(x) is programmed into the calculator by the user as a subroutine and is then used by the general solution to evaluate the integral. Execution time is dependent on the number of panels. Note f(x) should not have any singularities in the integration interval.

Reference:

Numerical Analysis
by
Kaiser S. Kunz

McGraw-Hill Book Company, Inc. (1967)

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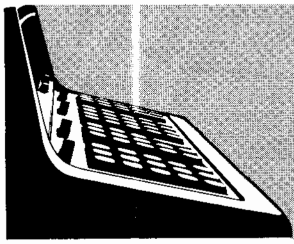
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



INTEGRAL OF THE FORM: $F(x) = \int_a^x f(u)\,du$ WITH PLOT

9100B ONLY
PART NO.
09100-70410

This program evaluates the integral of a function f(u) between any lower limit a and a successively incremented upper limit.

A modification of Simpson's one third rule is used to perform the integration. The following equations are used:

$$\text{For } j = 0, \quad \int_a^{x_j} f(u)\,du = h/6 \left[1/3\, f(a) + 4/3\, f(x_{j+1}) + 1/3\, f(x_{j+2}) \right]$$

$$\text{For } j \geq 3, \quad \int_{x_{j-1}}^{x_j} f(u)\,du = h/24 \left[f(x_{j-3}) + 5f(x_{j-2}) + 19\, f(x_{j-1}) + 9f(x_j) \right]$$

Notes:

Due to the manner in which the program is written, the function will initially be integrated over an area formed by twice the increment; thereafter, integration will proceed by one increment at a time.

To integrate with some constant (a) as a lower limit (other than zero), in the program steps in the f(u) subroutine add a to u to form u' and form f(u'). For a lower limit of zero simply form f(u). In each case the upper limit will exceed the lower by some multiple of the increment depending on the number of times x is incremented by depression of the CONTINUE key.

To integrate from some lower limit to a specified upper limit without continually depressing the CONTINUE key, enter a Pause at step (5) (7). Then just before the incremented upper limit (shown in a flashing display) reaches the desired upper limit, depress PAUSE--this will stop the program after the next integration.

If a plotter is available a plot of the function being integrated may be obtained by inserting a plot subroutine at (-9) (0). This plot subroutine scales the calculated X and F(X) by multiplying them by X_{coef} and Y_{coef} respectively.

Reference: Numerical Analysis
by
Faiser S. Kunz
McGraw-Hill Book Company, Inc.

Using origin controls, position pen at (0, Y) *

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: GO TO

PRESS: –

PRESS: 0

PRESS: 0

SET: [PROGRAM |]

Starting at (-0) (0) enter the program steps to form the function f(u) to be integrated; u is located in the Y register. After forming f(u) place it in the Y register. The last step of the subroutine must be:

RETURN

NOTE: Registers (-0) thru (-8) are available for programming and storage of f(u).

SET: [| RUN]

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $Y_{coef} \rightarrow Y$, $X_{coef} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: Increment h $\rightarrow$ X

PRESS: CONTINUE

DISPLAY

$\int_a^x f(u)\,du$	Z
(X - a)	Y
0	X

Each successive "CONTINUE" increments the upper limit and evaluates the integral using the increment upper limit. The integral is plotted after each x incrementation.

If no plot is desired, place a RETURN in (-9)(0) and CONTINUES in (0)(1) and (0)(2).

EXAMPLE

Obtain the cumulative distribution function of the normally distributed random variable X of variance 1 and mean value of 0

The probability density function of X is

$$P(X) = \frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}}$$

The cumulative distribution function is given by

$$Q(X) = \int_{-\infty}^{x} P(u)\,du$$

The lower limit ($-\infty$) can be replaced by a = -4 without loss of accuracy.

