HP-85

WAVEFORM ANALYSIS PAC

HEWLETT PACKARD

HP-85

Waveform Analysis Pac

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Introduction

The Waveform Analysis Pac has been designed to provide you with immediate solutions to a wide variety of problems encountered in many diverse technological disciplines.

A knowledge of programming is not needed to use the programs in the Waveform Analysis Pac. However, you should be familiar with sections 1 through 5 of the Owner's Manual.

For each program in the Waveform Analysis Pac, there is a description, a set of user instructions and examples. Also included is a remarks program which contains program listing comments and variable definitions, to enable you to follow program flow if you so desire.

The Waveform Analysis Pac programs define the output peripherals in the standard manner: i.e., CRT is 1 and printer is 2 and uses PRINT and DISP statements accordingly. If you want to ensure that the peripherals are defined as the programs assume, press (RESET) before running a program. The currently defined key labels are obtainable at any time while a program is running by pressing (KEY LABEL). Remember to press (CLEAR -LINE) before pressing (END LINE) if the key labels are in the input line. All files on the Waveform Analysis Pac cartridge have been secured using a code of HP and a security type of 2. To store a changed version of a program, you must first unsecure the file using HP as the security code and 2 as the security type.

We hope that the Waveform Analysis Pac will assist you in the solution of numerous problems in your discipline.

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Contents

Format of User Instructions

The user instructions are your guide to operating the programs in this Pac.

Certain key words have been used to indicate specific types of operations. You should become familiar with the meanings of these words so that the intent of the user instructions can then easily be followed.

Key Word	Meaning/Use
INSERT	Put the tape cartridge into the tape transport.
PRESS	Push an immediate executable key, e.g., (END LINE) or (RUN) .
TYPE	Push a series of keys which form a command, e.g., Type: (REW LOAD) "S-WAVE".
ENTER	Push a series of keys as a response to a machine prompt, e.g., Enter: The number of data samples (END LINE) .
GO TO Step n	Change the flow in the user instructions.
REPEAT	Designates a repeatable group of instructions.
NOTE:	Extra comments concerning instructions for this step.

The user instructions are written in outline form so that you can easily follow the instructions and the flow of operation.

Whenever a special function key is labeled "HELP", the program includes a "HELP" section which displays a short description of the function of each special function key. After solving a few problems using the written User Instructions, you should be able to solve problems rapidly, referring only to the "HELP" program to refresh your memory.

The program flow will often ask for a "YES" or "NO" answer to a question. A "YES" answer requires that you enter "Y" or "YES" before pressing (END LINE) . A "NO" answer requires that you enter "N" or "NO" before pressing (END LINE) .

Program Operation Hints

These programs have been designed to execute with a minimum amount of difficulty, but problems may occur which you can easily solve during program operation. There are four different types of errors or warnings that can occur while executing a program: input errors, math errors, tape errors and image format string errors.

The input errors include errors 43, 44, and 45. All of these errors will cause a message to be output followed by a new question mark as a prompt for the input. You should verify your mistake and then enter the corrected input. The programs will not proceed until the input is acceptable. There is a more complete discussion of INPUT in your Owner's Manual.

The second type of error which might occur is the math errors (1 through 13). With DEFAULT ON, the first eight errors listed in appendix E of your Owner's Manual cause a warning message to be output, but program execution will not be halted. The cause of these errors can usually be attributed to specific characteristics of your

data and the type of calculations being performed. In most cases, there is no cause for alarm, but you should direct your attention to a possible problem. An example of such a case is found in the Standard Pac when the curve fitting program computes a curve fit to your data which has a value of 1 for the coefficient of determination, r^2. The computation of the F ratio results in a divide by zero, Warning 8.

The third type of error, tape errors (60 through 75) may be due to several different problems. Some of the most likely causes are the tape being write-protected, the wrong cartridge (or no cartridge) being inserted, a bad tape cartridge, or wrong data file name specification during program execution. Appendix E of your Owner's Manual should be consulted for a complete listing.

The fourth type of error is due to generalizing the output to anticipated data ranges. In many cases, the output has assumed ranges which may or may not be appropriate with your data. Adjusting the image format string for your data will solve this type of problem. You may also want to change the image string if you require more digits to the right of the decimal point.

Whenever a running program is interrupted from the keyboard by inadvertently pressing a key, the system beeps. To continue program execution, press $\boxed{\text{CONT}}$.

These are the more common problems which may occur during program operation. Your Owner's Manual should be consulted if you need more assistance.

Theory of Fourier Transforms

The Fourier transform, as used for waveform analysis, is a powerful tool for solving problems and analyzing data in many fields of engineering, physics and mathematics.

As originally developed, the transform was defined for continuous (or analog) functions. However, recent developments have made possible its use in digital form. The Fast Fourier Transform, developed by Cooley and Tukey, made practical the calculations necessary for implementation of the Discrete Fourier Transform with modern instrumentation and digital computers.

We are specifically concerned with use of the Discrete Fourier Transform, since we have only a finite number of data points obtained by sampling an analog waveform. By means of these data points, we must obtain enough information to mathematically recreate the waveform, thereby allowing mathematical processing of the signal.

Data Domains

Prior to discussion of the Fourier transform, we must define the terms used to describe time domain and frequency domain data.

Time Domain:

The parameters for time domain data are as follows:

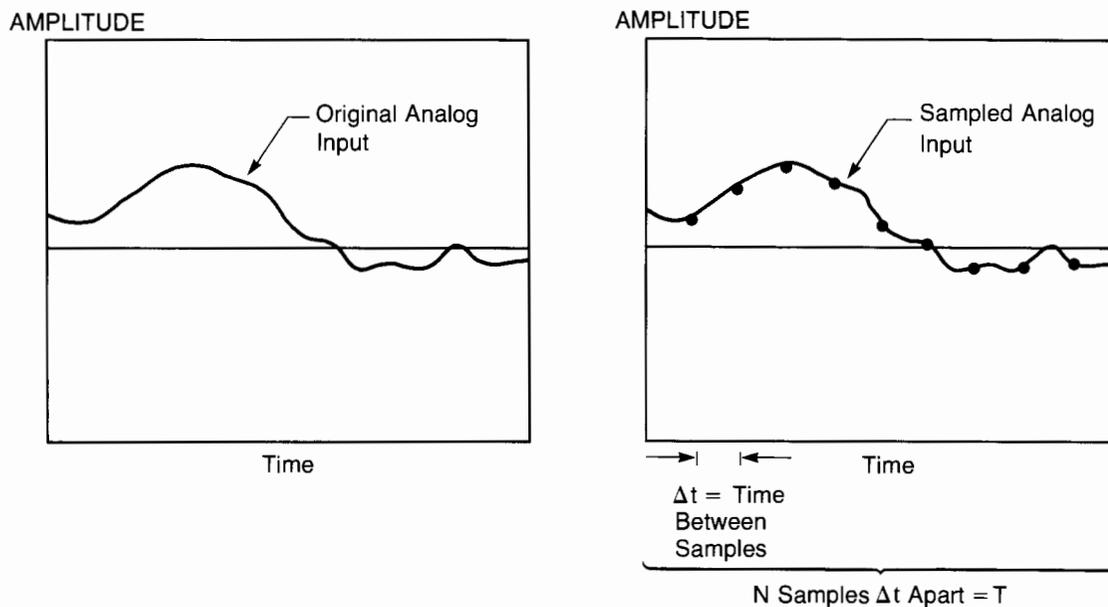Δt = the time interval, (i.e., the time between samples)

N = the data block size, (i.e., the number of samples taken)

T = the time window, (i.e., the total time of the data sample record)

From Figure 1, it can be seen that:

Time window = number of samples $\times$ time interval, or $T = N \Delta t$.

Figure 1: Time Domain

AMPLITUDE   AMPLITUDE

— Original Analog   — Sampled Analog
Input   Input

Time   Time

Δt = Time
Between
Samples

N Samples Δt Apart = T

Frequency Domain:

By performing a Fourier transform on the time domain data described in the previous section, we obtain a set of data in the frequency domain defined by the following parameters:

Δf = the frequency interval, (i.e., the number of Hz between frequency points). The origin is 0 Δf (the DC component of real (cosine) displays only). The next point is 1 Δf (the fundamental frequency); next 2 Δf (first harmonic); next 3 Δf (2nd harmonic), etc.

Since frequencies between the harmonics are not visible, they can only be shown by choosing a smaller frequency interval, Δf (i.e., increasing the frequency resolution). (Limits to this procedure are described on page xii.)

$N/2$ = The number of frequency points, (i.e., ½ the data block size). Each frequency component is broken into two parts: the real and the imaginary.

F_{max} = The maximum frequency.

From Figure 2: $F_{max} = N/2 \times \Delta f$

Figure 2: Frequency Domain

AMPLITUDE

Fourier Transform
of Sampled Input:
Cosine or Real Values

DC + (N/2) Frequency Points, Δf Apart

AMPLITUDE

Fourier Transform
of Sampled Input:
Sine or Imaginary Values

(N/2) − Frequency Points, Δf Apart
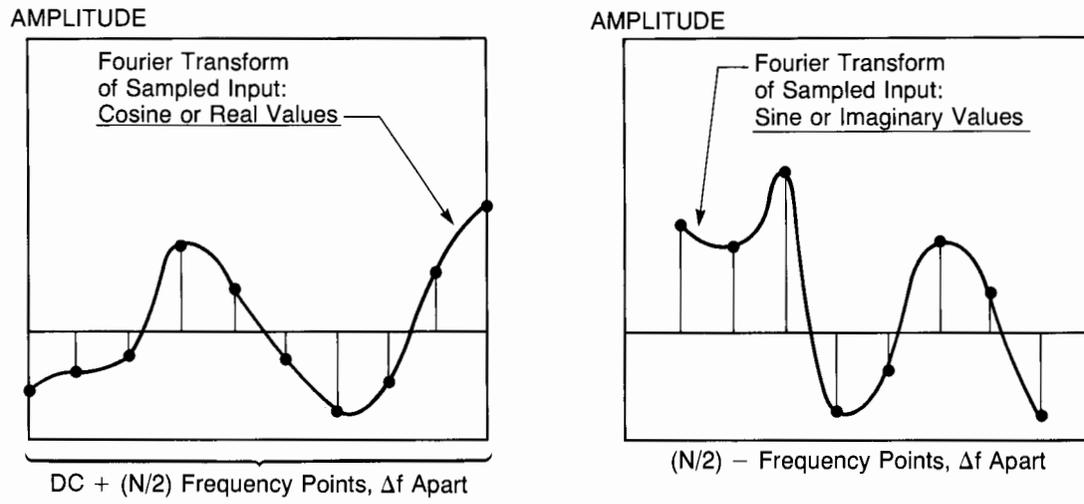
The relationships between the time and frequency domains are shown in Figure 3.
These relationships are:

the time interval, $\Delta t = \dfrac{1}{2\ F_{max}}$

the frequency interval, $\Delta f = \dfrac{1}{T}$

illustrating the dependence of a parameter in one domain to that in another.

Figure 3: The Relationship Between Time and Frequency Domains.

It is often desirable to make trade-offs in sampling the data to obtain the information in the most efficient manner. Table 1 summarizes the above equations for this purpose.
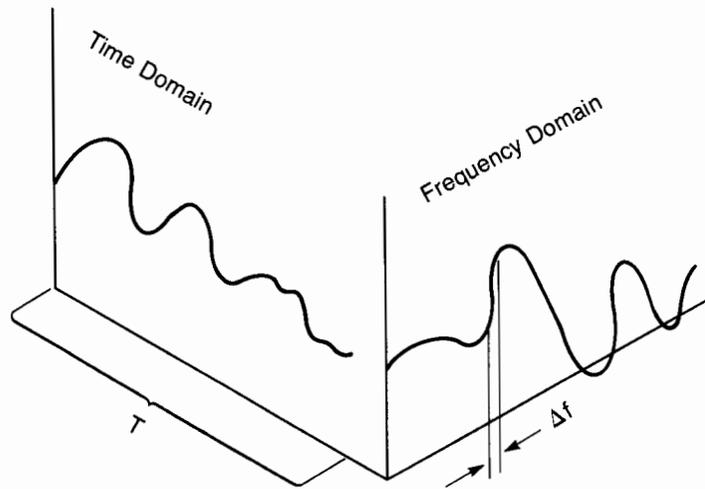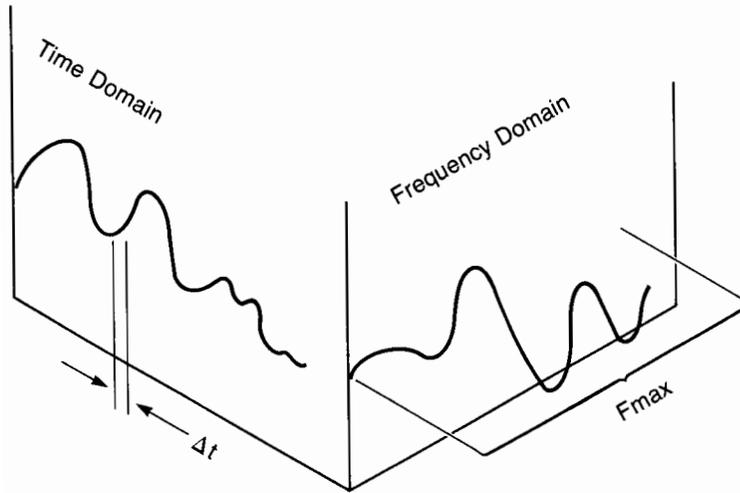
Table 1: Selecting Values for Data Sampling Parameters

Choose convenient round number for parameter shown.	Chosen parameter automatically fixes the value of parameter below, because of relationship in parentheses.	Then make either of the remaining two parameters (cannot be both) as close as possible to the desired value by choosing N* in the relaships shown.
1. Δt	F_{max} ($F_{max} = \dfrac{1}{2\,\Delta t}$)	T ($T = N\Delta t$) Δf ($\Delta f = \dfrac{1}{N\,\Delta t}$)
2. F_{max}	Δt ($\Delta t = \dfrac{1}{2F_{max}}$)	T ($T = N\Delta t$) Δf ($\Delta f = \dfrac{1}{N\,\Delta t}$)
3. Δf	T ($T = \dfrac{1}{\Delta f}$)	Δt ($\Delta t = \dfrac{T}{N}$) F_{max} ($F_{max} = \dfrac{N}{2} \cdot \Delta f$)
4. T	Δf ($\Delta f = \dfrac{1}{T}$)	Δt ($\Delta t = \dfrac{T}{N}$) F_{max} ($F_{max} = \dfrac{N}{2} \cdot \Delta f$)

*N, the data block size, is always a power of 2.

For instance:

If a user needs a 4 Hz frequency resolution (Δf) and a 2 kHz maximum frequency (F_{max}) the required number of samples may be calculated:

$$\Delta f = 4$$

$$F_{max} = (N/2) \Delta f \quad \text{or} \quad N = \frac{2 \, F_{max}}{\Delta f}$$

$$N = \frac{2(2 \times 10^3)}{4}$$

$$N = 1,000$$

When the data is in the frequency domain, i.e., is associated with a spectrum or some other function of frequency, the data points are stored differently from time domain data. A time series of N independent points results in a frequency spectrum containing N/2 independent frequencies.

In the Waveform Analysis Pac, N/2 positive frequencies (including DC and F_{max} terms) are computed, stored, and displayed from an N-point real time series. Each frequency has two independent values—a "real" (cosine) value, and an "imaginary" (sine) value. For DC and F_{max}, only the real values terms are stored. The imaginary values are zero and are not stored. The actual arithmetic is as follows: There are $N/2 + 1$ real points and there are $(N/2) - 1$ imaginary frequency values (since there are no imaginary values for DC or F_{max}). Adding the number of real and imaginary points together, we get:

$$(N/2) + 1 + (N/2) - 1 = N$$

points in the frequency domain from N points in the time domain.

Fourier Series

The French mathematician Jean Baptiste Fourier discovered that periodic time functions can be expressed as an infinite sum of weighted sine and cosine functions of the proper frequencies. This discovery allows convenient interpretation of time functions by analysis of their frequency content.

Mathematically, this may be expressed as:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \left(\frac{2\pi nt}{T} \right) + b_n \sin \left(\frac{2\pi nt}{T} \right) \right) \tag{1}$$

where T is the period of the periodic time function $x(t)$, i.e., $x(t) = x(t + T)$.