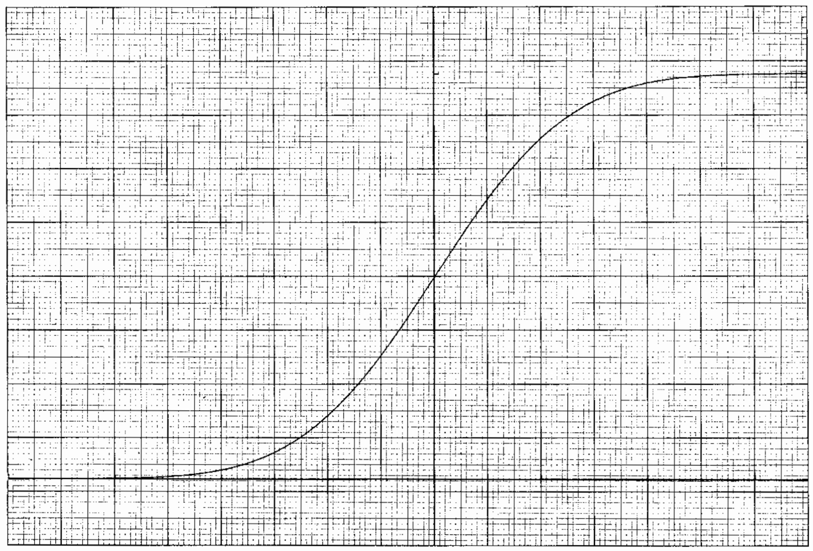
The program steps to generate P(u) are given on page 2 .

Data: Ycoef = 3760, Xcoef = 1000, h = .1

Results

X + 4	$\int_{-4}^{x} P(u)du$	X
0.5	.000201	-3.5
1.0	.001318	-3.0
1.5	.006178	-2.5
2.0	.022719	-2.0
2.5	.066775	-1.5
3.0	.158622	-1.0
3.5	.308504	-0.5
4.0	.499968	-0.0
4.5	.691432	0.5
5.0	.841314	1.0
5.5	.933162	1.5
6.0	.977218	2.0
6.5	.993758	2.5
7.0	.998618	3.0
7.5	.999736	3.5
8.0	.999937	4.0

*The Y (inches) must be determined by trial.



Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	FMT	42			
2	↓	25			
3	1	01	ENTER		
4	STOP	41	X_{coef}	Y_{coef}	0
5	x→( )	23			
6	—	34			
7	f	15			
8	y→( )	40			
9	—	34			
a	e	12			
b	CLEAR	20			
c	2	02	ENTER		
d	STOP	41	h	0	0
1 0	x→( )	23			
(+) 1	d	17			
2	GO TO ( )	44			
3	▲SUB▼	77			
4	—	34			
5	0	00			
6	0	00			
7	2	02			
8	4	04			
9	÷	35			
a	e	12			
b	↑	27			
c	0	00			
d	IF x=y	50			
2 0	6	06			
(+) 1	3	03			
2	1	01			
3	IF x=y	50			
4	7	07			
5	0	00			
6	2	02			
7	IF x=y	50			
8	7	07			
9	8	10			
a	0	00			
b	x⇄y	30			
c	9	11			
d	ROLL ↑	22			