By calculating the coefficients a_n and b_n using equations derived by Fourier, the amplitudes of the sine and cosine waves in the series can be found. Knowledge of the coefficients allows calculations of the magnitude and phase at each frequency in the function $x(t)$. The amplitude at frequency $f_n = (n/T)$ is $\sqrt{a_n^2 + b_n^2}$ and the corresponding phase is $\tan^{-1}(b_n/a_n)$.

The Fourier Transform

A restriction in the use of the Fourier series to find the frequency content of a time signal is that the signal must be periodic. One may ease this restriction by allowing the period to approach infinity. The resultant evaluation of the Fourier series is known as the Fourier transform, which is defined by the following pair of equations:

$$S_x(f) = \int_{-\infty}^{\infty} x(t)e^{-i2\pi ft}\, dt \quad \text{(Forward Fourier Transform)} \tag{2}$$

$$x(t) = \int_{-\infty}^{\infty} S_x(f)e^{i2\pi ft}\, df \quad \text{(Inverse Fourier Transform)} \tag{3}$$

(The expression $e^{\pm 2\pi ft} = \cos(2\pi ft) \pm i\sin(2\pi ft)$ is called the kernel of the Fourier transform.)

$S_x(f)$ is known as the Fourier transform of $x(t)$. It contains the amplitude and phase information for all the frequencies which make up $x(t)$ even though $x(t)$ is not periodic.

One may view the techniques of using the Fourier series and the Fourier transform as mathematical filtering operations.

The Discrete Finite Transform

In order to calculate Fourier transforms of time signals on a digital computer, the continuous time input signal must be converted to a set of discrete data signals. This is usually accomplished by sampling the waveform, defined as $x(t)$, at a given time interval. Usually, these will be evenly spaced at an interval, Δt. However, to perform the integration which defines the Fourier transform:

$$S_x(f) = \int_{-\infty}^{\infty} x(t)\, e^{-i2\pi ft}\, dt \tag{4}$$

the interval, Δt, must be infinitesimal (i.e., $\Delta t \rightarrow dt$). Thus, the samples would be separated by an infinitesimal amount of time. This type of sampling is obviously not possible. Therefore, we use instead:

$$S_x''(f) = \Delta t \sum_{n=-\infty}^{n=+\infty} x(n\Delta t)e^{-i2\pi fn\Delta t} \tag{5}$$

where $x(n\Delta t)$ are the measured values of the signal, and Δt is a practical time interval.

According to equation (5), we may still calculate a valid Fourier transform, even though we are dealing with a discrete sample with sampling intervals, Δt, large compared to the infinitesimal dt. However, the magnitude and phase information for all of the frequencies contained in $S_x(f)$ are not accurate for a Fourier transform calculated in this manner. Instead, the function $S_x''(f)$ accurately describes the spectrum of $x(t)$ only up to some maximum frequency (F_{max}). This maximum frequency is dependent upon the sampling interval, Δt, and can be determined in a manner to be shown later.

Examination of (5) shows that to accurately calculate $S_x''(f)$, one must take an infinite number of samples of the input signal. This is clearly impossible since each sample is separated by a finite time interval.

We must choose a finite number of samples accumulated over a finite time. If the time is T seconds, N is the number of samples and Δt is the time interval, then $N = T/\Delta t$.

Restricting the sampling time to T is thus equivalent to truncating equation (5) and we cannot expect to calculate magnitude and phase values for an unlimited number of frequencies between zero and F_{max}. The truncated version of equation (5) will not produce a continuous spectrum.

We may write this discrete finite transform as:

$$S_x''\ (m\Delta f) = \Delta t \sum_{n=0}^{N-1} x(n\Delta t)e^{-i2\pi m \Delta fn \Delta t}\ \text{ for } m = 0,\ \dots\ ,\ N-1 \tag{6}$$

Equation (6) requires that our input function be periodic since only periodic functions have such a discrete frequency spectrum. Thus, equation (6) assumes that the observed function repeats itself with a period T forever. (This assumption is made even though x(t) may not actually be periodic.)

It can be seen that equation (6) is actually a sampled Fourier series with N real-valued time domain points. To describe this data in the frequency domain, two values (i.e., magnitude and phase, or real and imaginary parts) must be obtained. Therefore, N points in the time domain allow us to define only $N/2$ complex quantities in the frequency domain.

Shannon's Sampling Theorem

Shannon's sampling theorem states that it requires slightly more than two samples per period to uniquely define a sinusoid. Therefore, we must sample slightly more than twice per period of the highest frequency we wish to resolve.

The theorem may be stated by the following equation:

$$F_{max} < \frac{1}{2\Delta t} \tag{7}$$

or for convenience: $\quad F_{max} = \frac{1}{2\Delta t.} \tag{8}$

From our definitions of frequency domain data, we recall that $F_{max}/(N/2) = \Delta f$, where Δf is the frequency resolution. (The maximum frequency that can be resolved accurately is obviously $F_{max} - \Delta f$.)

Substituting the definition in equation 8:

$$\Delta f = F_{max}/(N/2) = \frac{1}{2}\ \Delta t/(N/2) = \frac{1}{N\Delta t} = \frac{1}{T}\quad \text{ or } \Delta f = \frac{1}{T} \tag{9}$$

This equation, obtained as a direct result of Shannon's Sampling Theorem, is a physical law.

Convolution Theorem

A useful relationship between time domain and frequency domain data is given by the convolution function:

If $S_x(f) = \int_{-\infty}^{\infty} x(t)e^{-i2\pi ft} \, dt$

and $S_y(f) = \int_{-\infty}^{\infty} y(t)e^{-i2\pi ft} \, dt$

then we define $x(t)*y(t) = \int_{-\infty}^{\infty} x(t) \cdot y \, (\psi - t) \, dt$

where the convolution operation is denoted by $*$. The convolution theorem states that

$$\int_{-\infty}^{\infty} \left[x(t) * y(t) \right] e^{-i2\pi ft} \, dt = S_x(f) \cdot S_y(f)$$

and conversely: $\int_{-\infty}^{\infty} \left[S_x(f)*S_y(f) \right] e^{+i2\pi ft} \, dt = x(t) \cdot y(t)$

In other words: convolution in one domain is equivalent to multiplication in the other domain.

The convolution performed in the HP-85 Waveform Analysis Pac is the time domain convolution. Time domain data is converted to the frequency domain, a multiplication is performed and then the data is reconverted to the time domain (via the IFT).

Conjugate Multiplication

A point in the frequency domain, consisting of both real and imaginary components, may be described as a $+$ ib. The complex conjugate this point is a $-$ ib, (where a and b have the same values as in a $+$ ib). In the following discussions, we will denote the complex conjugate by use of a superscript $*$, (i.e., the complex conjugate of $S(f)$ is $S^*(f)$).

Conjugate multiplication consists of multiplying a complex point by its complex conjugate and results in $a^2 + b^2$. The conjugate multiplication operation causes loss of all phase information since the result is composed entirely of real values (there is no i in $a^2 + b^2$). The expression $S(f) \cdot S^*(f)$ denotes the conjugate multiplication operation. The usefulness of the operation will be seen in the following paragraphs.

Figure 4: A Point in the Power Spectrum
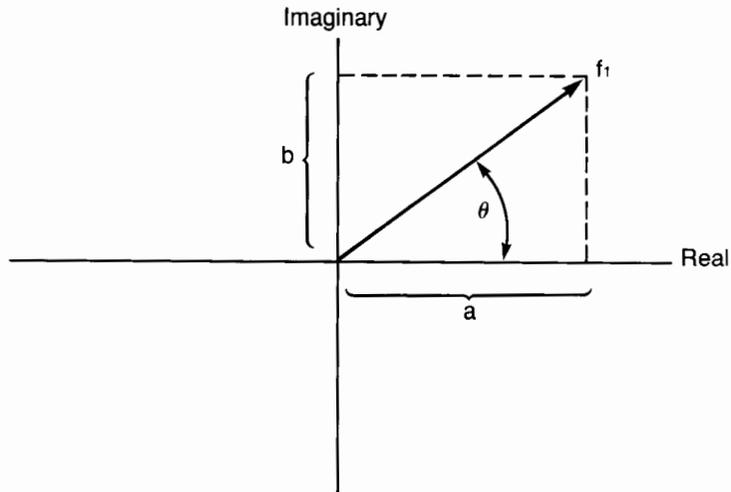
Hanning Function

As previously discussed, the Discrete Fourier Transform is always used in conjunction with truncated data. Truncation of a periodic function at a point other than a multiple of the period causes a discontinuity in the time domain and this results in "leakage" (i.e., the addition of side lobes in the frequency domain).

By properly choosing a truncation function, it is possible to minimize the "leakage" or additional added frequency components. One of the simplest and most effective of these truncation functions is the Hanning Function given by:

$$x(t) = \frac{\left(1 - \cos \frac{2\pi t}{T_c}\right)}{2} \qquad 0 \le t \le T_c$$

where T_c is the truncation interval.

From Reference (1), the magnitude of the Fourier Transform of the Hanning Function is given by

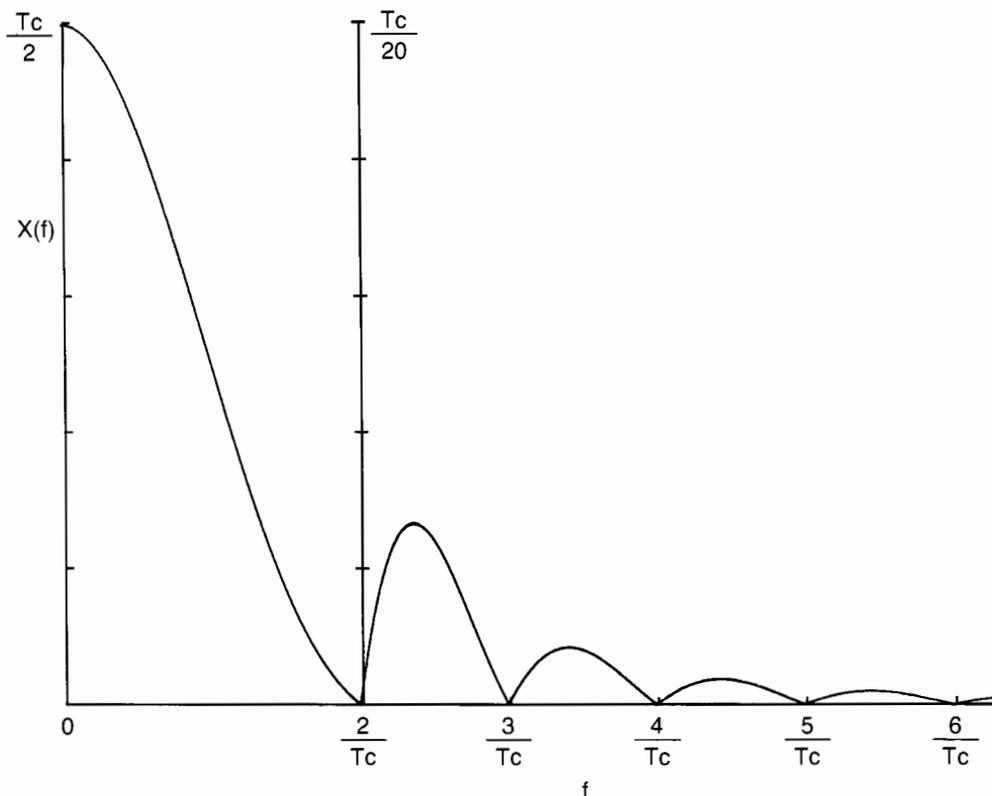$$X(f) = \left| \frac{1}{2} Q(f) + \frac{1}{4} \left[Q\left(f + \frac{1}{T_c}\right) + Q\left(f - \frac{1}{T_c}\right) \right] \right|$$

where $Q(f) = \dfrac{\sin(\pi T_c f)}{\pi f}$

The side lobes of this frequency function are very small, as shown in figure 5.

The Hanning Function may be used to minimize leakage which occurs in the various spectrum and correlation functions calculated in the HP-85 Waveform Analysis Pac.

Figure 5: Hanning Function

Tc/2

Tc/20

X(f)

0

$\dfrac{2}{Tc}$ $\dfrac{3}{Tc}$ $\dfrac{4}{Tc}$ $\dfrac{5}{Tc}$ $\dfrac{6}{Tc}$

f

Auto-Power Spectrum

The auto-power spectrum of a function x(t) is given by:

$$G_{xx} = S_x(f) \cdot S_x*(f)$$

Thus, the auto-power spectrum is just the result of conjugate multiplication of the frequency domain points. All the frequency components of G_{xx} are therefore real and positive.

Voltage Spectrum

The voltage spectrum is the positive square root of the auto-power spectrum and is defined as $\left| S_x(f) \right|$.

Auto-Correlation (Auto-Covariance) Functions

The auto-correlation, R_{xx}, is defined as the inverse Fourier transform of the auto-power spectrum, G_{xx}.

$$R_{xx}(t) = \int_{-\infty}^{\infty} \left| S_x(f) \right|^2 e^{i2\pi ft} \, df = \int_{-\infty}^{\infty} \left[S_x(f) \cdot S_x*(f) \right] e^{i2\pi ft} \, df$$

xv

Since only real input functions are being considered:

$$R_{xx}(t) = \int_{-\infty}^{\infty} x(\psi) \cdot x(\psi - t) d\psi$$

$R_{xx}(t)$ is sometimes referred to as the auto-covariance function and the auto-correlation function is then defined as:

$$\rho_{xx} = R_{xx}(t)/R_{xx}(0)$$

The correlation integral is similar to the convolution integral except that the operation does not include ''flipping'' the function $x(\psi)$ as is the case in a convolution. Therefore the auto-correlation of a pulse appears as:

Figure 6: Auto Correlation of a Pulse
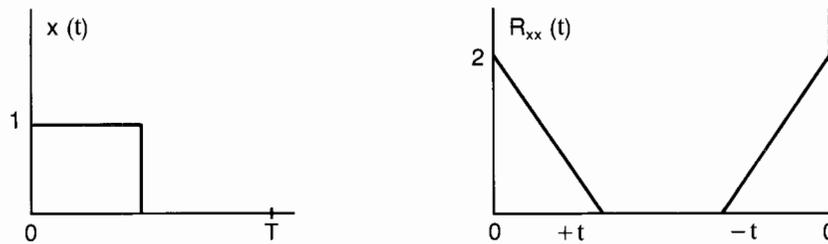
Cross-Power Spectrum

The cross-power spectrum of two signals is defined as $G_{yx}(f) = S_y(f) \cdot S_x^*(f)$ (where $S_x^*(f)$ is the complex conjugate of $S_x(f)$).

G_{yx} may have either positive or negative values and the relative phase between the signals is preserved when calculating the cross-power spectrum.

Since the Discrete Fourier Transform assumes periodicity of the function, leakage effects may arise such as those encountered in auto-spectral analysis. Use of the Hanning function will diminish leakage and enhance the amplitude information as it does in the auto-spectral case.

If a cross-spectrum value is zero, then either one or both of the individual spectra are zero at that frequency. Likewise, a large value of the cross-spectrum indicates that both of the individual spectra have large values at that frequency. Thus, the cross-spectrum indicates the relationship between two signals.

The transfer function of the system can be determined from cross spectral analysis by the following relationships:

If $S_y(f) = S_x(f) \cdot H(f)$ then $G_{yx}(f) = \left[S_x(f) \cdot H(f) \right] S_x^*(f)$

or $H(f) = G_{yx}(f)/G_{xx}(f)$

where $x(t)$ and $y(t)$ are input and output functions of the system.

Thus, both the magnitude and phase of the transfer function can be described by using cross-spectral analysis.

Cross-Correlation (Cross-Covariance) Function

The relationship between input and output is not sufficient to determine accurate transfer functions in cross-spectrum analysis. In addition we need to know whether or not the output was caused solely by the input. For this purpose, cross-correlation is used. It is defined as the inverse Fourier transform of the cross-power spectrum:

$$R_{yx}(t) = \int_{-\infty}^{\infty} x(\psi) \cdot y(\psi - t) d\psi$$

Although the power spectrum and the correlation function theoretically have equal information content, it may require more samples per period to properly interpret the time domain (correlation) function.

REFERENCES:

1. Brigham, E. O., *The Fast Fourier Transform*, Prentice-Hall, Inc., (1974), pp. 141-144.

2. Richardson, Mark H., *"Fundamentals of the Discrete Fourier Transform"*, Sound and Vibration, March 1978.

3. Hewlett-Packard Model 9845 Waveform Analysis Pac, Part No. 09845-12501.

Single Data Block Analysis

This program allows analysis of single blocks of either time or frequency domain data. The data block may consist of up to 512 data points. (The total number of data points must be an integer power of 2.)

The data may be entered either from the keyboard or from a tape file. After entry it may be edited as desired. Storage of keyboard entered or edited data is readily accomplished.

Routines allow modulation of time domain data or calculation of the Hanning function for frequency domain data.

After the appropriate fast Fourier transform is calculated, the transform may be plotted. In the case of the forward transform, plots of both magnitude and phase can be obtained.

The auto-power spectrum and auto-correlation function may also be calculated and plotted.

The use of specially defined and labelled keys provides convenience and flexibility in performing any of the calculations or printing, plotting or storing the data.

Special Function Key Information:

The following special function keys are used to perform desired operations during single data block analysis:

HELP (KEY #5): This key prints a summary of the key functions available to the user.

ENTER (KEY #1): This key loads a data entry program and redefines the special function keys. The data entry routine allows any of four modes of data entry:

1. Enter time domain data from the keyboard.
2. Enter frequency domain data from the keyboard.
3. Enter time domain data from a tape file.
4. Enter frequency domain data from a tape file.

PRINT (KEY #2): This key prints the data on the internal thermal printer. The data is printed according to its current type (i.e., time or frequency domain, power, or correlation).

FFT or IFT (KEY #3): This key performs a Fast Fourier Transform to convert time domain data to frequency domain data (FFT) or frequency domain data to time domain data (IFT) depending upon which data type is currently resident in the computer.

PLOT (KEY #4): This key plots the data on the CRT. The data is plotted according to its current type (i.e., time or frequency domain, power, or correlation). When plotting frequency domain data, both phase and magnitude plots may be made. LOG plots are available for magnitude or power data. Any of three types of plots may be chosen: line, dot, or bar. In addition, the plots may be copied on the internal thermal printer.

EDT/STO (KEY # 5): This key loads a data editing and storage routine and redefines two of the special function keys. (KEY # 1) now becomes an EDIT key used to edit the time or frequency domain data as desired. (KEY # 5) is now a STORE key used to store the current data on a tape file.

MODULATE (KEY # 6): If the data is in the time domain, (KEY # 6) allows the user to modulate the data by inputting amplitude and frequency.

HANNING (KEY # 6): If the data is in the frequency domain, (KEY # 6) calculates the Hanning function.*

POWER (KEY # 7): This key calculates the power function.*

CORRLTN (KEY # 8): This key calculates the auto-correlation.*

User Instructions

1. To load the program:
   a. Insert the Waveform Analysis Pac cartridge into the tape transport.
   b. Type: $\boxed{\text{REW LOAD}}$ "S-WAVE" $\boxed{\text{END LINE}}$.
2. When the program has been loaded:
   a. Press: $\boxed{\text{RUN}}$.
3. When the keys are labelled:

   HELP
   ENTER
   a. Press: KEY # 5 (HELP) to print a description of the key functions.
   b. Go to step 3.
   OR:
   a. Press: KEY # 1 (ENTER) to begin data entry.
4. When ENTRY MODE? is displayed:
   a. Enter: 1, 2, 3 or 4 $\boxed{\text{END LINE}}$ for the entry mode you wish to use.
   Note: The entry modes are listed on the CRT.
   b. If you entered 1 or 3, go to step 5.
   c. If you entered 2 or 4, go to step 8.

5. When NUMBER OF SAMPLES? is displayed:
   a. Enter: The number of data samples $\boxed{\text{END LINE}}$. (Caution: if using data file, be sure number of points corresponds with the number stored on the file.)
   Note: The value must be an integer power of 2, min. = 8, max. = 512.
6. When TIME INTERVAL (IN SEC.)? is displayed:
   a. Enter: The time interval $\boxed{\text{END LINE}}$ (in seconds).
   b. If you entered 1 as the entry mode in step 4a, go to step 7.
   c. If you entered 3 as the entry mode in step 4a, go to step 13.
7. When DATA POINT i ? is displayed:
   a. Enter: The value of the ith data point $\boxed{\text{END LINE}}$.
   b. Repeat step 7a as often as necessary.
   c. When all the data has been entered go to step 14.

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* Caution: Performing this calculation destroys the original data. Care should be taken to save any critical data before using this key.

8. When `NUMBER OF VALUES`
    `([COEFF. PAIRS]*2+2)?` is dis-
    played:
    a. Enter: The number of values in the fre-
        quency domain data (END LINE).
    (Caution: if using data file, be sure number of
    points corresponds with the number stored
    on the file.)
    Note: The value must be an integer power of 2,
    min. $= 8$, max. $= 512$. This value is the
    same as the number of data points which
    result from performing an IFT (conver-
    sion to time domain).
9. When `FREQUENCY INTERVAL`
    `[Hz]?` is displayed:
    a. Enter: The frequency interval (END LINE) (in Hz).
    b. If you entered 2 as the entry mode in step
        4a, go to step 10.
    c. If you entered 4 as the entry mode in step
        4a, go to step 13.
10. When `DC TERM?` is displayed:
    a. Enter: The DC term (END LINE).
11. When `MAX FREQ. TERM?` is displayed:
    a. Enter: The maximum frequency term (real
        part only) (END LINE).
12. When `COEFF. (Coeff.#) REAL,`
    `IMAG?` is displayed:
    a. Enter:The real and imaginary components
        of the indicated data element separated by a
        comma (END LINE) (i.e., 4, 6).
    b. Repeat step 12a until all the data has been
        entered.
    c. Go to step 14.
13. When `DATA FILE NAME?` is displayed:
    a. Enter: The name of the data file (END LINE) you
        wish to use.
14. When `SELECT OPTION` is dis-
    played and the keys are labelled: (for time
    domain data)

```
EDT/STO MODULATE POWER CORRLTN.
ENTER   PRINT    FFT   PLOT
```

OR:

(for frequency domain data)
```
EDT/STO HANNING  POWER CORRLTN.
ENTER   PRINT    IFT   PLOT
```

You may select any of the following functions by
pressing the appropriate key:
a. Press: KEY #1 (`ENTER`) to
    enter data.
b. Go to step 4.

OR:

a. Press: KEY #2 (`PRINT`) to print the
    data.
b. Go to step 14.

OR:

a. Press: KEY #3 (`FFT` or `IFT`) to perform
    the Fast Fourier Transform (time domain
    data) or Inverse Fourier Transform (fre-
    quency domain data).
    1) `FFT (or IFT) CALCULATING`
        is displayed.
    2) When transform is completed program
        stops with above display.
b. Go to step 14.

OR:

a. Press KEY #4 (`PLOT`) and go to step 15.

OR:

a. Press KEY #5 (`EDT/STO`) and go to
    step 20.

OR:

a. Press KEY #6 (`MODULATE` or
    `HANNING`)
    1) If frequency domain data, Hanning
        function is calculated and program
        stops with above display. (CAUTION:
        Destroys original data.)
    2) If time domain data (`MODULATE`),
        then go to step 31.

OR:

a. Press KEY #7 (`POWER`) to calculate
    Power function. When function has been

calculated, program stops with above display. (CAUTION: Destroys original data.)

b.   Go to step 14.

OR:

a.   Press KEY #8 (CORRLTN) to calculate the correlation. When correlation is calculated, program stops with above display. (CAUTION: Destroys original data.)

b.   Go to step 14.

15.   If 1---MAG 2---PHASE
*** SELECT NUMBER ? is displayed
(after FFT):

a.   Enter: 1 or 2 (END LINE), depending upon whether you want a magnitude or a phase plot.

16.   When 1---LINE 2---POINT
3---BAR
*** SELECT NUMBER ? is displayed:

a.   Enter: 1, 2, or 3 (END LINE), depending upon which type of plot you want.

17.   If LOG PLOT (YES/NO)? is displayed:

a.   Enter: Y (END LINE) if you want a LOG plot.

OR:

a.   Enter: N (END LINE) if you do not want a LOG plot.

18.   When the plot is completed, COPY PLOT:
Y/N? is displayed:

a.   Enter: Y (END LINE) if you want a copy of the plot.

OR:

a.   Enter: N (END LINE) if you do not want a copy of the plot.

19.   When MORE PLOTS: Y/N? is displayed:

a.   Enter: Y (END LINE) if you want another type of plot.

b.   Go to step 15.

OR:

a.   Enter: N (END LINE) if you do not want other plots.

b.   Go to step 14.

20.   When the keys are labelled:

```
STORE     MODULATE   POWER  CORRLTN.
EDIT      PRINT      FFT    PLOT
```

OR:

```
STORE     HANNING    POWER  CORRLTN.
EDIT      PRINT      IFT    PLOT
```

a.   To edit the data, press: KEY #1 (EDIT) and go to step 21.

b.   To store the data, press: KEY #5 (STORE) and go to step 29.

21.   a.   If the data is in the time domain, go to step 22.

b.   If the data is in the frequency domain, go to step 24.

22.   When WHICH DATA POINT TO
CHANGE? is displayed:

a.   Enter: The subscript of the data point (END LINE) you wish to change.

23.   When ENTER NEW VALUE? is displayed:

a.   Enter: The new data value (END LINE).

b.   Go to step 28.

24.   When WHICH COEFFICIENT? is displayed:

a.   Enter: The subscript of the coefficient (END LINE) you wish to change.

b.   If the value entered in step 24a is greater than 0 and less than the maximum frequency coefficient, go to step 25.

c.   If the value entered in step 24a is 0 (DC term), go to step 26.

d.   If the value entered in step 24a is the maximum frequency coefficient, go to step 27.

25.   When NEW COEFFICIENT:REAL,
IMAGINARY? is displayed:

a.   Enter: The real and imaginary components (END LINE) of the new coefficient, separated by a comma (i.e., 4, 6).

b.   Go to step 28.

26. When PLEASE ENTER THE NEW
    DC TERM? is displayed:
    a. Enter: The new DC term $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$.
    b. Go to step 28.

27. When PLEASE ENTER THE NEW
    MAX FREQUENCY TERM
    [REAL]? is displayed:
    a. Enter: The new max. frequency term $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$.

28. When EDIT MORE POINTS:
    Y/N? is displayed:
    a. Enter: Y $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$ if you wish to edit more
        points.
    b. Go to step 21.

    OR:
    a. Enter: N $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$ if you do not wish to edit
        more points.
    b. Go to step 14.

29. When NEW FILE: Y/N? is displayed:
    a. Y $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$ if a new file is to be created.
    OR:
    a. Enter: N $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$ if an old file is to be used.

30. When ENTER FILE NAME? is
    displayed:
    a. If a new file is being created, enter:
        The new file name $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$.
        (Max: 6 characters)
    b. Go to step 14.

    OR:
    a. If an old file is being used, enter:
        The old file name $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$.
    b. Go to step 14.

31. When H*COS(2*PI*F) *** H=?
    is displayed:
    a. Enter: The value of H $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$ in the displayed
        equation (H is the amplitude of the carrier
        wave).

32. When H*COS(2*PI*F) ***
    F [Hz]=? is displayed:
    a. Enter: The value of F $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$ in the displayed
        equation. (F is the frequency of the carrier
        wave).
    b. Go to step 14.

Example 1:

The file named "DEMOD" contains a single block of 64 time domain data points taken at a time interval of 1 millisecond. Obtain a printout and a plot of the data, then convert it to the frequency domain and obtain a copy of the data. Make plots of both the magnitude and phase of the frequency domain data. Perform the auto correlation calculation, and plot its results.

Press KEY #1, (ENTER).

QUESTION	RESPONSE
ENTRY MODE?	3
NUMBER OF SAMPLES?	64
TIME INTERVAL (IN SEC.)?	.001
DATA FILE NAME?	DEMOD

Press KEY #2, (PRINT), to obtain the following printout of the data.

```
TIME DOMAIN DATA
NUMBER OF DATA POINTS =   64
TIME=0 TO  6.40000E-002[SEC]
TIME INTERVAL= 1.00000E-003[SEC]
DATA PT.   TIME[SEC]      DATA
     1   0.0000E+000   0.00000E+000
     2   1.0000E-003   1.00000E-001
     3   2.0000E-003   2.00000E-001
     4   3.0000E-003   3.00000E-001
     5   4.0000E-003   4.00000E-001
     6   5.0000E-003   5.00000E-001
     7   6.0000E-003   6.00000E-001
     8   7.0000E-003   7.00000E-001
     .        .             .
     .        .             .
     .        .             .
    60   5.9000E-002  -5.00000E-001
    61   6.0000E-002  -4.00000E-001
    62   6.1000E-002  -3.00000E-001
    63   6.2000E-002  -2.00000E-001
    64   6.3000E-002  -1.00000E-001
```
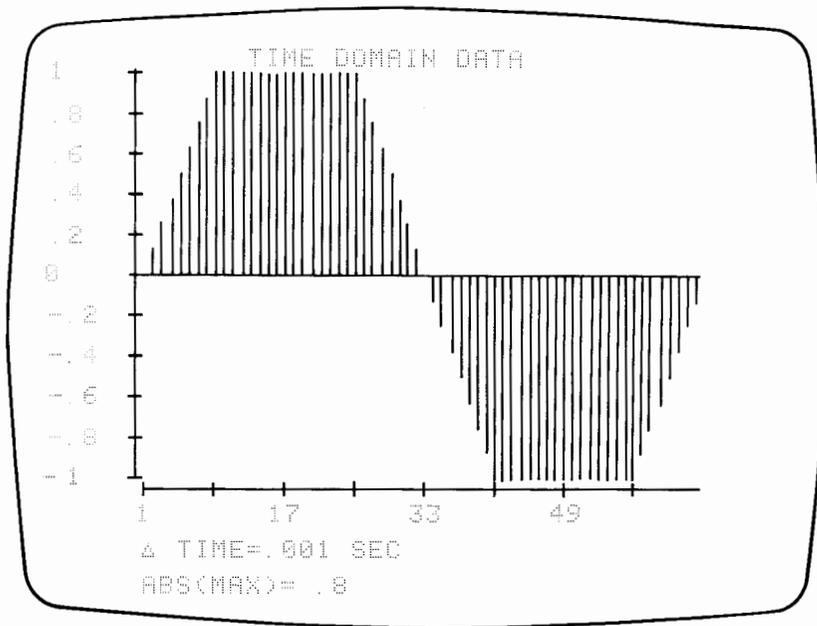
Press KEY #4, (PLOT), for a plot of the data.