Step	Key	Code	Display x	Display y	Display z
3 0	×	36			
(+) 1	ROLL ↓	31			
2	ACC +	60			
3	1	01			
4	9	11			
5	ROLL ↓	31			
6	y⇄( )	24			
7	a	13			
8	ROLL ↓	31			
9	×	36			
a	ROLL ↓	31			
b	ACC +	60			
c	5	05			
d	ROLL ↓	31			
4 0	y⇄( )	24			
(+) 1	b	14			
2	ROLL ↓	31			
3	×	36			
4	ROLL ↓	31			
5	ACC −	63			
6	↓	25			
7	1	01			
8	y⇄( )	24			
9	c	16			
a	x⇄y	30			
b	ACC +	60			
c	RCL	61			
d	↑	27			
5 0	d	17			
(+) 1	×	36			
2	ROLL ↑	22			
3	×	36			
4	d	17			
5	—	34			
6	CLEAR x	37	DISPLAY		
7	STOP	41	0	X-a	$\int_a^x$
8	CONT	47			
9	GO TO ( )	44			
a	▲SUB▼	77			
b	—	34			
c	9	11			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
6 0	GO TO ( )	44			
(+) 1	8	10			
2	3	03			
3	↓	25			
4	y→( )	40			
5	c	16			
6	8	10			
7	×	36			
8	1	01			
9	x⇄y	30			
a	ACC +	60			
b	GO TO ( )	44			
c	8	10			
d	3	03			
7 0	↓	25			
(+) 1	y→( )	40			
2	b	14			
3	3	03			
4	2	02			
5	GO TO ( )	44			
6	6	06			
7	7	07			
8	↓	25			
9	y→( )	40			
a	a	13			
b	8	10			
c	×	36			
d	1	01			

Storage

	+	−
f	$f(x_j)$	X coef
e	j	Y coef
d	h	
c		
b		
a		
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
8 0	GO TO ( )	44			
(+) 1	4	04			
2	a	13			
3	e	12			
4	↑	27			
5	d	17			
6	×	36			
7	GO TO ( )	44			
8	1	01			
9	2	02			
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
9 0	x←( )	67			
(−) 1	−	34			
2	f	15			
3	×	36			
4	ROLL ↑	22			
5	x⇄y	30			
6	x←( )	67			
7	−	34			
8	e	12			
9	×	36			
a	x⇄y	30			
b	ROLL ↑	22			
c	FMT	42			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
0 0	4	04			
(−) 1	CHG SIGN	32			
2	+	33			
3	↓	25			
4	↑	27			
5	×	36			
6	2	02			
7	÷	35			
8	↓	25			
9	CHG SIGN	32			
a	e^x	74			
b	↑	27			
c	π	56			
d	↑	27			
1 0	2	02			
(−) 1	×	36			
2	↓	25			
3	√x	76			
4	÷	35			
5	RETURN	77			
6					
7					
8					
9					
a					
b					
c					
d					

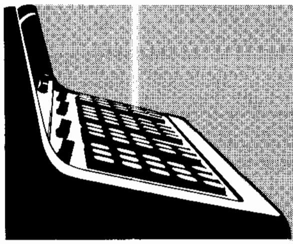
} EXAMPLE (steps 0 0 – 1 5)

Step	Key	Code	Display x	Display y	Display z
a 0	RETURN	77			
(−) 1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



MAXIMUM AND MINIMUM OF Z = Z(X, Y)

This program will calculate the approximate maximum and minimum of a function of two independent variables over specified ranges of these two variables. The method involved is a direct evaluation of the function at a specified number of points matrixly distributed about the X - Y grid.

The program was written mainly for use in conjunction with program 09100-70412 but may also be used for such purposes as obtaining upper and lower bounds within a particular area.

It is recommended that the program be run at least twice within one range (with the sampling number increased the second time) to verify the results of the first run.

USER INSTRUCTIONS

ENTER PROGRAM: Starting Address is (-0) (0)

PRESS: GO TO (+) (3) (a)

SET: [PROGRAM |]

Enter the program steps which take X from the +e register and Y from the +9 register and generate Z(X, Y) in the Y register and program a RETURN for the final step. The flag is not available. (Leave +e and +9 unchanged)

NOTE: Locations + (3) (a) through + (8) (d) are available. (74 steps)

SET: [| RUN]

→ PRESS: GO TO (-) (0) (0)

PRESS: CONTINUE

DISPLAY

0	Z
1	Y
1	X

ENTER RANGE OF X: X_{max} → Y, X_{min} → X

PRESS: CONTINUE

DISPLAY

0	Z
2	Y
2	X

ENTER RANGE OF Y: Y_{max} → Y, Y_{min} → X

PRESS: CONTINUE

DISPLAY

0	Z
3	Y
3	X

ENTER NUMBER OF SAMPLES:
Total number of samples to be taken → X

PRESS: CONTINUE

NOTE: The screen may be blanked for several minutes, the actual time depending upon the specified number of samples and the particular function involved.