QUESTION	RESPONSE
1---LINE 2---POINT 3---BAR	
*** SELECT NUMBER ?	3
COPY PLOT: Y/N?	Y

MORE PLOTS: Y/N?   N

Press KEY #3, (FFT), to convert to frequency domain. Then press KEY #2, (PRINT), to obtain a printout of the
of the data.


```
FREQUENCY DOMAIN DATA
FREQ. WINDOW=0 TO 5.000E+002[Hz]
FREQ. INTERVAL= 1.563+001[Hz]
DC TERM= 0.00000E+000
MAX FREQ. TERM= 0.00000E+000
    1  F=    1.56250E+001
    R=-4.063E-012  I=-9.178E-001
    M= 9.178E-001  θ=   -90.00
    2  F=    3.12500E+001
    R= 0.000E+000  I= 0.000E+000
    M= 0.000E+000  θ=    0.00
    3  F=    4.68750E+001
    R=-9.219E-013  I=-1.026E-001
    M= 1.026E-001  θ=   -90.00
        .            .            .
        .            .            .
        .            .            .
```
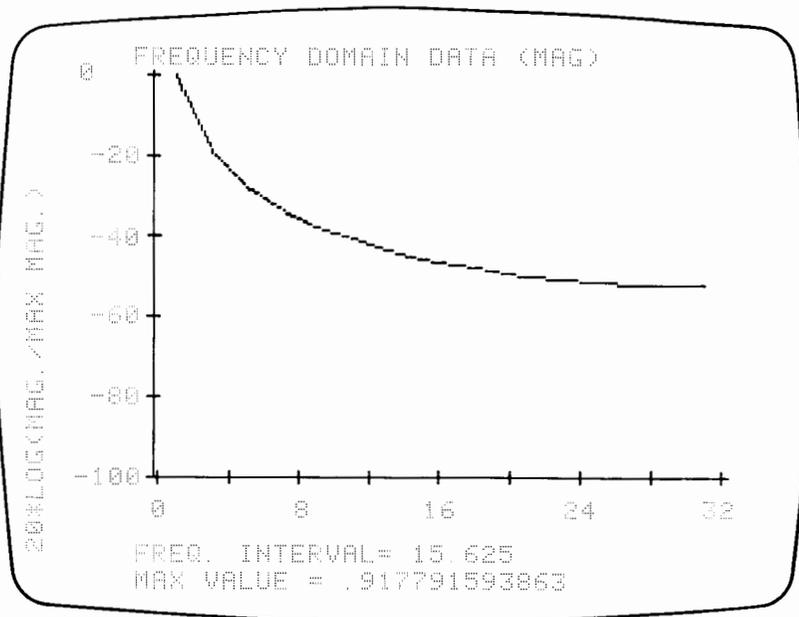
```
          .         .              .
          .         .              .
          .         .              .
   29   F=      4.53125E+002
      R=-1.563E-014   I= 2.258E-003
      M= 2.258E-003   θ=    90.00
   30   F=      4.68750E+002
      R= 0.000E+000   I= 0.000E+000
      M= 0.000E+000   θ=     0.00
   31   F=      4.84375E+002
      R= 9.375E-013   I= 2.215E-003
      M= 2.215E-003   θ=    90.00
```
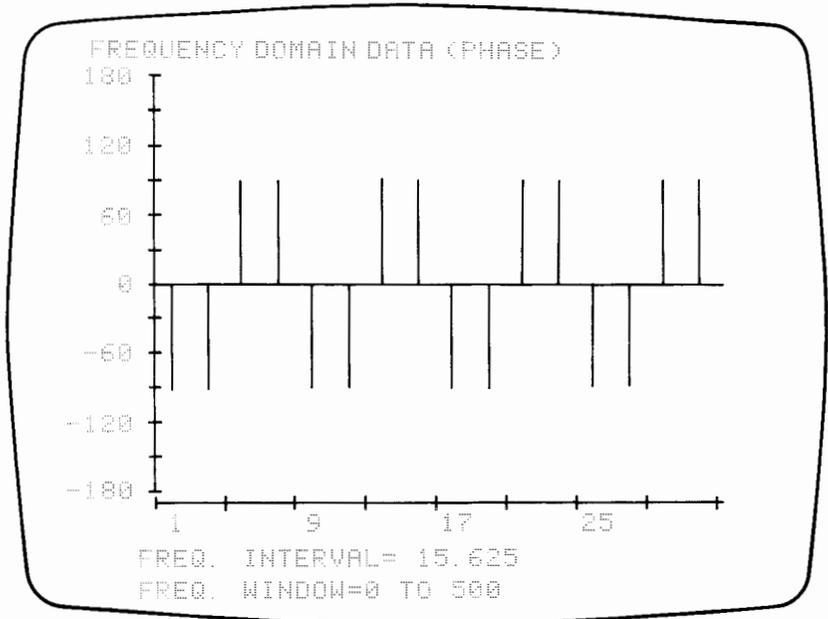
Press KEY #4, (PLOT).

QUESTION	RESPONSE
1---MAG 2---PHASE	
*** SELECT NUMBER ?	1
1---LINE 2---POINT 3---BAR	
*** SELECT NUMBER ?	1
LOG PLOT (YES/NO)?	Y
COPY PLOT: Y/N?	Y

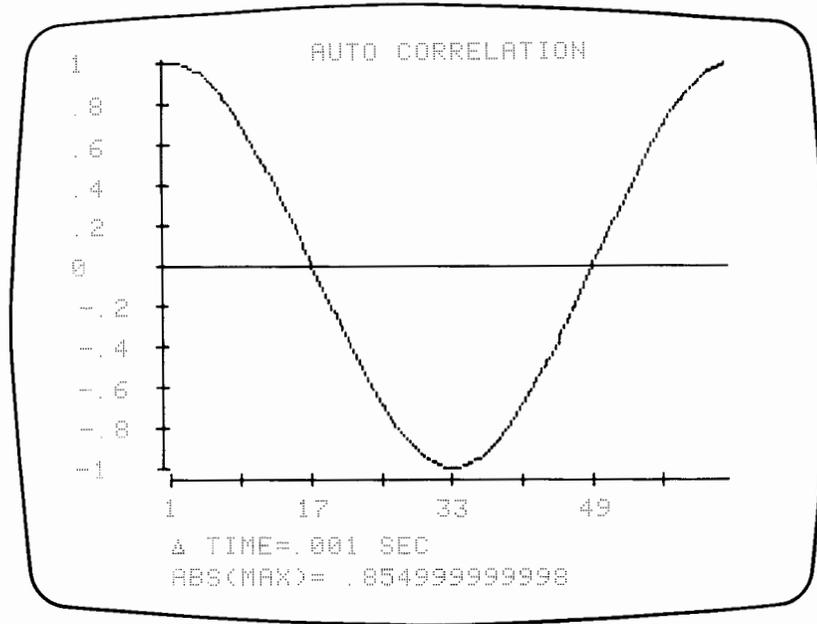MORE PLOTS: Y/N?   Y

1---MAG   2---PHASE

*** SELECT NUMBER ?   2

1---LINE   2---POINT   3---BAR

*** SELECT NUMBER ?   3

COPY PLOT: Y/N?   Y

MORE PLOTS: Y/N?   N

Press KEY #8, (CORRLTN), then press KEY #4, (PLOT).

**QUESTION**   **RESPONSE**

1---LINE   2---POINT   3---BAR

*** SELECT NUMBER ?   1

COPY PLOT: Y/N?   Y

```
                    AUTO CORRELATION
  1
  .8
  .6
  .4
  .2
   0
 -.2
 -.4
 -.6
 -.8
  -1
        1        17        33        49
     Δ TIME=.001 SEC
     ABS(MAX)= .85499999998
```

MORE PLOTS: Y/N?   N

Example 2:

Loran-C pulses are broadcast on a center frequency of 100 kHz with a pulse shape such that 99% of the radiated energy is contained between the frequencies of 90 and 110 kHz. The data file "LORAN" contains 256 data points of time domain data, taken at intervals of 2.5 microseconds, of the function:

$$f(t) = 10^{12}t^2 e^{-\frac{2 \times 10^6 t}{65}} \sin\left(\frac{2\pi t}{10^{-5}}\right)$$

Perform a waveform analysis (single data block) to obtain information on the nature of the pulse in both time and frequency domains. Calculate the power spectrum to obtain an estimate of the validity of the statement regarding the radiated energy.

QUESTION	RESPONSE
ENTRY MODE?	3
NUMBER OF SAMPLES?	256
TIME INTERVAL (IN SEC.)?	.0000025
DATA FILE NAME?	LORAN
Press KEY #4, (PLOT), for a plot of the data.	
1---LINE 2---POINT 3---BAR	
*** SELECT NUMBER ?	1
COPY PLOT: Y/N?	Y
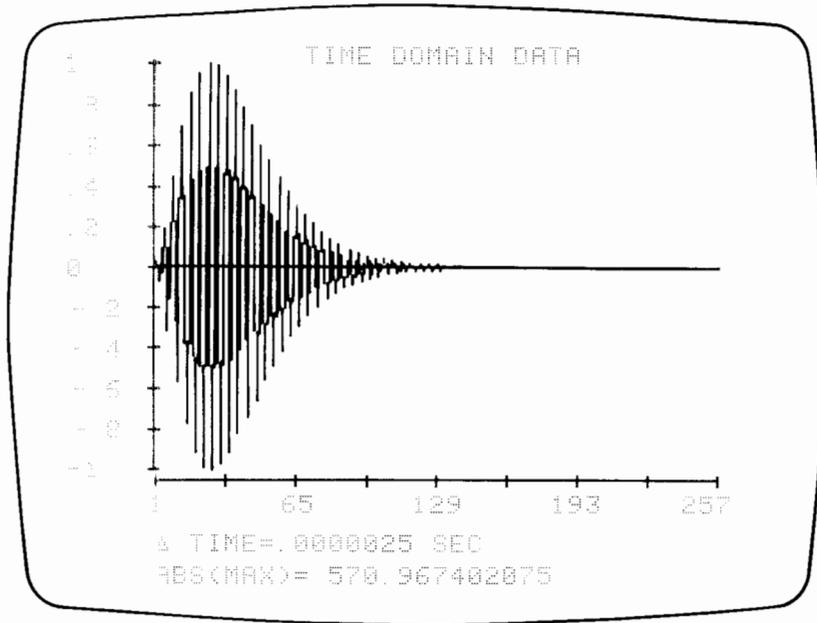
MORE PLOTS: Y/N?   N

Press KEY #3, (FFT), then KEY #4, (PLOT).

1---MAG   2---PHASE
*** SELECT NUMBER ?   1
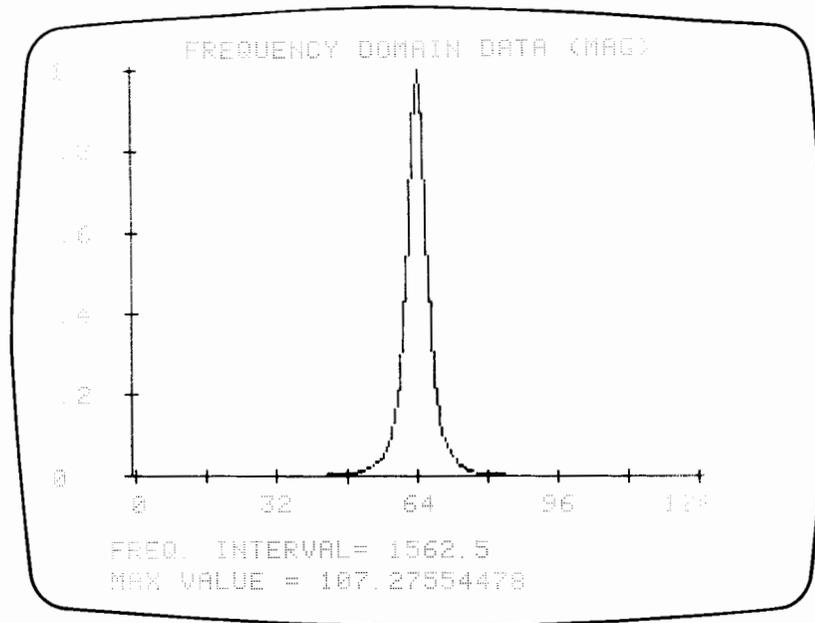1---LINE   2---POINT   3---BAR
*** SELECT NUMBER ?   1
LOG PLOT (YES/NO)?   N
COPY PLOT: Y/N?   Y

FREQUENCY DOMAIN DATA (MAG)

FREQ. INTERVAL= 1562.5
MAX VALUE = 107.27554478

The bandwidth of interest, 90 to 110 kHz, lies between data points 57.6 and 70.4 on the horizontal axis of the frequency domain (magnitude) plot. It appears likely from the plot that 99% of the radiated power does lie within this bandwidth.

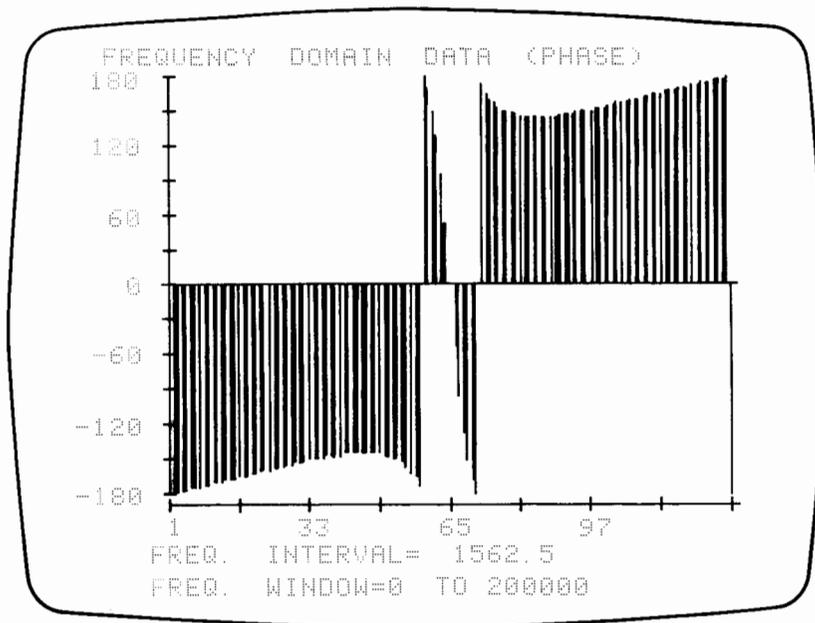Several other plots of possible interest for this data are shown below.

MORE PLOTS: Y/N?	Y
1---MAG 2---PHASE	
*** SELECT NUMBER ?	2
1---LINE 2---POINT 3---BAR	
*** SELECT NUMBER ?	3
COPY PLOT: Y/N?	Y

MORE PLOTS: Y/N?   N

Press KEY #7, (POWER), then KEY #4, (PLOT).

1---LINE   2---POINT   3---BAR
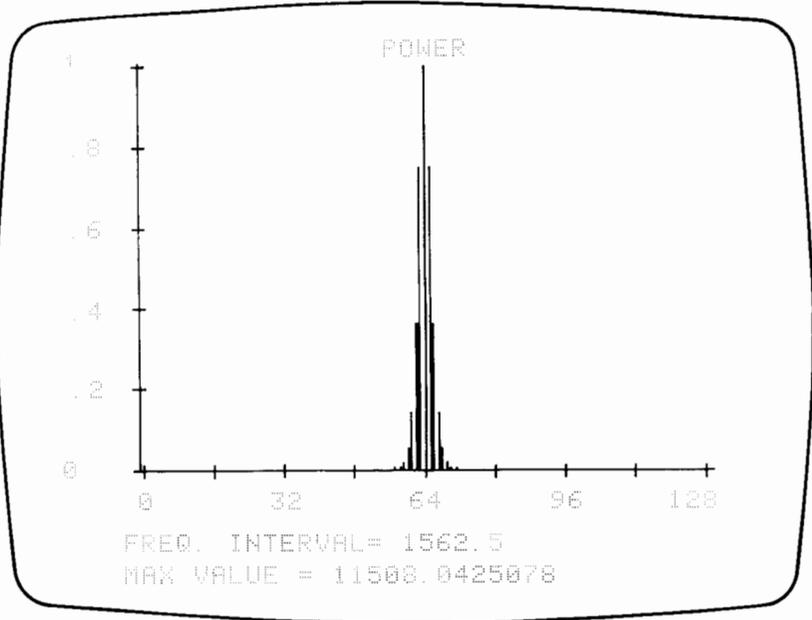*** SELECT NUMBER ?   3
LOG PLOT (YES/NO)?   N
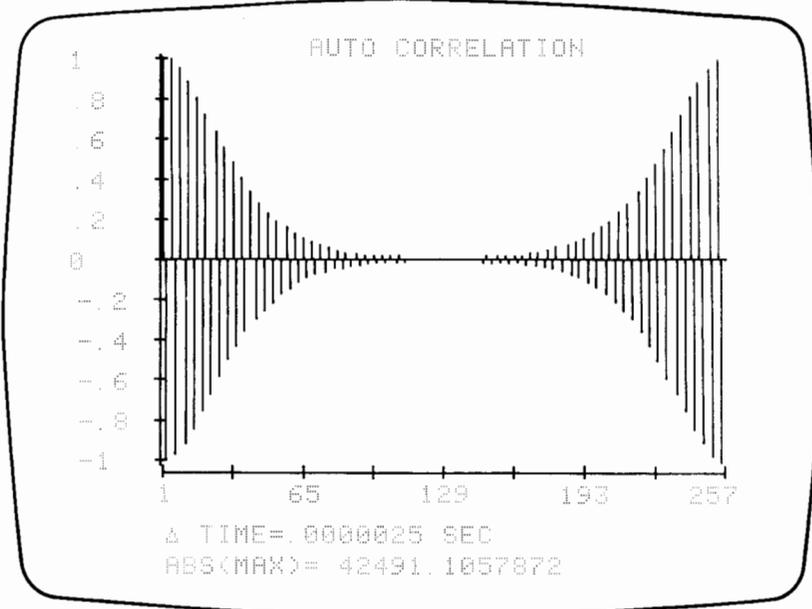COPY PLOT: Y/N?   Y

MORE PLOTS: Y/N?   N

Press KEY #8, (CORRLTN), and KEY #4 (PLOT).

1---LINE   2---POINT   3---BAR
*** SELECT NUMBER ?   3
COPY PLOT: Y/N?   Y

Double Data Block Analysis

This program allows analysis of double blocks of time or frequency domain data. Each data block may consist of up to 256 data points. (The total number of data points in each block must be an integer power of 2.)

The data may be entered either from the keyboard or from tape files. After entry, it may be edited as desired or the data domain may be changed. Storage of keyboard entered or edited data on a tape file is easily accomplished. Routines allow for modulation of the data and for various data manipulations (performing arithmetic operations between the data blocks, etc.).

By pressing the appropriate key, the user may calculate the cross-power or cross-correlation of the two data blocks. The time convolution between the two blocks may also be performed.

Plots of the data or the results of the various calculations can be obtained, including either magnitude or phase plots, where appropriate.

Use of the specially defined and labelled keys provides convenience and flexibility in performing any of the calculations and in printing, plotting and storing the data.

Special Function Key Information:

The following special function keys are used to perform desired functions during double data block analysis:

HELP (KEY #5): This key prints a summary of the key functions available to the user.

ENTER (KEY #1): This key loads entry program and redefines the special function keys. The data entry routine allows any of four modes of data entry for each data block:

1. Enter time domain data from the keyboard.
2. Enter frequency domain data from the keyboard.
3. Enter time domain data from a tape file.
4. Enter frequency domain data from a tape file.

PRINT (KEY #2): This key prints the data on the internal thermal printer. If the data is in the frequency or the time domain the user may choose which block to print. If the data is cross-power, cross-correlation, convolution, or power data, data block 1 is printed. (To move data from one block to another, use the Data Handling key described later.)

CONVOL (KEY #3): This key calculates the time convolution between data blocks 1 and 2. After this operation the convolution data is stored in data block 1 and data block 2 is cleared.

PLOT (KEY #4): This key plots the data on the CRT. If the data is in the time or frequency domain the user may select which block is to be plotted. If the data is cross-power, cross-correlation or convolution it must be

in data block 1 (normal location after calculation). Both phase and magnitude plots are available for frequency domain data. LOG plots are available for magnitude and cross-power data. Any of three types of plots may be chosen: line, dot, or bar. In addition, the plots may be copied on the internal thermal printer.

CHNG/STO (KEY #5): This key calls data editing, manipulation, and storage routines and redefines the special function keys:

    DONE (KEY #1) exits the data editing and handling routine and returns operation to the calculations mode.

    EDIT (KEY #2) may be used to edit the time or frequency domain data in either block 1 or 2.

    STORE (KEY #3) may be used to store the data blocks on a tape file.

    CHNG. DOM (KEY #4). This key may be used for changing from the time domain to the frequency domain or vice versa, for both data blocks. Both blocks should be in the same domain.

    DATA HAN (KEY #4) is used to perform various data handling operations between the two blocks. The various manipulations available to the user are displayed after pressing the key.

MOD. (KEY #6): This key is used to modulate the time domain data by inputting amplitude and frequency. The user may choose which data block to modulate.

X-POWER (KEY #7): This key calculates the cross-power of data blocks 1 and 2. The data in blocks 1 and 2 may be time or frequency domain or data block 1 may be cross-correlation data with data block 2 cleared. After this operation the cross-power data will be in data block 1 and data block 2 will be cleared.

X-CORREL (KEY #8): This key calculates the cross-correlation of data blocks 1 and 2. The data in block 1 and 2 may be time or frequency domain or data block 1 may be cross-power data with data block 2 cleared. After this operation the cross-correlation data will be in data block 1 and data block 2 will be cleared.

User Instructions

1. To load the program:

    a. Insert the Waveform Analysis Pac cartridge into the tape transport.

    b. Type: `REW/LOAD` "D-WAVE" `END LINE`.

2. When the program has been loaded:

    a. Press: `RUN`.

3. When the keys are labelled:

```
-------------------------------------------------
HELP
ENTER
```

    a. Press: KEY #5 (HELP) to print a summary of the key functions.

OR:

    a. Press: KEY #1 (ENTER) to begin data entry.

4. When NUMBER OF SAMPLES? is displayed:

    a. Enter: The number of samples `END LINE`. (Caution: if using data file be sure number of points corresponds with the number stored on the file.)

    Note: The value must be an integer power of 2, min.=8, max.=256.

5. When TIME INTERVAL (IN SEC.)? is displayed:

    a. Enter: The time interval `END LINE` (in seconds).

6. When ENTRY MODE? is displayed:

    a. Enter: 1, 2, 3 or 4 `END LINE` for the entry mode you wish to use.

    Note: The entry modes are listed on the CRT.

    b. If you entered 1, go to step 7.

    c. If you entered 2, go to step 8.

    d. If you entered 3 or 4, go to step 11.

7. When $\texttt{DATA POINT i?}$ is displayed:
   a. Enter: The value of the ith data point $\boxed{\texttt{END LINE}}$.
   b. Repeat step 7a as often as necessary.
   c. When all the data has been entered go to step 12.
8. When $\texttt{DC TERM?}$ is displayed:
   a. Enter: The DC term $\boxed{\texttt{END LINE}}$.
9. When $\texttt{MAX FREQ. TERM?}$ is displayed:
   a. Enter: The maximum frequency term $\boxed{\texttt{END LINE}}$.
10. When $\texttt{COEFF. (coeff.\#) REAL,}$ $\texttt{IMAG?}$ is displayed:
   a. Enter: The real and imaginary components of the indicated data element, separated by a comma $\boxed{\texttt{END LINE}}$. (i.e., 2,8)
   b. Repeat step 10a as often as necessary.
   c. When all the data has been entered, go to step 12.

11. When $\texttt{DATA FILE NAME?}$ is displayed:
   a. Enter: The name of the data file $\boxed{\texttt{END LINE}}$ you wish to use.

12. If only the data for the first data block has been entered, go back to step 6 to enter the data for the second block. Otherwise, when $\texttt{SELECT OPTION}$ is displayed, go to step 13.

13. When the keys are labelled:
   ```
   CHNG/STO MOD.   X-POWER X-CORREL
   ENTER    PRINT   CONVOL   PLOT
   ```
   You may select any of the following functions by pressing the appropriate key:
   a. Press: KEY #2 ($\texttt{PRINT}$) and go to step 14.

   OR:
   a. Press: KEY #3 ($\texttt{CONVOL}$) to perform convolution. When convolution has been performed, go to step 13. (CAUTION: Destroys original data.)

   OR:
   a. Press: KEY #4 ($\texttt{PLOT}$) and go to step 15.

OR:
   a. Press: KEY #5 ($\texttt{CHNG/STO}$) and go to step 25.
OR:
   a. Press: KEY #6 ($\texttt{MOD.}$) and go to step 21.
OR:
   a. Press: KEY #7 ($\texttt{X-POWER}$) to calculate cross power. When cross power calculation has been calculated go to step 13. (CAUTION: Destroys original data.)
OR:
   a. Press: KEY #8 ($\texttt{X-CORREL}$) to calculate cross correlation. When cross correlation has been calculated go to step 13. (CAUTION: Destroys original data.)

14. When $\texttt{DATA BLOCK TO PRINT:1}$ $\texttt{OR 2?}$ is displayed:
   a. Enter: The number of the data block $\boxed{\texttt{END LINE}}$.
   b. Go to step 13.

15. If $\texttt{WHICH DATA BLOCK: 1 or}$ $\texttt{2?}$ is displayed:
   a. Enter: The number of the data block $\boxed{\texttt{END LINE}}$.

16. If $\texttt{1---MAG 2---PHASE}$ $\texttt{*** SELECT NUMBER ?}$ is displayed:
   a. Enter: 1 or 2 $\boxed{\texttt{END LINE}}$ depending upon whether you want a magnitude or a phase plot.

17. When $\texttt{1---LINE 2---POINT}$ $\texttt{3---BAR *** SELECT NUMBER ?}$ is displayed:
   a. Enter: 1, 2 or 3 $\boxed{\texttt{END LINE}}$ depending upon the type of plot you want.

18. If $\texttt{LOG PLOT (YES/NO)?}$ is displayed:
   a. Enter: Y $\boxed{\texttt{END LINE}}$ if you want a LOG plot.
   OR:
   a. Enter: N $\boxed{\texttt{END LINE}}$ if you do not want a LOG plot.

19. When the plot is completed, `COPY PLOT: Y/N?` is displayed:
    a. Enter: `Y` (END LINE) if you want a copy of the plot.
    OR:
    a. Enter: `N` (END LINE) if you do not want a copy of the plot.
20. When `MORE PLOTS: Y/N?` is displayed:
    a. Enter: `Y` (END LINE) if you want another type of plot.
    b. Go to step 15.
    OR:
    a. Enter: `N` (END LINE) if you do not want other plots.
    b. Go to step 13.
21. When `MODULATE DATA BLOCK 1: Y/N?` is displayed:
    a. Enter: `Y` (END LINE) if you wish to modulate data block 1.
    b. Go to step 23.
    OR:
    a. Enter: `N` (END LINE) if you do not wish to modulate data block 1.
22. When `MODULATE DATA BLOCK 2: Y/N?` is displayed:
    a. Enter: `Y` (END LINE) if you wish to modulate data block 2.
    b. Go to step 23.
    OR:
    a. Enter: `N` (END LINE) if you do not wish to modulate data block 2.
    b. Go to step 13.
23. When `H*COS(2*PI*F) *** H=?` is displayed:
    a. Enter: The value of H (END LINE) in the displayed equation. (H is the amplitude of the carrier wave.)
24. When `H*COS(2*PI*F) *** F [Hz]=?` is displayed:
    a. Enter: The value of F (END LINE) in the dis-

played equation. (F is the frequency of the carrier wave.)
    b. Go to step 13.
25. When the keys are labelled:

`DONE     EDIT     STORE     CHNG. DOM`

You may select any of the following functions by pressing the appropriate key:
    a. Press: KEY # 1 (`DONE`) and go to step 13.
    OR:
    a. Press: KEY # 2 (`EDIT`) and go to step 26.
    OR:
    a. Press: KEY # 3 (`STORE`) and go to step 36.
    OR:
    a. Press: KEY # 4 (`CHNG. DOM`) and go to step 25.
26. When the keys are labelled:

`DONE     EDIT               DATA HAN`

You may select any of the following functions by pressing the appropriate key:
    a. Press: KEY # 1 (`DONE`) and go to step 13.
    OR:
    a. Press: KEY # 2 (`EDIT`) and go to step 27.

    Note: This key available for time and frequency domain data only. Not available after convolution, cross-power or cross-correlation calculations.

    OR:
    a. Press: KEY # 4 (`DATA HAN`) and go to step 35.
27. When `DATA BLOCK TO EDIT: 1 OR 2?` is displayed:
    a. Enter: The number (END LINE) of the data block you wish to edit.
       1. If time domain data go to step 28.
    OR:
       2. If frequency domain data go to step 30.

28. When WHICH DATA POINT TO CHANGE? is displayed:
    a. Enter the number (END LINE) of the data point you want to change.

29. When ENTER NEW VALUE? is displayed:
    a. Enter: The new data value (END LINE).
    b. Go to step 34.

30. When WHICH COEFFICIENT? is displayed:
    a. Enter: The subscript (END LINE) of the coefficient you wish to change.
    b. If the value entered in step 30a is greater than 0 and less than the maximum frequency coefficient, go to step 31.
    c. If the value entered in step 30a is 0, (DC term), go to step 32.
    d. If the value entered in step 30a is the maximum frequency coefficient, go to step 33.

31. When NEW COEFFICIENT: REAL, IMAGINARY? is displayed:
    a. Enter: The real and the imaginary components (END LINE) of the new coefficient, separated by a comma (i.e., 2,−5).
    b. Go to step 34.

32. When PLEASE ENTER THE NEW DC TERM? is displayed:
    a. Enter: The new DC term (END LINE).
    b. Go to step 34.

33. When PLEASE ENTER THE NEW MAX FREQUENCY TERM [REAL]? is displayed:
    a. Enter: The new max. frequency term (END LINE).

34. When EDIT MORE POINTS: Y/N? is displayed:
    a. Enter: Y (END LINE) if you wish to edit more points.
       1) If time domain data go to step 28.
       2) If frequency domain data go to step 30.

OR:
    a. Enter: N (END LINE) if you do not wish to edit more points.
    b. Go to step 26.

35. When DATA BLOCK MANIPULATION descriptions and ENTER OPERATION CODE? as shown here:

```
 DATA BLOCK MANIPULATION
 1:BLK1=BLK1+BLK2,SAME DOMAIN
 2:BLK1=BLK1-BLK2,SAME DOMAIN
 3:BLK1=BLK1*BLK2,SAME DOMAIN
 4:BLK1=BLK1/BLK2,SAME DOMAIN
 5:BLOCK2=BLOCK1
 6:BLOCK1=BLOCK2
 7:BLK1=BLK1*CONJ.BLK2,BOTH FREQ
 8:EXCHANGE BLOCK1 & BLOCK2
 9:BLK1=BLK1*CONJ.BLK1,FREQ DOM
10:BLK1=BLK1+CONST.,TIME DOM
11:BLK1=BLK1-CONST.,TIME DOM
12:BLK1=BLK1*CONST.
```

are displayed:
    a. Enter: The operation code (END LINE) for the manipulation you want to perform.
    Note: Codes consist of numbers 1 through 12 as defined by their descriptions.
    b. If the operation code was 10, 11 or 12, go to step 39.
    c. Go to step 26.

36. When DATA BLOCK TO SAVE: 1 OR 2? is displayed:
    a. Enter: 1 or 2 (END LINE) depending on which data block is to be stored.

37. When NEW FILE: Y/N? is displayed:
    a. Enter: Y (END LINE) if a new file is to be used.
OR:
    a. Enter: N (END LINE) if an old file is to be used.

38. When ENTER FILE NAME? is displayed:
    a. If a new file is being created enter: The new file name (END LINE). (Max: 6 characters.)
    b. Go to step 13.

OR:

a.   If an old file is being used, enter:
     The old file name $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$.

b.   Go to step 13.

39.   When ENTER THE CONSTANT
      VALUE? is displayed:

a.   Enter: The constant value $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$ you wish to
     use.

b.   Go to step 26.

Example 1:

A radar pulse with a duration of 8 microseconds is transmitted beginning at T = 4 microseconds. Some time later, a signal containing the reflected pulse buried in random noise is received. Use waveform analysis (double data block) to find the time delay and calculate the distance to the target. (The pulse and signal data each consist of 64 time domain data points taken at time intervals of 1 microsecond. They are stored in data files "PULSE" and "SIGNAL" respectively.)

Press KEY #1, (ENTER).

QUESTION	RESPONSE
NUMBER OF SAMPLES?	64
TIME INTERVAL (IN SEC.)?	$10 \wedge -6$
ENTRY MODE?	3
DATA FILE NAME?	PULSE
ENTRY MODE?	3
DATA FILE NAME?	PULSE

Press KEY #4, (PLOT).