USER INSTRUCTIONS (Con't)

DISPLAY

0	Z
Z_{max}	Y
Z_{min}	X

← To re-run this program with different inputs.

EXAMPLE

Find the max - min of $Z = y^2 - x^2$ where

$-2 \le X \le 2$

$-2 \le Y \le 2$

Allow 100 samples of the function

Solution:

$-4 \le Z \le 4$

Program steps to generate function

3 a	x←( )	67			
(+) b	9	11			
c	↑	27			
d	×	36			
4 0	e	12			
(+) 1	↑	27			
2	×	36			
3	↓	25			
4	−	34			
5	RETURN	77			

 PART NO. 09100-70411 9100B ONLY

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
01	1	01			
02	↑	27	ENTER		
03	STOP	41	X_{min}	X_{max}	0
04	x→( )	23			
05	c	16			
06	y→( )	40			
07	d	17			
08	CLEAR	20			
09	2	02			
0a	↑	27	ENTER		
0b	STOP	41	Y_{min}	Y_{max}	0
0c	x→( )	23			
0d	9	11			
10	y→( )	40			
11	b	14			
12	CLEAR	20			
13	3	03			
14	↑	27	ENTER		
15	STOP	41	N	0	0
16	√x	76			
17	int x	64			
18	↑	27			
19	d	17			
1a	↑	27			
1b	c	16			
1c	−	34			
1d	ROLL ↓	31			
20	x⇄y	30			
21	÷	35			
22	ROLL ↓	31			
23	−	34			
24	ACC +	60			
25	b	14			
26	x⇄y	30			
27	x←( )	67			
28	9	11			
29	−	34			
2a	↓	25			
2b	x⇄y	30			
2c	÷	35			
2d	y→( )	40			
30	a	13			
31	ENTER EXP	26			
32	9	11			
33	9	11			
34	x→( )	23			
35	−	34			
36	e	12			
37	CHG SIGN	32			
38	x→( )	23			
39	−	34			
3a	f	15			
3b	RCL	61			
3c	ACC −	63			
3d	+	33			
40	ACC +	60			
41	d	17			
42	IF x<y	52			
43	5	05			
44	d	17			
45	GO TO ( )	44			
46	▲SUB▼	77			
47	+	33			
48	3	03			
49	a	13			
4a	x←( )	67			
4b	−	34			
4c	e	12			
4d	IF x>y	53			
50	y→( )	40			
51	−	34			
52	e	12			
53	x←( )	67			
54	−	34			
55	f	15			
56	IF x<y	52			
57	y→( )	40			
58	−	34			
59	f	15			
5a	GO TO ( )	44			
5b	3	03			
5c	b	14			
5d	y⇄( )	24			
60	9	11			
61	a	13			
62	+	33			
63	b	14			
64	IF x<y	52			
65	7	07			
66	5	05			
67	y→( )	40			
68	9	11			
69	RCL	61			
6a	ACC −	63			
6b	x⇄y	30	DISPLAY		
6c	c	16	Z_{min}	Z_{max}	0
6d	x⇄y	30			
70	−	34			
71	ACC +	60			
72	GO TO ( )	44			
73	3	03			
74	b	14			
75	CLEAR	20			
76	y⇄( )	24			
77	−	34			
78	f	15			
79	x←( )	67			
7a	−	34			
7b	e	12			
7c	END	46			
7d					

Storage

Register	+	−
f	ΔX	Z max
e	X	Z min
d	X max	
c	X min	
b	Y max	
a	ΔY	
9	Y min	
8		
7		
6		
5		
4		
3		
2		
1		
0		