WHICH DATA BLOCK: 1 or 2?	1
1---LINE 2---POINT 3----BAR	
*** SELECT NUMBER ?	1
COPY PLOT: Y/N?	Y

TIME DOMAIN DATA

△ TIME=.000001 SEC
ABS(MAX) = 1
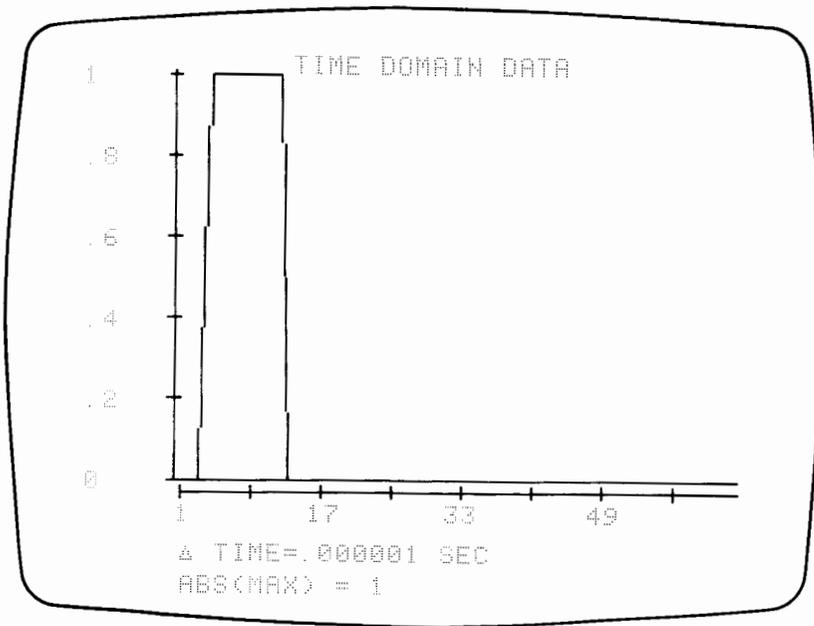
MORE PLOTS: Y/N?	N

Press KEY #3, (CONVOL), then KEY #4, (PLOT).

| **QUESTION**	**RESPONSE**
1---LINE 2---POINT 3---BAR	
*** SELECT NUMBER ?	1
COPY PLOT: Y/N?	Y

CONVOLUTION

△ TIME=.000001 SEC
ABS(MAX) = .261230468751
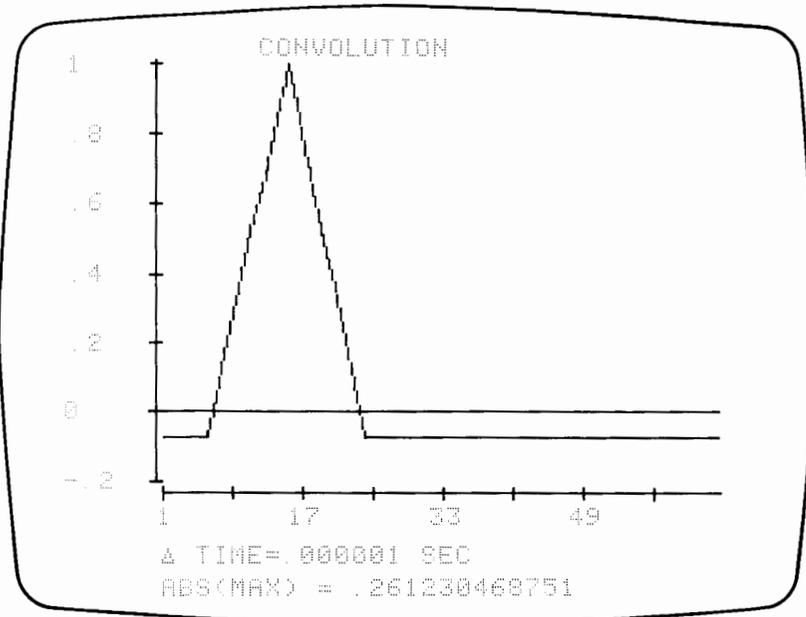
MORE PLOTS: Y/N?   N

(Now enter ''PULSE'' and ''SIGNAL'' data.)

Press KEY #1, (ENTER).

QUESTION	**RESPONSE**
NUMBER OF SAMPLES?	64
TIME INTERVAL (IN SEC.)?	10∧−6
ENTRY MODE?	3
DATA FILE NAME?	PULSE
ENTRY MODE?	3
DATA FILE NAME?	SIGNAL

Press KEY #4, (PLOT).

WHICH DATA BLOCK: 1 or 2?   2

1---LINE   2---POINT   3---BAR

*** SELECT NUMBER ?   1
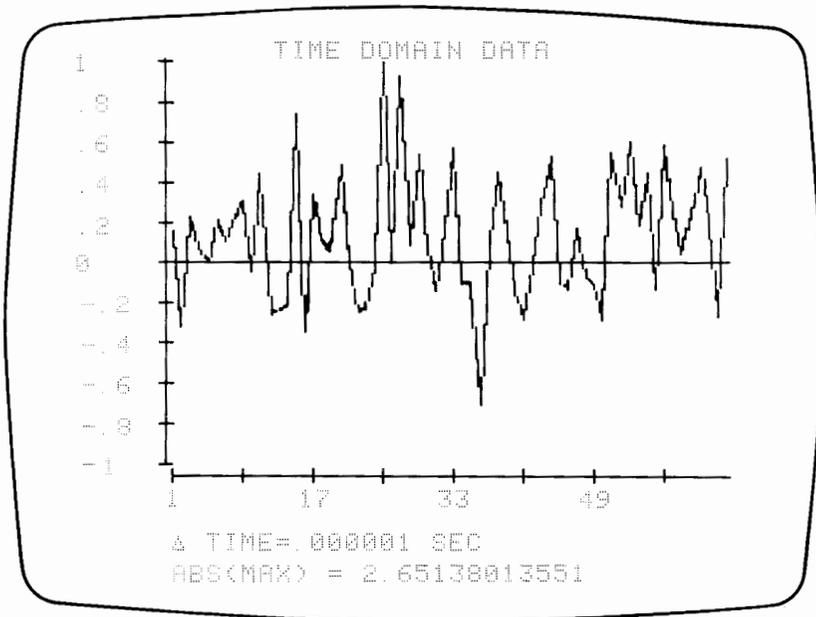
COPY PLOT: Y/N?   Y

MORE PLOTS: Y/N?   N

Press KEY #3, (CONVOL) then KEY #4, (PLOT).

1---LINE 2---POINT 3---BAR
*** SELECT NUMBER ?   1
COPY PLOT: Y/N?   Y
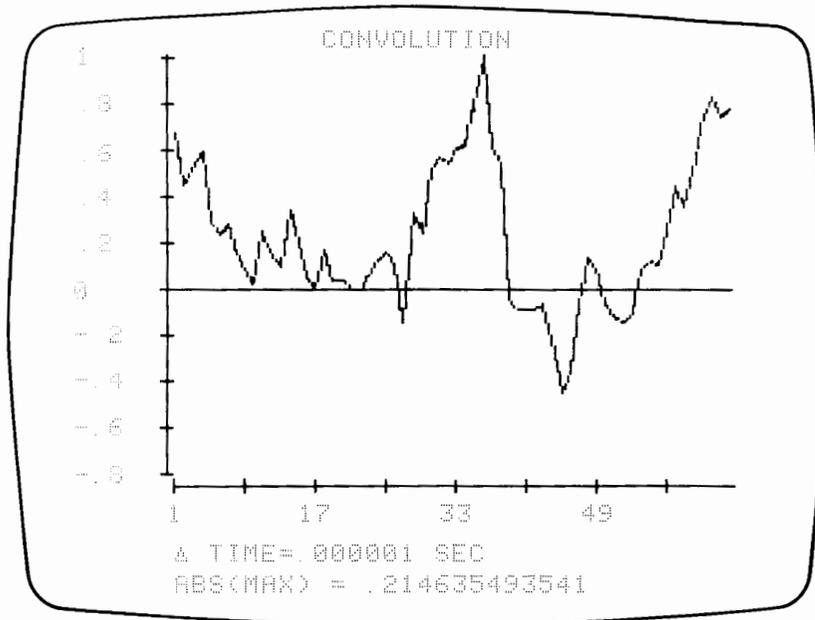
MORE PLOTS: Y/N?   N

The peak of the pulse-pulse convolution occurs at approximately 15 microseconds. The peak of the pulse-signal convolution occurs at about 36 microseconds; therefore, the pulse to signal delay is approximately 21 microseconds giving a target distance of approximately 10,326 feet.

(Note: Similar applications are commonly found in acoustics, vibration testing, seismic studies, etc.)

Notes

Fourier Series Coefficients For Equally Spaced Data

From a set of equally–spaced data points (x_i, y_i) $(i = 1,..,n)$, where Δx is the spacing between points, it is possible to compute a set of coefficients a_i and b_i of the Fourier series which approximates the behavior of the data points.

For a finite Fourier series given by the formula:

$$\frac{a_0}{2} + \sum_{i=1}^{N} \left(a_i \cos \left(\frac{2\pi i x}{T} \right) + b_i \sin \left(\frac{2\pi i x}{T} \right) \right)$$

where T is the period of the function, given by $x_n - x_1$
and N is the number of coefficients to be computed;

$$\text{If: } g(x_j) = y_j \cos \left(\frac{2\pi i x}{T} \right)$$

and

$$h(x_i) = y_j \sin \left(\frac{2\pi i x}{T} \right)$$

then $\quad a_i \approx \frac{2\Delta x}{3T} \left\{ g(x_1) + 4g(x_2) + 2g(x_3) + 4g(x_4) + \ldots + 4g(x_{n-1}) + g(x_n) \right\}$

$$b_i \approx \frac{2\Delta x}{3T} \left\{ h(x_1) + 4h(x_2) + 2h(x_3) + 4h(x_4) + \ldots + 4h(x_{n-1}) + h(x_n) \right\}.$$

Sine and cosine functional values are computed recursively with the following formula:

$$\sin \left(\frac{2\pi x_i}{\Delta x} (J+1) \right) = \sin \left(\frac{2\pi x_i}{\Delta x} \right) \cos \left(\frac{2\pi x_i}{\Delta x} J \right) + \cos \left(\frac{2\pi x_i}{\Delta x} \right) \sin \left(\frac{2\pi x_i}{\Delta x} J \right)$$

$$\cos \left(\frac{2\pi x_i}{\Delta x} (J+1) \right) = \cos \left(\frac{2\pi x_i}{\Delta x} \right) \cos \left(\frac{2\pi x_i}{\Delta x} J \right) - \sin \left(\frac{2\pi x_i}{\Delta x} \right) \sin \left(\frac{2\pi x_i}{\Delta x} J \right)$$

This program allows calculation of Fourier series coefficients using up to 255 data points. The number of data points must be odd and the number of Fourier series coefficients calculated may be no more than the number of data points.

The data may be entered either from the keyboard or from a tape file. After entry it may be edited as desired. Storage of the data on a tape file is easily accomplished.

After calculation of the Fourier series coefficients the data or the coefficients may be printed or plotted as desired.

The use of specially defined and labelled keys provides convenience and flexibility in running the program.

Special Function Key Information:

The following special function keys are used to perform desired operations during input of data and calculation of the Fourier series coefficients:

ENTER (KEY #1): This key loads a data entry program. The data may be entered from the keyboard or from a tape file.

PRINT (KEY #2): Either the data or the Fourier series coefficients may be printed on the internal thermal printer by use of this key.

COMPUTE (KEY #3): This key calculates the Fourier series coefficients from the data.

PLOT (KEY #4): This key calls the plotting routines and redefines some of the special function keys as follows:

DATA(KEY #1) plots the data on the CRT

MAG. (KEY #2) plots the magnitude on the CRT

PHASE (KEY #3) plots the phase on the CRT

COPY (KEY #4) makes a copy of the plot on the internal thermal printer

DONE (KEY #5) exits the plotting routine and returns the keys to their earlier definition.

Any of three types of plots: line, dot or bar, may be obtained.

EDIT (KEY #5): This key allows you to edit the data as desired.

STORE (KEY #6): This key allows storage of the data on a tape file.

User Instructions

1. To load the program:
   a. Insert the Waveform Analysis Pac cartridge into the tape transport.
   b. Type: (REW LOAD) "FSC-EQ" (END LINE).
2. To start the program:
   a. Press: (RUN).
3. When the keys are labelled:
   ```
   -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- --
   EDIT        STORE
   ENTER       PRINT      COMPUTE  PLOT
   ```
   You may select any of the following functions by

pressing the appropriate key:

a. Press: KEY #1 (ENTER) to enter the data. Go to step 4.

OR:

a. Press: KEY #2 (PRINT) to print data or coefficients. Go to step 11.

OR:

a. Press: KEY #3 (COMPUTE) to compute the Fourier coefficients. Go to step 3.

REFERENCES:

1. Hamming, R.W., *Numerical Methods for Scientists and Engineers*, (McGraw-Hill, 1962), pp. 67-80.

2. Acton, Forman S., *Numerical Methods that Work*, (Harper and Row, 1970), pp. 221-257.

OR:

a.   Press: KEY #4 (PLOT) to plot the data or coefficients. Go to step 12.

OR:

a.   Press: KEY #5 (EDIT) to edit the data. Go to step 17.

OR:

a.   Press: KEY #6 (STORE) to store the data. Go to step 18.

4.   When NO. OF DATA POINTS? is displayed:

a.   Enter: The number of data points you have $\binom{END}{LINE}$.

(Caution: If using data file, be sure number of points corresponds with the number stored on the file.)

Note: It is necessary to enter an odd number of data points, min.=3, max.=255.

5.   When ENTER NUMBER OF FOURIER COEF.? is displayed:

a.   Enter: The number of coefficients $\binom{END}{LINE}$ you want the program to compute.

Note: The number of coefficients may be no greater than the number of data points.

6.   When ENTRY MODE? is displayed:

a.   Enter: 1 or 2 $\binom{END}{LINE}$ depending upon whether you wish to enter the data manually or from a tape file.

7.   When INITIAL TIME (IN SEC)? is displayed:

a.   Enter: The initial time (in sec.) $\binom{END}{LINE}$.

8.   When TIME INCREMENT (IN SEC)? is displayed:

a.   Enter: The time increment (in sec.) $\binom{END}{LINE}$ (i.e., the spacing in the X direction between data points).

b.   If you entered a 1 in step 6, go to step 9.

c.   If you entered a 2 in step 6, go to step 10.

9.   When Y (i)? is displayed:

a.   Enter: The Y value $\binom{END}{LINE}$ of the ith data point.

b.   Repeat step 9a until all data points have been entered, then go to step 3.

10.   When FILE NAME? is displayed:

a.   Enter: The name of the data file $\binom{END}{LINE}$ you wish to use.

b.   Go to step 3.

11.   If coefficients have not yet been computed, the data is printed automatically. Return to step 3. If coefficients have been computed, when PRINT DATA/COEFFICIENTS: D/C? is displayed:

a.   Enter: D or C $\binom{END}{LINE}$ depending on whether data or coefficients are to be printed.

b.   Go to step 3.

12.   If coefficients have been computed go to step 14. If coefficients have not yet been computed,
1---LINE 2---POINT 3---BAR *** SELECT NUMBER? is displayed:

a.   Enter: 1, 2 or 3 $\binom{END}{LINE}$ depending on the type of data plot you want.

13.   When the keys are labelled:

```
DONE
DATA                    COPY
```

a.   Press: KEY #1 (DATA) for another type of plot and go to step 15.

OR:

a.   Press: KEY #4 (COPY) for a copy of the plot and go to step 16.

OR:

a.   Press: KEY #5 (DONE) and go to step 3.

14.   The keys are labelled:

```
DONE
DATA       MAG      PHASE      COPY
```

You may select any of the following functions by pressing the appropriate key.

a.   Press: KEY #1 (DATA) to plot the data, go to step 15.

OR:

a.   Press: KEY #2 (MAG.) to plot the magnitude, go to step 15.

OR:

a.   Press: KEY #3 (PHASE) to plot the phase, go to step 15.

OR:

a.   Press: KEY #4 (COPY) to make a copy of the plot, go to step 16.

OR:

a.   Press: KEY #5 (DONE) and go to step 3.

15.   When 1---LINE 2---POINT 3---BAR *** SELECT NUMBER? is displayed:

a.   Enter: 1, 2 or 3 (END LINE) depending on the type of plot you want.

b.   After the plot is displayed, press (KEY LABEL) and return to step 13 or step 14.

16.   When the copy is completed press (KEY LABEL) and return to step 13 or step 14.

17.   When DATA POINT TO CHANGE? is displayed:

a.   Enter: The subscript of the data point (END LINE) you wish to change.

When Y(i)? is displayed:

a.   Enter: The new data value (END LINE).

b.   Go to step 3.

18.   When FILE NAME? is displayed:

a.   Enter: The name of the file in which you wish to store the data.

19.   When NEW FILE:Y/N? is displayed:

a.   If a new file is being created:

1) Enter: Y (END LINE).

2) Go to step 3.

OR:

a.   If you are using an old file:

1) Enter: N (END LINE).