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Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

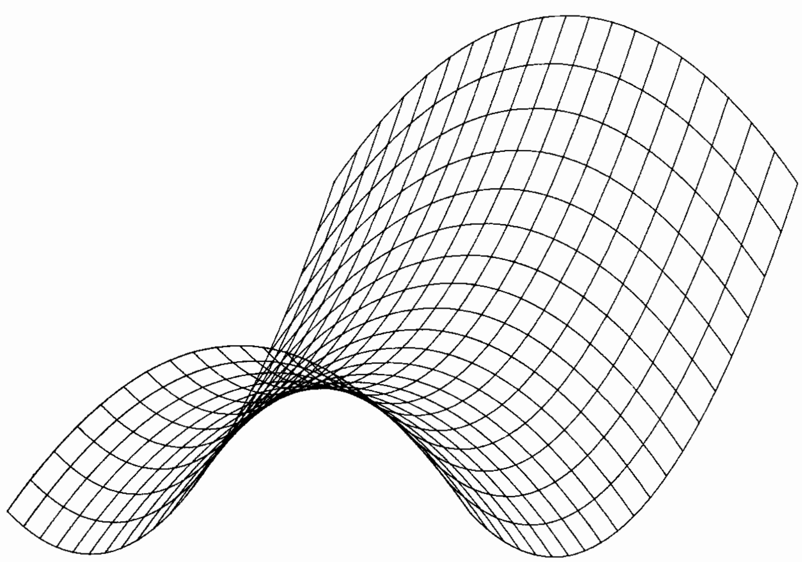
9100B ONLY
PART NO.
09100-70412

PLOT OF Z = Z(X, Y)

This program will plot in three dimensions a wide variety of functions of two independent variables. The user specifies the ranges of the three variables X, Y and Z. If the range of Z is unknown, program 09100-70411, Maximum and Minimum of Z = Z(X, Y) may be used to determine its range for particular ranges of X and Y.

The algorithm incorporated forms an X - Y grid of the independent variables, scales the grid and rotates it in the X - Y plane. It then proceeds to plot the evaluation of the function as incremented lines of elevation at scaled distances above the grid. The function is plotted in a cross hatched pattern, holding one variable as a parameter and incrementing the other variable then reversing these roles to plot in an orthogonal direction.

The angle of rotation and degree of resolution are built into the program and the three necessary scale factors are generated before the plot begins. The instructions for changing any of these to obtain a different perspective of the plot are given at the end of the user instructions.



USER INSTRUCTIONS

PRESS: STOP

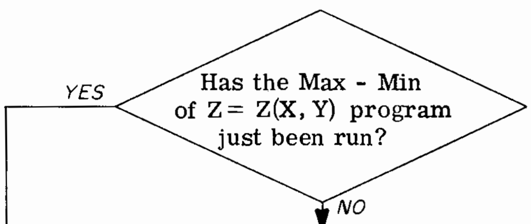
Using the origin controls, locate the pen in the lower left corner of the paper.

SET: Decimal Wheel at 6 or less

SET: | DEGREES | |

PRESS: END

ENTER PROGRAM: Side A followed by Side B



PRESS: GO TO (+) (3) (a)

SET: | PROGRAM | |

Enter the program steps which take X from the + e register and Y from the +9 register and generate Z(X, Y). These variables must be left in their respective registers (+e and +9). Leave the programmed value of Z(X, Y) in the Y register and program a RETURN for the final step. The flag is not available.

NOTE: Locations + (3) (a) through + (8) (d) are available. (74 steps)

SET: | | RUN |

→ PRESS: END

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
1 ———— X

ENTER RANGE OF X: X_{max}→Y, X_{min}→X

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
2 ———— X

ENTER RANGE OF Y: Y_{max}→Y, Y_{min}→X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0 ———— Z
0 ———— Y
3 ———— X

ENTER RANGE OF Z: Z_{max}→Y, Z_{min}→X

PRESS: CONTINUE

The function will now be plotted. It is self-terminating.

NOTE: Execution of this program destroys the −0 and the −1 registers of program steps. To re-run the program, side B must be re-entered at location −(0)(0).

EXAMPLE

Plot the saddle function $Z = y^2 - x^2$
where
$-2 \leq X \leq 2$

$-2 \leq Y \leq 2$

The range of Z is
$-4 \leq Z \leq 4$

Program steps to generate function

3 (+)	a	x←( )	67			
	b	9	11			
	c	↑	27			
	d	×	36			
4 (+)	0	e	12			
	1	↑	27			
	2	×	36			
	3	↓	25			
	4	−	34			
	5	RETURN	77			

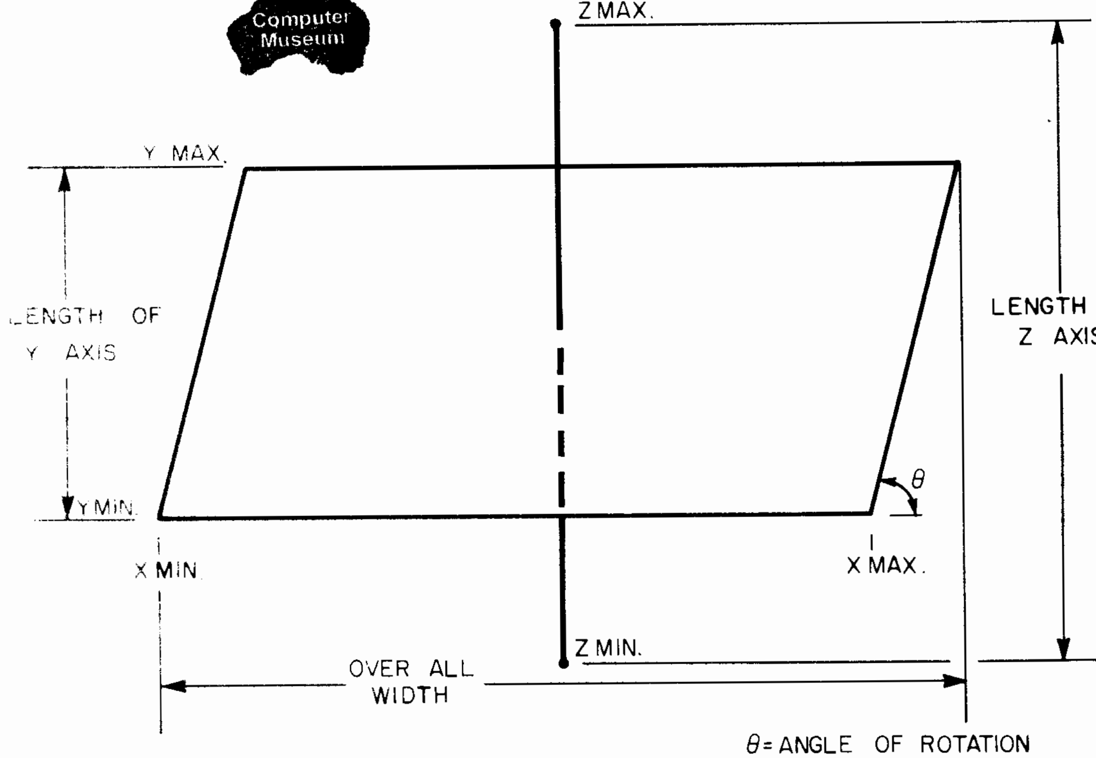


FIGURE 1

The instructions below explain how the user may modify the perspective of a plot. (Refer to Figure 1 for a geometrical representation of the terms.)