2) Go to step 3.

Example:

Calculate 10 Fourier series coefficients for the following 33 data points obtained, starting at time zero, at 0.1 second intervals. Plot the data and the magnitude and phase of the coefficients.

Press KEY #1, (ENTER).

QUESTION	RESPONSE
NO. OF DATA POINTS?	33
ENTER NUMBER OF FOURIER COEF.?	10
ENTRY MODE?	1
INITIAL TIME (IN SEC.)?	0
TIME INCREMENT (IN SEC.)?	.1

```
# OF DATA POINTS= 33
NUMBER OF FOURIER SERIES
COEFFICIENTS= 10
INITIAL TIME (SEC.)= 0
INCREMENT(SEC.)= .1
```

```
RANGE VALUES:
   Y( 1)= 0.000000E+000
   Y( 2)= 1.000000E-001
   Y( 3)= 2.000000E-001
   Y( 4)= 3.000000E-001
   Y( 5)= 4.000000E-001
   Y( 6)= 5.000000E-001
   Y( 7)= 6.000000E-001
   Y( 8)= 7.000000E-001
   Y( 9)= 8.000000E-001
   Y(10)= 8.000000E-001
   Y(11)= 8.000000E-001
   Y(12)= 8.000000E-001
   Y(13)= 8.000000E-001
   Y(14)= 8.000000E-001
   Y(15)= 8.000000E-001
   Y(16)= 8.000000E-001
   Y(17)= 8.000000E-001
   Y(18)= 8.000000E-001
   Y(19)= 8.000000E-001
   Y(20)= 8.000000E-001
   Y(21)= 8.000000E-001
   Y(22)= 8.000000E-001
   Y(23)= 8.000000E-001
   Y(24)= 8.000000E-001
   Y(25)= 8.000000E-001
   Y(26)= 7.000000E-001
   Y(27)= 6.000000E-001
   Y(28)= 5.000000E-001
   Y(29)= 4.000000E-001
   Y(30)= 3.000000E-001
   Y(31)= 2.000000E-001
   Y(32)= 1.000000E-001
   Y(33)= 0.000000E+000
```
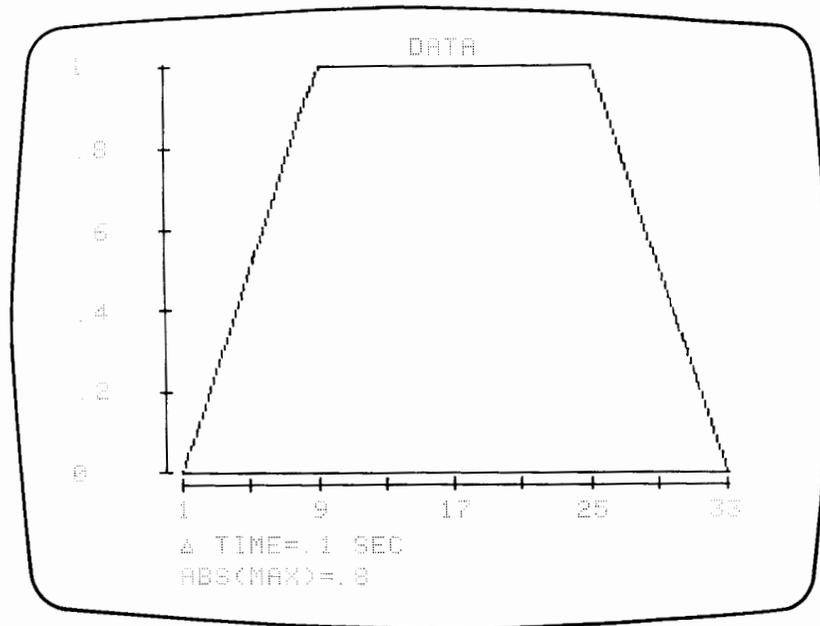
After all data has been entered, press KEY #4, (PLOT).

1---LINE 2---POINT 3---BAR

*** SELECT NUMBER ?   1

Press KEY #4, (COPY), to obtain a copy, if desired.

Press $\boxed{\substack{\text{KEY} \\ \text{LABEL}}}$, then KEY #5, (DONE).

Then press KEY #3, (COMPUTE).

When calculation is complete, press KEY #2, (PRINT).

QUESTION   **RESPONSE**

PRINT DATA/COEFFICIENTS: D/C?   C

DC TERM = 6.00000E-001

  1
    R=-3.242E-001   I= 0.000E+000
    M= 3.24220E-001   θ= 180.00

  2
    R=-1.620E-001   I= 0.000E+000
    M= 1.62048E-001   θ= 180.00

  3
    R=-3.595E-002   I= 0.000E+000
    M= 3.59478E-002   θ= 180.00

```
 4
  R= 0.000E+000   I= 0.000E+000
  M= 0.00000E+000      θ=   0.00

 5
  R=-1.272E-002   I= 0.000E+000
  M= 1.27237E-002      θ= 180.00

 6
  R=-1.724E-002   I= 0.000E+000
  M= 1.72355E-002      θ= 180.00

 7
  R=-6.022E-003   I= 0.000E+000
  M= 6.02160E-003      θ= 180.00

 8
  R= 0.000E+000   I= 0.000E+000
  M= 0.00000E+000      θ=   0.00

 9
  R=-2.641E-003   I= 0.000E+000
  M= 2.64145E-003      θ= 180.00

10
  R=-2.291E-003   I= 0.000E+000
  M= 2.29075E-003      θ= 180.00
```

Press KEY #4, (PLOT), then KEY #2, (MAG).

1---LINE 2---POINT 3---BAR
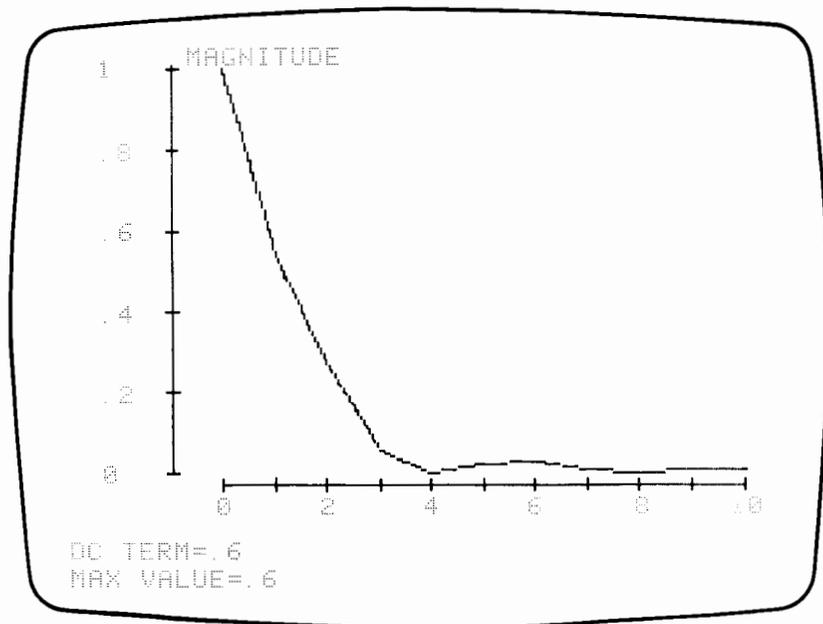
*** SELECT NUMBER ?   1

Press (KEY/LABEL), (if a copy is desired, press KEY #4, (COPY), (KEY/LABEL)).

Press KEY #3 (PHASE).

1---LINE 2---POINT 3---BAR

*** SELECT NUMBER ?   3
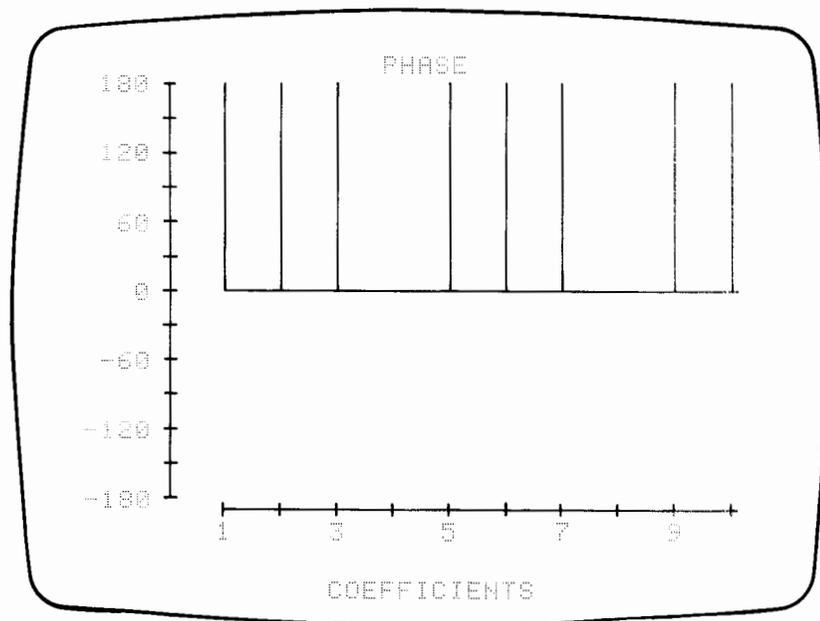
Fourier Series Coefficients For Unequally Spaced Data

This program calculates the Fourier series coefficients a_i and b_i of the Fourier series corresponding to a function F(x) which is approximated by n discrete data points (x(j), y(j)) for j = 1, ..., n.

For a finite Fourier series given by:

$$\frac{a_0}{2} + \sum_{i=1}^{N}\left[a_i \cos\left(\frac{2\pi ix}{T}\right) + b_i\sin\left(\frac{2\pi ix}{T}\right)\right]$$

where the Fourier coefficients a_i and b_i are:

$$a_i = \frac{2}{T}\int_{x_1}^{x_1+T} F(x) \cos\left(\frac{2\pi ix}{T}\right) dx \ \ \text{for}\, i = 0, N.$$

$$b_i = \frac{2}{T}\int_{x_1}^{x_1+T} F(x) \sin\left(\frac{2\pi ix}{T}\right) dx \ \ \text{for}\, i = 1, N.$$

T specifies the period, given by $(x_n - x_1)$, and N indicates the number of coefficients desired. The coefficients are evaluated by numerically integrating a parabola passing through three successive points. Execution time depends on the number of coefficients calculated.

This program allows calculation of Fourier series coefficients using up to 255 data points. The number of data points must be odd and the number of Fourier series coefficients calculated may be no more than the number of data points.

The data may be entered either from the keyboard or from a tape file. After entry it may be edited as desired. Storage of the data on a tape file is easily accomplished.

After calculation of the Fourier series coefficients, the data on the coefficients may be printed or plotted as desired.

The use of specially defined and labelled keys provides convenience and flexibility in running the program.

Special Function Key Information:

The following special function keys are used to perform desired operations during input of data and calculation of the Fourier series coefficients.

ENTER (KEY # 1): This key loads a data entry program. The data may be entered from the keyboard or from a tape file.

PRINT (KEY # 2): Either the data or the Fourier series coefficients may be printed on the internal thermal printer by use of this key.

COMPUTE (KEY #3): This key calculates the Fourier series coefficient from the data.

PLOT (KEY #4): This key calls the plotting routines and redefines some of the special function keys as follows:

DATA (KEY #1) plots the data on the CRT.

MAG. (KEY #2) plots the magnitude on the CRT.

PHASE (KEY #3) plots the phase on the CRT.

COPY (KEY #4) makes a copy of the plot on the internal thermal printer.

DONE (KEY #5) exits the plotting routine and returns the keys to their earlier definition.

Any of three types of plots: line, dot or bar, may be obtained.

EDIT (KEY #5): This key allows you to edit the data as desired.

STORE (KEY #6): This key allows storage of the data on a tape file.

User Instructions

1. To load the program:

   a. Insert the Waveform Analysis Pac cartridge into the tape transport.

   b. Type $\left(\begin{smallmatrix} REW \\ LOAD \end{smallmatrix}\right)$ "FSC-UN" $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$.

2. To start the program:

   a. Press: $\left(RUN \right)$.

3. When the keys are labelled:

   ```
   ----------------------------------------
   EDIT      STORE
   ENTER     PRINT     COMPUTE PLOT
   ```

   You may select any of the following functions by pressing the appropriate key:

   a. Press: KEY #1 (ENTER) to enter the data. Go to step 4.

   OR:

   a. Press: KEY #2 (PRINT) to print data or coefficients. Go to step 10.

   OR:

   a. Press: KEY #3 (COMPUTE) to compute the Fourier coefficients. Go to step 3.

   OR:

   a. Press: KEY #4 (PLOT) to plot the data or coefficients. Go to step 11.

OR:

   a. Press: KEY #5 (EDIT) to edit the data. Go to step 16.

OR:

   a. Press: KEY #6 (STORE) to store the data. Go to step 16.

4. When NO. OF DATA POINTS? is displayed:

   a. Enter: The number of data points $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$. (Caution: If using data file, be sure number of points corresponds with the number stored on the file.)

   Note: It is necessary to enter an odd number of data point, min.=3, max.=175.

5. When ENTER NUMBER OF FOURIER COEF. ? is displayed:

   a. Enter: The number of coefficients $\left(\begin{smallmatrix} END \\ LINE \end{smallmatrix}\right)$ you want the program to compute.

   Note: The number of coefficients may be no greater than the number of data points.

REFERENCES:

1. Hewlett-Packard 9820A Math Pac, pp. 43-50.
2. Hamming, R.W. *Numerical Methods for Scientists and Engineers* (McGraw-Hill, 1962), pp. 67-80.
3. Acton, Forman S., *Numerical Methods that Work* (Harper and Row, 1970), pp. 221-257.

6. When ENTRY MODE? is displayed:
   a. Enter: 1 or 2 $\left(\substack{\text{END}\\\text{LINE}}\right)$ depending upon whether you wish to enter the data manually or from a tape file.
   b. If you entered a 1 in step 6a, go to step 7.
   c. If you entered a 2 in step 6a, go to step 9.
7. When X(i)? is displayed:
   a. Enter: The X value $\left(\substack{\text{END}\\\text{LINE}}\right)$ of the ith data point.
8. When Y(i)? is displayed:
   a. Enter: The Y value $\left(\substack{\text{END}\\\text{LINE}}\right)$ of the ith data point.
   b. Repeat steps 7 and 8 until all data points have been entered. Go to step 3.
9. When FILE NAME? is displayed:
   a. Enter: The name of the data file $\left(\substack{\text{END}\\\text{LINE}}\right)$ you wish to use.
   b. Go to step 3.
10. If coefficients have not yet been computed, the data is printed automatically, go to step 3. If coefficients have been computed, when PRINT DATA/COEFFICIENTS: D/C? is displayed:
    a. Enter: D or C $\left(\substack{\text{END}\\\text{LINE}}\right)$ depending on whether data or coefficients are to be printed.
    b. Go to step 3.
11. If coefficients have been computed go to step 13. If coefficients have not been computed 1---LINE 2---POINT 3---BAR *** SELECT NUMBER? is displayed.
    a. Enter: 1, 2 or 3 $\left(\substack{\text{END}\\\text{LINE}}\right)$ depending upon the type of plot you want.
12. When the plot is completed the keys are labelled:

    ```
    DONE
    DATA                    COPY
    ```

    a. Press: KEY #1 (DATA) for another type of plot, go to step 14.

    OR:

    a. Press: KEY #4 (COPY) for a copy of the plot, go to step 15.

OR:
a. Press: KEY #5 (DONE) and go to step 3.
13. The keys are labelled:

    ```
    DONE
    DATA      MAG.      PHASE      COPY
    ```

    You may select any of the following functions by pressing the appropriate key.
    a. Press: KEY #1 (DATA) to plot the data, go to step 14.

    OR:

    a. Press: KEY #2 (MAG.) to plot the magnitude, go to step 14.

    OR:

    a. Press: KEY #3 (PHASE) to plot the phase, go to step 14.

    OR:

    a. Press: KEY #4 (COPY) to make a copy of the plot, go to step 15.