1. LENGTH OF Z AXIS: The distance from the minimum value of Z to the maximum value of Z of the plot is specified in plotter variables in locations (−)(2)(3) through (−)(2)(6). The present value is 2500 representing $\frac{2500}{500}$ = 5 inches $\left(\frac{2500}{200} = 12.5 \text{ cm}\right)$ on the plot surface.

2. LENGTH OF Y AXIS: The distance from the minimum value of Y to the maximum value of Y of the plot is specified in plotter variables in locations (+)(1)(b) through (+)(2)(0). The present value is 2500 representing $\frac{2500}{500}$ = 5 inches $\left(\frac{2500}{200} = 12.5 \text{ cm}\right)$ on the plot surface.

3. OVERALL WIDTH OF PLOT: The overall width of the plot is specified in plotter variables in locations (−)(1)(1) through (−)(1)(4). The present value is 6000 representing $\frac{6000}{500}$ = 12 inches $\left(\frac{6000}{200} = 30\text{cm}\right)$ on the plot surface.

4. NUMBER OF GRID LINES: The algorithm plots the function in two directions — across the X axis and across the Y axis. The number of grid lines plotted in each direction is specified by the number in storage (−)(b)(8) and (−)(b)(9). This programmed number is actually one less than the desired number. At present, a 20 is stored giving 21 grid lines in each direction.

5. RESOLUTION: The algorithm plots each grid line in a specified number of increments. This number is stored in (−)(3)(8) and (−)(3)(9). The present programmed increment factor is 40. This should be increased for higher resolution.

6. ANGLE OF ROTATION: The plot must be rotated to gain a useful view of the function. This angle is specified in storage locations (+)(2)(a) through (+)(2)(c). The present angle is 45°.

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
01	1	01	ENTER		
02	STOP	41	X_{min}	X_{max}	0
03	$x\to( )$	23			
04	c	16			
05	$y\to( )$	40			
06	d	17			
07	CLEAR	20			
08	2	02	ENTER		
09	STOP	41	Y_{min}	Y_{max}	0
0a	$x\to( )$	23			
0b	a	13			
0c	$x\to( )$	23			
0d	9	11			
10	$y\to( )$	40			
11	b	14			
12	−	34			
13	↑	27			
14	↓	25			
15	GO TO ( )	44			
16	▲SUB▼	77			
17	2	02			
18	a	13			
19	sin x	70			
1a	×	36			
1b	2	02			
1c	5	05			
1d	0	00			
20	0	00			
21	$x\rightleftarrows y$	30			
22	÷	35			
23	$y\to( )$	40			
24	−	34			
25	e	12			
26	GO TO ( )	44			
27	−	34			
28	0	00			
29	0	00			
2a	CONT	47			
2b	CONT	47			
2c	4	04			
2d	5	05			
30	RETURN	77			
31	SET FLAG	54			
32	$y\rightleftarrows( )$	24			
33	9	11			
34	$y\rightleftarrows( )$	24			
35	e	12			
36	$y\rightleftarrows( )$	24			
37	9	11			
38	RETURN	77			

Storage

	+	−
f	ΔX/ ΔY	X Grid
e	X/ Y	Y Grid
d	X max	
c	X min	
b	Y max	
a	Y min	
9	X min/ Y min	
8		
7		
6		
5		
4		
3		
2		
1		Z Grid
0		Z min

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0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
a				
b				
c				
d				

0				
1				
2				
3				
4				
5				
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7				
8				
9				
a				
b				
c				
d				

0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
a				
b				
c				
d				

HEWLETT·PACKARD

0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
a				
b				
c				
d				

0				
1				
2				
3				
4				
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6				
7				
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9				
a				
b				
c				
d				

0				
1				
2				
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4				
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6				
7				
8				
9				
a				
b				
c				
d				

HEWLETT·PACKARD

0				
1				
2				
3				
4				
5				
6				
7				
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9				
a				
b				
c				
d				

0				
1				
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4				
5				
6				
7				
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9				
a				
b				
c				
d				

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	