    OR:

    a. Press: KEY #5 (DONE) and go to step 3.
14. When 1---LINE 2---POINT 3---BAR *** SELECT NUMBER? is displayed:
    a. Enter: 1, 2 or 3 $\left(\substack{\text{END}\\\text{LINE}}\right)$ depending on the type of plot you want.
    b. After the plot is displayed, press $\left(\substack{\text{KEY}\\\text{LABEL}}\right)$ and return to step 12 or step 13.
15. When the copy is completed, press $\left(\substack{\text{KEY}\\\text{LABEL}}\right)$ and return to step 12 or step 13.
16. When DATA POINT TO CHANGE? is displayed:
    a. Enter: The subscript of the data point $\left(\substack{\text{END}\\\text{LINE}}\right)$ you wish to change.
    b. When X(i)? is displayed:
       1) Enter: The new data value $\left(\substack{\text{END}\\\text{LINE}}\right)$.
    c. When Y(i)? is displayed:
       1) Enter: The new data value $\left(\substack{\text{END}\\\text{LINE}}\right)$.
    d. Go to step 3.
17. When FILE NAME? is displayed:
    a. Enter: The name of the file in which you

wish to store the data. (Max: 6 characters).

18. When NEW FILE: Y/N? is displayed:

    a. If a new file is being created:

       1) Enter: Y (END LINE).

2) Go to step 3.

OR:

    a. If you are using an old file:

       1) Enter: N (END LINE).

       2) Go to step 3.

Example:

Calculate 10 Fourier series coefficients from the 37 unequally spaced data points given below. Obtain plots of the data and the magnitude and phase of the coefficients.

Press KEY #1, (ENTER).

QUESTION	RESPONSE
NO. OF DATA POINTS?	37
ENTER NUMBER OF FOURIER COEF.?	10
ENTRY MODE?	1

```
# OF DATA POINTS= 37
NUMBER OF FOURIER SERIES
COEFFICIENTS= 10
DATA VALUES
   I       X(I)            Y(I)
   1    0.0000E+000    1.5000E+000
   2    1.0000E-001    3.0000E+000
   3    1.5000E-001    3.0000E+000
   4    2.0000E-001    3.0000E+000
   5    2.5000E-001    3.0000E+000
   6    3.0000E-001    3.0000E+000
   7    4.0000E-001    3.0000E+000
   8    5.0000E-001    3.0000E+000
   9    6.0000E-001    3.0000E+000
  10    7.0000E-001    3.0000E+000
  11    8.0000E-001    3.0000E+000
  12    9.0000E-001    3.0000E+000
  13    1.0000E+000    1.5000E+000
  14    1.1000E+000    0.0000E+000
  15    1.2000E+000    0.0000E+000
  16    1.3000E+000    0.0000E+000
  17    1.5000E+000    0.0000E+000
```

```
18    1.7000E+000    0.0000E+000
19    1.9000E+000    0.0000E+000
20    2.0000E+000    0.0000E+000
21    2.5000E+000    0.0000E+000
22    2.7500E+000    0.0000E+000
23    3.0000E+000    0.0000E+000
24    3.5000E+000    0.0000E+000
25    3.7000E+000    0.0000E+000
26    3.9000E+000    0.0000E+000
27    4.0000E+000    0.0000E+000
28    4.2500E+000    0.0000E+000
29    4.5000E+000    0.0000E+000
30    4.8000E+000    0.0000E+000
31    5.0000E+000    0.0000E+000
32    5.2000E+000    0.0000E+000
33    5.4000E+000    0.0000E+000
34    5.8000E+000    0.0000E+000
35    5.8500E+000    0.0000E+000
36    5.9000E+000    0.0000E+000
37    6.0000E+000    1.5000E+000
```

Press KEY #3, (COMPUTE). When calculation is completed, press KEY #2, (PRINT).

QUESTION	RESPONSE
PRINT DATA/COEFFICIENTS: D/C?	C

```
DC TERM = 5.00000E-001

   1
   R= 8.262E-001   I= 4.768E-001
   M= 9.53956E-001     θ=   29.99

   2
   R= 4.120E-001   I= 7.132E-001
   M= 8.23612E-001     θ=   59.99

   3
   R= 0.000E+000   I= 6.307E-001
   M= 6.30736E-001     θ=   90.00

   4
   R=-2.036E-001   I= 3.520E-001
   M= 4.06661E-001     θ=  120.05
```

```
 5
  R=-1.614E-001   I= 9.247E-002
  M= 1.86017E-001     θ= 150.19

 6
  R= 0.000E+000   I=-7.231E-004
  M= 7.23070E-004     θ= -90.00

 7
  R= 1.123E-001   I= 6.1417E-002
  M= 1.29333E-001     θ=  29.75

 8
  R= 9.651E-002   I= 1.666E-001
  M= 1.92577E-001     θ=  59.92

 9
  R= 0.000E+000   I= 1.936E-001
  M= 1.93603E-001     θ=  90.00

10
  R=-7.366E-002   I= 1.276E-001
  M= 1.47307E-001     θ= 120.00
```

Press KEY #4, (PLOT). Then press KEY #1, (DATA).

1---LINE 2---POINT 3---BAR
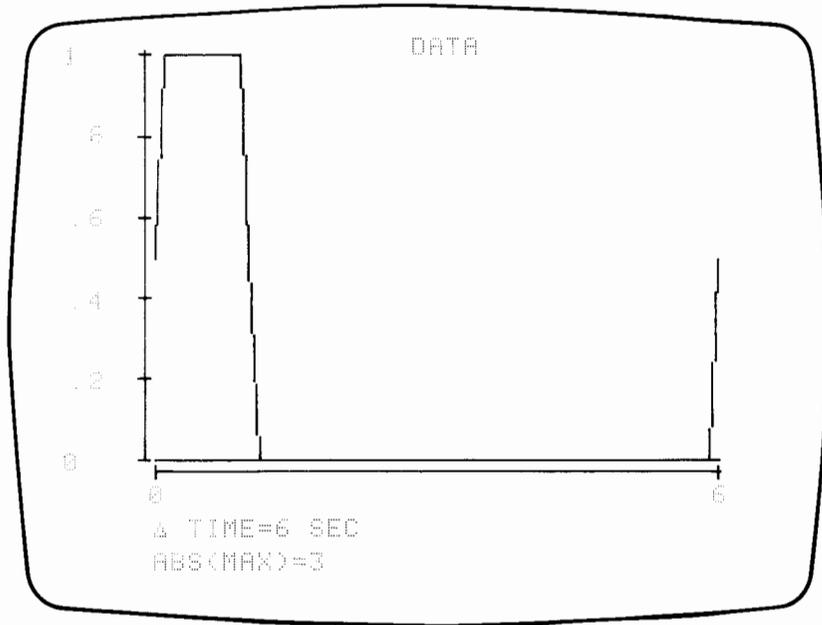
*** SELECT NUMBER ?   1

Press [KEY LABEL] (if a copy is desired, press KEY #4,
(COPY), [KEY LABEL].)
Press KEY #2, (MAG).

1---LINE 2---POINT 3---BAR
*** SELECT NUMBER ?   1
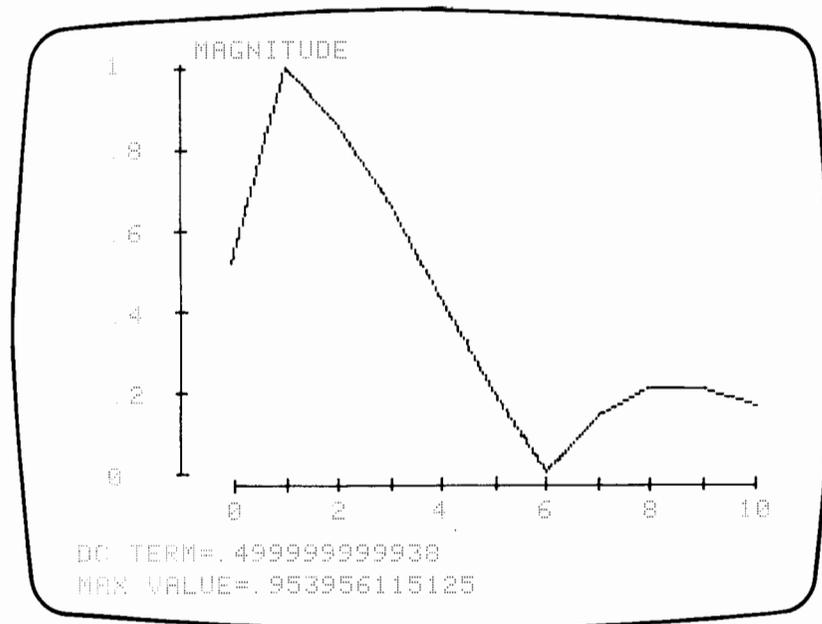
Press $\boxed{\text{KEY}\atop\text{LABEL}}$ (if you wish a copy, press KEY #4, (COPY), $\boxed{\text{KEY}\atop\text{LABEL}}$.
Press KEY #3, (PHASE).
1---LINE 2---POINT 3---BAR
*** SELECT NUMBER ?   3
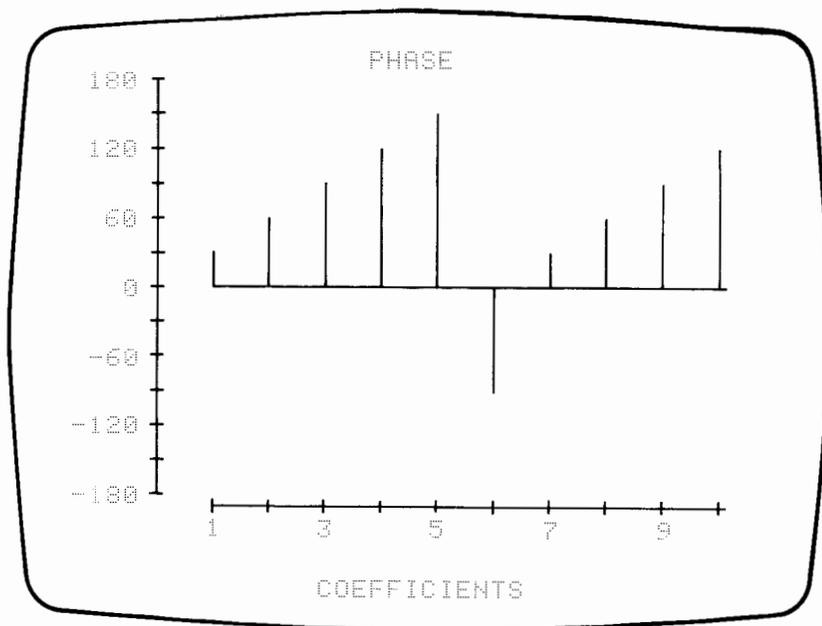
Appendix A
Use of the HP-85 Waveform Analysis Pac
With the Model 82903A Memory Module

When the Model 82903A Memory Module is present on the HP-85, the system memory is significantly increased. This allows you to greatly increase the number of data points you may process with the HP-85 Waveform Analysis Pac.

The increased data capacities are summarized below:

	HP-85 Configuration	
Type of Calculation	Standard	With Model 82903A Memory Module
Single Data Block Analysis	512	2048
Double Data Block Analysis	256	1024
Fourier Series Coeff., (Equally Spaced)	255	921
Fourier Series Coeff., (Unequally Spaced)	175	675

The following modifications must be made in the programs:

	Program Title	Line:	Change To:
Single Data Block Analysis	"S-WAVE"	20 COM INTEGER C1, ... R1(256), I1(256)	20 COM INTEGER C1, ... R1(1024), I1(1024)
	"WAVE"	20 COM INTEGER C1, ... R1(256), I1(256) 40 P0=256 2630 CREATE F$,17	20 COM INTEGER C1, ... R1(1024), I1(1024) 40 P0=1024 2630 CREATE F$,65
	"WAVPLT"	20 COM INTEGER C1, ... R1(256), I1(256)	20 COM INTEGER C1, ... R1(1024), I1(1024)
Double Data Block Analysis	"D-WAVE"	20 COM INTEGER C1, ... R1(128), I1(128), R2(128), I2(128)	20 COM INTEGER C1, ... R1(512), I1(512) R2(512), I2(512)
	"WAVED"	20 COM INTEGER C1, ... R1(128), I1(128), R2(128), I2(128) 200 P0=128 2540 CREATE F$,17	20 COM INTEGER C1, ... R1(512), I1(512), R2(512), I2(512) 200 P0=512 2540 CREATE F$,65
	"WAVDED"	20 COM INTEGER C1, ... R1(128), I1(128) R2(128), I2(128)	20 COM INTEGER C1, ... R1(512), I1(512) R2(512), I2(512)
	"WVDPLT"	20 COM INTEGER C1, ... R1(128), I1(128) R2(128), I2(128)	20 COM INTEGER C1, ... R1(512), I1(512) R2(512), I2(512)

Program Title		Line:	Change To:
Fourier Series Coefficients: Equally Spaced	"FSC-EQ"	10 DIM A(256), B(256), Y(256), A$[33] 30 FOR I=0 TO 256 @ A(I), B(I), Y(I)=0 @ NEXT I 140 IF N2>2 AND N2<256 THEN 170 1030 CREATE A$,9	10 DIM A(922), B(922), Y(922), A$[33] 30 FOR I=0 TO 922 @ A(I), B(I), Y(I)=0 @ NEXT I 140 IF N2>2 AND N2<922 THEN 170 1030 CREATE A$,30
Fourier Series Coefficients Unequally Spaced	"FSC-UN"	10 DIM A(175), B(175), X(175), Y(175), A$[33] 30 FOR I=0 TO 175 @ A(I), B(I), X(I), Y(I)= 0 @ NEXT I 140 IF N2<=2 OR N2>175 THEN ... 890 CREATE A$, 12	10 DIM A(675), B(675), X(675), Y(675), A$[33] 30 FOR I=0 TO 675 @ A(I), B(I), X(I),Y(I)= 0 @ NEXT I 140 IF N2<=2 OR N2>675 THEN ... 890 CREATE A$, 44

Appendix B
Obtaining Program Listings

To obtain a list of program names and data files stored on the tape:

    a. Insert the Waveform Analysis Pac cartridge into the tape transport.

    b. Type: $\boxed{\text{CAT}}$ $\boxed{\substack{\text{END} \\ \text{LINE}}}$.

To obtain a listing of one of the Waveform Analysis programs:

    a. Insert the Waveform Analysis Pac cartridge into the tape transport.

    b. Type: $\boxed{\substack{\text{REW} \\ \text{LOAD}}}$ "(program name)" $\boxed{\substack{\text{END} \\ \text{LINE}}}$.

    Note: If program name is "WAVE" or "WAVED", go to step c, otherwise go to step d.

    c. Type: LOADBIN "FFTBIN" $\boxed{\substack{\text{END} \\ \text{LINE}}}$

    d. To display the listing, press: $\boxed{\substack{\text{P LST} \\ \text{LIST}}}$ or, to print the listing, press: $\boxed{\text{SHIFT}}\boxed{\substack{\text{P LST} \\ \text{LIST}}}$.

To purge a data file from tape:

    a. Insert the Waveform Analysis Pac cartridge into the tape transport.

    b. Type: PURGE "(file name)" $\boxed{\substack{\text{END} \\ \text{LINE}}}$.

Notes

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Appendix C
Program Remarks

To help you understand the flow of the programs in the Waveform Analysis Pac, abbreviated remarks for each of the programs in the pac, as well as definitions of variables used, are contained in a program named "WAVDOC".

User Instructions

1. To load the program:
   a. Insert the Waveform Analysis Pac cartridge into the tape transport.
   b. Type: (REW/LOAD) "WAVDOC" (END LINE).

2. When the program has been loaded:
   a. Press: (RUN).

3. When PRINT OR DISPLAY OUTPUT (P/D)? is displayed:
   a. Enter: P (END LINE) to print the output.

   OR:

   a. Enter: D (END LINE) to display the output.
   Note: Contents of the display may be printed at any time by pressing (SHIFT) (COPY).

4. When the keys are labelled:

   WAVED   WAVDED   WVDPLT   FSC
   S-WAVE   WAVE   WAVPLT   D-WAVE

   a. Press: The special function key corresponding to the program name in which you are interested.

   b. If you pressed KEY #8, (FSC); when the keys are labelled:

   TITLES
   FSC-EQ   FSC-UN

   1) Press KEY #1, (FSC-EQ).

   OR:

   2) Press KEY #2, (FSC-UN).

   OR:

   3) Then press KEY #5, (TITLES), and go to step 4.

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For additional information please contact your local Hewlett-Packard Sales Office.